

A CYCLIC COHERENT METHOD FOR WIDEBAND SOURCE LOCATION

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ABSTRACT

The problem of source location of wideband signals impinging on an array of sensors is addressed. The proposed method exploits the cyclostationarity exhibited by most communication signals to discriminate signals of interest from noise and interfering signals. The new method performs coherent combination of the spatial contributions at different frequencies and exploits signal-subspace properties of the resulting focused matrix. Numerical results show that the proposed technique is superior to existing algorithms and assures good performances also when the signals of interest are fully correlated.

1 INTRODUCTION

Conventional wideband source location methods assume a wide-sense stationary model for the source signals and exploit the properties of appropriate spatio-temporal matrices derived from the data received by an array of sensors to estimate the directions-of-arrival (DOAs) of the impinging wavefronts. They, however, suffer from the following main drawbacks: i) they need a post-processing of the estimated DOAs to separate those relative to signals of interest (SOIs) from those relative to signals not of interest (SNOIs); ii) they require that the total number of impinging signals be less than the number of sensors; iii) they assume that the spatio-temporal characteristics of noise and interference be (almost) perfectly known.

In recent years, with reference mainly to the narrowband-signal case, the *cyclic* approach (see [1] and references therein) has been proposed to circumvent these drawbacks. The new approach is based on the assumption that the SOIs exhibit *cyclostationarity* or *spectral correlation* [2], a property that arises principally from periodic signal processing operations (e.g., sampling, modulation, coding). Based on the new approach, the *cyclic* methods can automatically discriminate in favor of the SOIs against the SNOIs on the basis of their known different spectral correlation properties. Unlike conventional methods, they operate properly also when the total number of impinging signals is greater

than the number of sensors and/or the spatio-temporal characteristics of noise and interference are unknown.

More recently, some efforts have been made to apply the cyclic approach to wideband source location problems. A time-domain approach is considered in [3], where high-resolution DOA estimation is performed by signal-subspace decomposition of a matrix whose elements are samples of the cyclic correlation functions. Instead, the non-eigenstructure based methods proposed in [4,5] and the eigenstructure-based Wideband Cyclic MUSIC (WCM) [6] perform DOA estimation in the frequency domain by resorting to the properties of the array cyclic spectral density matrix (ACSDM). However, they make rather an inefficient use of SOI bandwidth, since the spatial spectra are constructed on the basis of the ACSDM evaluated at a *single* frequency value. A more efficient use of bandwidth could be pursued by averaging spatial spectra obtained at different frequency values: however, signal-to-noise ratio (SNR) threshold effects would prevent this simple *noncoherent* averaging from being really effective. Finally, it should be noted that, with the exception of the low-resolution technique proposed in [4], all of the proposed high-resolution methods cannot handle the case of completely correlated source signals.

In this paper, we propose a new high-resolution cyclic method for wideband source location, which potentially exploits all the SOI bandwidth and can operate successfully also in the presence of fully correlated sources. Since in wideband array processing the signal subspace at one frequency is different from that at another frequency, the proposed method performs a *coherent* transformation or *focusing* of the individual ACSDMs evaluated at different frequencies to a common focusing frequency. The focused individual ACSDMs are then combined to generate a single matrix whose signal-subspace properties can be exploited to estimate the DOAs of the SOIs. The new method can be considered as the extension to the cyclostationary case of the coherent signal-subspace method proposed in [7] and, hence, it will be referred to as the *Cyclic Coherent Signal-Subspace* (CCSS) method.

2 CYCLOSTATIONARITY

A scalar real time series $x(t)$ is said to exhibit cyclostationarity or, equivalently, spectral correlation, if and only if the *cyclic autocorrelation function* (CAF)

$$R_{xx}^\alpha(\tau) \triangleq \langle x(t + \tau/2) x(t - \tau/2) e^{-j2\pi\alpha t} \rangle \quad (1)$$

exists and is not identically zero for some value of the cycle frequency $\alpha \neq 0$. In (1), $\langle \cdot \rangle$ denotes infinite time averaging.

