CHANNEL PREDICTION FOR ADAPTIVE CODED MODULATION IN RAYLEIGH FADING

Geir E. Øien, *Henrik Holm*, and *Kjell J. Hole*
Dept. of Telecommunications, NTNU, N-7491 Trondheim, Norway
Tel: +47 73 59 43 15; fax: +47 59 73 26 40; e-mail: oien@tele.ntnu.no

Abstract

Adaptive coded modulation (ACM) is a spectrally efficient wireless transmission scheme when the transmitter has perfect channel knowledge. We analyze how the performance of ACM schemes deteriorate when only partial channel state information (CSI) is available due to imperfect channel prediction. The results hold for Rayleigh fading, constant transmit power, perfect coherent detection in the receiver, an arbitrary number of independent receive antenna branches, maximum ratio combining (MRC), and any linear fading predictor based on periodically transmitted pilot symbols.

1 INTRODUCTION

Narrowband radio channels are often modelled by *flat-fading* models [1]. In [2], the channel capacity of a flat-fading channel with arbitrary fading distribution was derived for perfect CSI at the transmitter and receiver. The relevant CSI is the *channel-signal-to-noise-ratio* (CSNR), $\gamma = P_r/P_n$ where P_r is received signal power and P_n is noise power. The capacity can be approached using *rate adaptation*; letting the number of information bits per channel symbol be instantly and continuously updated according to the CSNR. The rate is high when CSNR is high, decreasing smoothly as CSNR decreases and going to zero below a threshold.

The optimal scheme may be approximated using *discrete* rate updating [3]. The transmitter switches between signal constellations and channel codes of varying size/rate at discrete time instants, such that the instantaneous rate is the highest possible which meets the given BER requirements for the available CSI—thus simultaneously ensuring maximum *average* spectral efficiency (ASE) and acceptable BER.

We study the baseband system shown in Figure 1. Each fading channel corresponds to a wireless link between the transmitter and one out of H receive antenna elements. The adaptive coded modulator/demodulator contains N transmitter-receiver pairs, indexed by $n = 1, \ldots, N$. Transmitter n has a rate of R_n information bits per symbol, such that $R_1 < R_2 < \ldots < R_N$. Transmitter-receiver pair n is used when $\gamma \in [\gamma_n, \gamma_{n+1}\rangle$. For each n, γ_n is computed as the lowest CSNR necessary for transmitter-receiver pair n to operate at a BER below some specified target BER_0, at transmit power P. Also, we let $\gamma_0 = 0$ and $\gamma_{N+1} = \infty$, so for all $n \in \{0, \ldots, N\}$, $\gamma_{n+1} > \gamma_n$. We assume that no available transmitter-receiver pair satisfies the BER requirement in $[0, \gamma_1\rangle$, so no information is transmitted when γ falls here.

When performing rate adaptation, the transmitter must rely on the accuracy of the CSNR *as predicted by the receiver* at discrete time k. The *true* channel quality at the transmitter update time $k+\tau$, where τ is the discrete return channel delay (corresponding to τT_s seconds where T_s [s] is the channel symbol duration), may deviate from the prediction; hence the transmitter may adapt to the wrong channel quality.

Denoting the transmitted complex baseband signal after pilot symbol insertion at time index k by $x(k)$, the received signal on the hth subchannel can be written $y_h(k) = z_h(k) \cdot x(k) + n_h(k)$. Here $z_h(k)$ is the *complex fading amplitude*, and $n_h(k)$ is complex-valued additive white gaussian noise (AWGN) with statistically independent real and imaginary components. $x(k)$ is the information signal, except for time instants $k = mL$ (m any integer, L a constant integer), when deterministic *pilot symbols* are periodically transmitted. We assume that the pilot symbols all have the same (absolute) value, $x(mL) = a$.

Assuming flat Rayleigh fading on each subchannel, $z_h(k)$ is complex-valued gaussian with zero mean, and will be assumed approximately constant between two successive pilot symbols (block fading). Its variance is $\Omega = E[|z_h|^2]$, independent of h, with Ω assumed time-invariant; i.e., we assume a wide-sense stationary (WSS) channel.