The Fourier transform $S_{xx}^\alpha(f)$ of $R_{xx}^\alpha(\tau)$ is referred to as the *cyclic spectrum* or the *cyclic spectral density function* (CSDF). Note that, for $\alpha = 0$, the CAF and CSDF reduce to the conventional autocorrelation function and the power spectral density function, respectively.

The CSDF can be given an alternative interpretation that sheds more light on the concept of spectral correlation. More precisely, let

$$X(t, f)_T \triangleq \int_{t-T/2}^{t+T/2} x(u) e^{-j2\pi f u} du \quad (2)$$

denote the finite-time Fourier transform (FTFT) of $x(t)$. Thus, it can be easily shown [2] that

$$S_{xx}^\alpha(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \langle X(t, f + \frac{\alpha}{2})_T X^*(t, f - \frac{\alpha}{2})_T \rangle, \quad (3)$$

where the superscript $*$ denotes complex conjugation. Equation (3) shows that the magnitude of the CSDF measures the strength of correlation between the spectral components of $x(t)$ at frequencies $f \pm \alpha/2$.

The previous definitions can be readily extended to vectors of real-valued time series. More specifically, given an M -column vector $\mathbf{x}(t)$, the $M \times M$ *cyclic correlation matrix* is defined as

$$\mathbf{R}_{xx}^\alpha(\tau) \triangleq \langle \mathbf{x}(t + \tau/2) \mathbf{x}^T(t - \tau/2) e^{-j2\pi\alpha t} \rangle, \quad (4)$$

where the superscript T denotes transpose operation. Its Fourier transform is the $M \times M$ *cyclic spectral density matrix* (CSDM)

$$\mathbf{S}_{xx}^\alpha(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \langle \mathbf{X}(t, f + \frac{\alpha}{2})_T \mathbf{X}^H(t, f - \frac{\alpha}{2})_T \rangle, \quad (5)$$

where $\mathbf{X}(t, f)_T$ is the FTFT of $\mathbf{x}(t)$ and the superscript H denotes Hermitian (conjugate transpose) operation.

3 THE PROPOSED CCSS METHOD

Let an array of M isotropic sensors with known arbitrary positions and characteristics receive $D_\alpha < M$ SOIs exhibiting cyclostationarity with a common cycle frequency α . If the signal sources are assumed to be in the far-field, the wavefronts impinging on the sensors can be approximated as planar. Moreover, though not necessary, it is assumed here that sensors and sources are coplanar, so that the positions of the

sources are completely described by the DOA vector $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_{D_\alpha})$ of the associated planewaves.

Under the previous assumptions, the signal $x_i(t)$ at the output of the i th sensor is given by

$$x_i(t) = \sum_{k=1}^{D_\alpha} s_k[t - \tau_i(\theta_k)] + n_i(t), \quad i = 1, 2, \dots, M. \quad (6)$$

In (6), $s_k(t)$ is the k th SOI associated to the wavefront impinging from direction θ_k as observed at a reference point, $\tau_i(\theta_k)$ is the time interval for the k th waveform to propagate from the reference point to the i th sensor and, finally, $n_i(t)$ is the zero-mean signal modeling noise plus interference at the i th sensor.

Under the assumptions that the interfering and noise signals are uncorrelated with the SOIs and do not exhibit cyclostationarity with the considered cycle frequency α , the ACSDFM is given by

$$\mathbf{S}_{xx}^\alpha(f) = \mathbf{A}(\boldsymbol{\theta}, f + \frac{\alpha}{2}) \mathbf{S}_{ss}^\alpha(f) \mathbf{A}^H(\boldsymbol{\theta}, f - \frac{\alpha}{2}), \quad (7)$$

where $\mathbf{S}_{ss}^\alpha(f)$ is the $D_\alpha \times D_\alpha$ CSDM of the SOIs and

$$\mathbf{A}(\boldsymbol{\theta}, f) \triangleq [\mathbf{a}(\theta_1, f), \mathbf{a}(\theta_2, f), \dots, \mathbf{a}(\theta_{D_\alpha}, f)] \quad (8)$$

is the $M \times D_\alpha$ matrix of the steering vectors.

The potential of the cyclic approach stems from the fact that in (7) any contribution from the SNOIs has been removed. Such a result allows one to predict satisfactory performances even in strongly adverse interference environments. In particular, cyclic methods can provide good accuracy also when the interfering signals, whose number can be greater than the number of sensors, exhibit an arbitrary degree of correlation among themselves and arrive from directions arbitrarily close to those of the SOIs. Moreover, cyclic techniques do not require knowledge of the noise statistics, which is generally needed by conventional methods.

In order to exploit the wideband nature of the SOIs, the proposed method performs a *coherent* combination of the ACSDFMs (7) evaluated in correspondence of J frequency values f_j . More precisely, the contribution at frequency f_j is *focused* to frequency f_F by means of a couple of transformation matrices $\mathbf{T}(\boldsymbol{\theta}, f_j \pm \alpha/2)$ such that

$$\mathbf{T}(\boldsymbol{\theta}, f_j \pm \frac{\alpha}{2}) \mathbf{A}(\boldsymbol{\theta}, f_j \pm \frac{\alpha}{2}) = \mathbf{A}(\boldsymbol{\theta}, f_F \pm \frac{\alpha}{2}). \quad (9)$$

The existence of such matrices can be easily shown following the guidelines given in [7], provided that the matrices $\mathbf{A}(\boldsymbol{\theta}, f_j \pm \alpha/2)$ and $\mathbf{A}(\boldsymbol{\theta}, f_F \pm \alpha/2)$ have full rank D_α , which is a common assumption in array processing.

The determination of appropriate transformation matrices allows one to construct the matrix

$$\mathcal{R}_{xx}^\alpha \triangleq \sum_{j=1}^J w(f_j) \mathbf{T}(\boldsymbol{\theta}, f_j + \frac{\alpha}{2}) \mathbf{S}_{xx}^\alpha(f_j) \mathbf{T}^H(\boldsymbol{\theta}, f_j - \frac{\alpha}{2}), \quad (10)$$

where $w(f)$ is a complex weight function. By substituting (7) into (10) and taking into account (9), one has

$$\mathcal{R}_{\mathbf{x}\mathbf{x}}^\alpha = \mathbf{A}(\boldsymbol{\theta}, f_F + \frac{\alpha}{2}) \mathcal{R}_{\mathbf{s}\mathbf{s}}^\alpha \mathbf{A}^H(\boldsymbol{\theta}, f_F - \frac{\alpha}{2}), \quad (11)$$

where

$$\mathcal{R}_{\mathbf{s}\mathbf{s}}^\alpha \triangleq \sum_{j=1}^J w(f_j) \mathbf{S}_{\mathbf{s}\mathbf{s}}^\alpha(f_j). \quad (12)$$

Since the matrices $\mathbf{A}(\boldsymbol{\theta}, f_F \pm \alpha/2)$ have rank D_α , under the assumption that also $\mathcal{R}_{\mathbf{s}\mathbf{s}}^\alpha$ has full rank, it results that $\mathcal{R}_{\mathbf{x}\mathbf{x}}^\alpha$ has rank D_α and, hence, any signal-subspace method can be applied to obtain high-resolution estimates of the DOAs.

The proposed method can be considered as the extension to the cyclostationary case of the coherent signal-subspace method [7] and, hence, it is called here the *Cyclic Coherent Signal-Subspace* method. In the conventional case, however, an additive contribution due to noise and interference is present in (11).

It is clear from (9) that construction of $\mathbf{T}(\boldsymbol{\theta}, f_j \pm \alpha/2)$ assuring *perfect* focusing requires knowledge of the true DOAs. In practice, transformation matrices can be constructed using preliminary estimates of the signal DOAs, which can be obtained at a modest computational cost by a low-resolution cyclic method like those proposed in [4,5]. Note also that the accuracy of the CCSS method can be improved by utilizing the obtained estimates as preliminary estimates in successive iterated steps of the algorithm. As to the structure of the transformation matrices, we observe that the same classes proposed in [8] for the conventional case can also be adopted in the cyclic case.

Finally, it should be noted that whereas in a multipath environment $\mathbf{S}_{\mathbf{s}\mathbf{s}}^\alpha(f)$ is singular at every value of frequency, the frequency smoothing (12) performed to obtain $\mathcal{R}_{\mathbf{s}\mathbf{s}}^\alpha$ removes the singularity, provided that the weight function $w(f)$ is properly chosen. To gain more insight into this aspect, observe that the continuous-frequency counterpart of (12) is

$$\mathcal{R}_{\mathbf{s}\mathbf{s}}^\alpha = \int_{-\infty}^{+\infty} w(f) \mathbf{S}_{\mathbf{s}\mathbf{s}}^\alpha(f) df. \quad (13)$$

Hence, by choosing $w(f) = e^{j2\pi f\tau}$ we get

$$\mathcal{R}_{\mathbf{s}\mathbf{s}}^\alpha = \mathbf{R}_{\mathbf{s}\mathbf{s}}^\alpha(\tau), \quad (14)$$

which has rank D_α even when some of the SOIs are fully correlated, provided that no signal is exactly equal to another and τ is chosen according to the signal characteristics.

4 SIMULATION RESULTS

We present here results of computer experiments aimed at testing the effectiveness of the proposed CCSS method and comparing it with both the WCM algorithm

and its noncoherent version (in the following, referred to as NC-WCM) based on geometric mean of WCM spatial spectra relative to different frequencies.

The method implementation starts from the digital baseband (complex) signals, that is, after frequency conversion from the center frequency $f_0 = 100$ Hz of the receiver band to baseband and quadrature sampling with frequency $f_s = 80$ Hz.

All experiments assume a passive uniform linear array of $M = 8$ isotropic sensors with spacing equal to one-half of the wavelength corresponding to f_0 . The Rayleigh angle resolution limit for this array is $\text{BW} \simeq 2/(M-1) \simeq 16^\circ$. The noise is stationary complex bandpass Gaussian with a flat power spectrum in the frequency interval of analysis $[-f_s/2, f_s/2]$ and uncorrelated from sensor to sensor. The SOIs and SNOIs are uncorrelated DSB-SC modulated signals with Gaussian stationary modulating signals and 20 % fractional bandwidth. The SNR for each signal is defined as the ratio of the power of this signal to that of the noise at each sensor.

In regard to the choice of α , note that a DSB-SC modulated signal with carrier frequency f_c exhibits cyclostationarity at cycle frequency $\alpha = 2f_c$. Since in all experiments the carrier frequency is 110 Hz for the SOIs and 120 Hz for the SNOIs, signal discrimination is obtained working with $\alpha = 220$ Hz. Moreover, ACSDM estimation is obtained, unless otherwise specified, via FFT processing of 64 data segments each consisting of 128 samples. Because of the frequency conversion performed on the received signals, an appropriate value of the focusing frequency for the CCSS algorithm is $f_F = 0$ and the frequency range for f_j is chosen to be $[-10 \text{ Hz}, 10 \text{ Hz}]$. The transformation matrices are those obtained by the rotational signal-subspace approach [8] with the focusing angles chosen as $(\beta_i - 0.25\text{BW}, \beta_i, \beta_i + 0.25\text{BW})$, where $\boldsymbol{\beta}$ is the vector of the DOA preliminary estimates. Finally, the Cyclic MUSIC algorithm [1] is applied to the estimated version of (11) to obtain high-resolution DOA estimates. Note that in all the experiments the true number D_α of SOIs is assumed to be known.

In the first experiment, we test the performance of the proposed CCSS method in the single group case, where all the DOAs of the SOIs are within one beamwidth of one focusing angle. Two equipower uncorrelated SOIs with 5-dB SNR impinge from DOAs 8° and 13° together with a SNOI with 5-dB SNR and DOA 30° . The focusing angles are 6.5° , 10.5° , and 14.5° . Figure 1 shows the estimated spatial spectra (in dB) versus the bearing (in degree) for five independent trials. The proposed technique is able to resolve the two SOIs whereas both WCM and NC-WCM exhibit poor performances. Nevertheless, note that all the three cyclic methods exhibit signal selectivity: no peak is present in correspondence of the DOA 30° of the SNOI.

The second experiment is aimed at testing the performance of the CCSS method in a challenging scenario where the DOAs of the SOIs are grouped within one

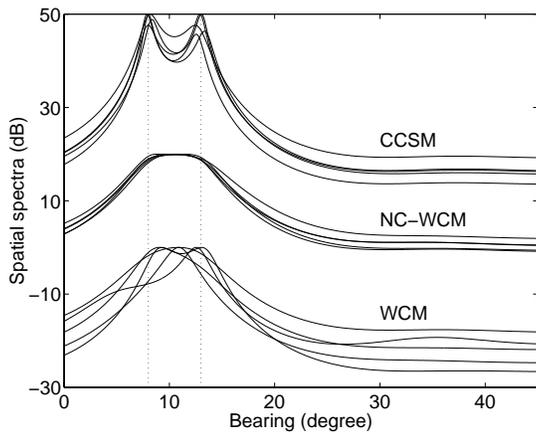


Figure 1

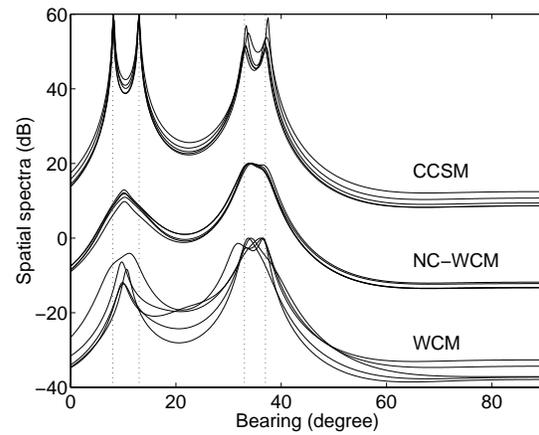


Figure 2

beamwidth of several angles (multigroup case) and furthermore some of the SOIs are fully correlated. Four equipower SOIs with 10-dB SNR impinge from DOAs 8° , 13° , 33° , and 37° together with two equipower uncorrelated SNOIs with 0-dB SNR and DOAs 20° and 40° . Moreover, the SOI coming from 13° is the 0.125-s delayed version of the SOI coming from 8° . The focusing angles are 6.5° , 10.5° , 14.5° , 31° , 35° , and 39° . All four SOIs (Fig. 2) are correctly resolved by CCSS method, whereas the other techniques fail. In particular, note the extremely poor performance of both WCM and NC-WCM in correspondence of the DOAs of the two fully correlated SOIs, since the CSDM of the SOIs turns out to be rank deficient at every frequency value.

Finally, in the last experiment we test the performance of the proposed method when the array is overloaded. Four equipower uncorrelated SOIs with 10-dB SNR impinge from DOAs 4° , 9° , 25° , and 30° together with six equipower uncorrelated SNOIs with 0-dB SNR and DOAs 10° , 20° , 30° , 40° , 50° , and 60° . The focusing angles are 2.5° , 6.5° , 10.5° , 23.5° , 27.5° , and 31.5° . Due to the extremely degraded interference environment, the number of data segments for ACSDM estimation is increased from 64 to 128. The results (Fig. 3) show again that the proposed CCSS method is the only one providing accurate estimates of all the DOAs of the SOIs.

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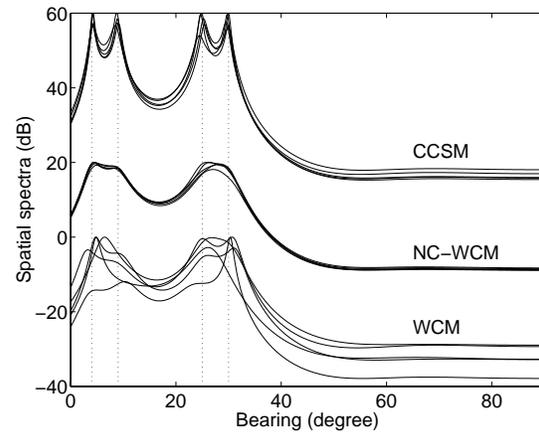


Figure 3

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