If a constant average transmit power P [W] is

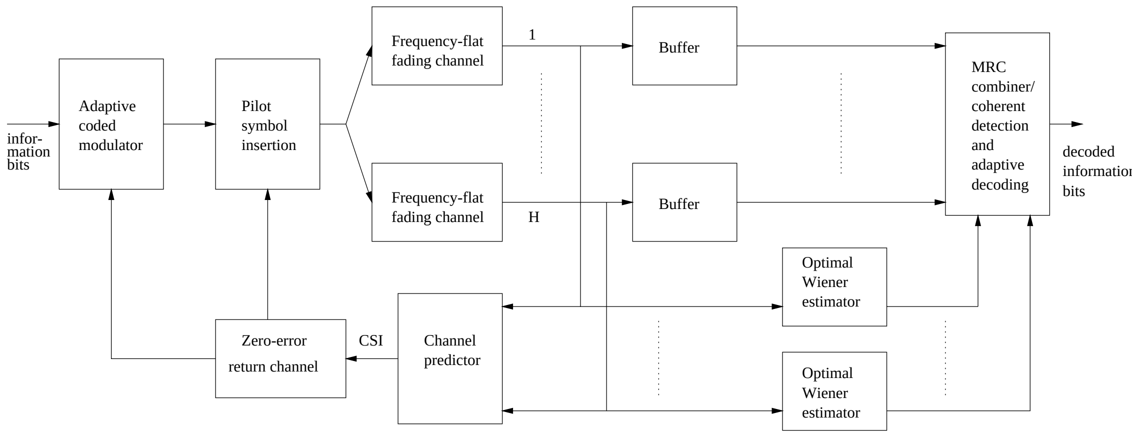

Figure 1: ACM system with pilot-aided channel estimation (for detection) and prediction (for adaptation).

used, and the one-sided power spectral density of the complex AWGN is N_0 [W/Hz] in every subchannel, the received CSNR on subchannel h at a given time k is $\gamma_h(k) = |z_h(k)|^2 \cdot P/(N_0 B)$, where B [Hz] is the one-sided information bandwidth. The expected CSNR is $E[\gamma_h] = \overline{\gamma}_h = \Omega P/(N_0 B)$. With H statistically independent antenna branches combined by MRC at the receiver, the *overall* received CSNR at time k is $\gamma(k) = \sum_{h=1}^{H} \gamma_h(k)$, which is *Gamma* distributed [4] with expectation $\overline{\gamma} = H\overline{\gamma}_h$. This CSNR is predicted and sent back to the transmitter for each received pilot symbol. The delay between CSNR prediction and the subsequent allowed transmitter update is an integer number of pilot symbol periods, $\tau = jL$. The return channel transmitting the code index n is assumed noiseless.

For the receiver signal detection, the signal may be buffered before channel estimation, and an optimal *noncausal Wiener interpolator filter* may be assumed used [5]. This will smooth the noise and allow for true coherent detection. Hence, we still assume that the CSI used during *detection* is perfect.

2 CHANNEL PREDICTION

For any pilot symbol time instant $k - lL$ (l positive integer), define

$$\tilde{z}_h(k - lL) = z_h(k - lL) + \frac{n_h(k - lL)}{a} \qquad (1)$$

which is the maximum-likelihood (ML) estimate of $z_h(k - lL)$ based on one received observation [5]. The two terms are statistically independent complex gaussians, so their sum is a complex gaussian with variance equal to the sum of their variances. At time $k + jL$ of transmitter update we have available $\tilde{z}_h(k), \tilde{z}_h(k-L), \tilde{z}_h(k-2L), \ldots, \tilde{z}_h(k-(K-1)L)$ from which to predict $z_h(k + jL)$. Here, $K \geq 1$ is the chosen *predictor order.* For gaussian processes, the *optimal* predictor in the *maximum a posteriori* (MAP) sense is a *linear* function of the observations [5]. *Any* linear predictor of order K can be written

$$\widehat{z}_h(k + jL) = \mathbf{f}_j^T \tilde{\mathbf{z}}_h \qquad (2)$$

where $\mathbf{f}_j = [f_j(0), \ldots, f_j(K-1)]^T$ is the predictor filter coefficient vector delay jL (we do not need a subchannel index h, since the optimal filter will depend only on the feedback delay when all subchannels have the same fading properties), and

$$\tilde{\mathbf{z}}_h = [\tilde{z}_h(k), \tilde{z}_h(k-L), \ldots, \tilde{z}_h(k-(K-1)L)]^T.$$

The predicted fading envelope is a linear combination of gaussians, so it is itself also a complex gaussian. Now, define $\widehat{\alpha}_h = |\widehat{z}_h|$, with associated $E[\widehat{\alpha}_h^2] = \widehat{\Omega}$. Then, there exists a constant r such that $\widehat{\Omega} = r \cdot \Omega$. It follows that $\widehat{\alpha}_h$ is Rayleigh distributed, and that the corresponding predicted overall CSNR $\widehat{\gamma} = \sum_{h=1}^{H} \widehat{\alpha}_h^2 P/(N_0 B)$ is Gamma distributed—with expected value $E[\widehat{\gamma}] = r\overline{\gamma}$. In [6] expressions are derived for r which apply directly also to the case of linear prediction. Let

$$[\mathbf{R}]_{kl} = \frac{\mathrm{Cov}(z_h(kL), z_h(lL))}{\Omega}, \qquad (3)$$

element (k, l) in the *normalized covariance matrix* $\mathbf{R}$ (dimension $K \times K$) of the fading—at *pilot* time instants—on the subchannel in question. Due to the WSS assumption this will only be a function of the lag between the two pilot symbol time instants kL and lL, $[\mathbf{R}]_{kl} = R(\tau_{kl})$, with $\tau_{kl} = |k - l|LT_s$. For

$a = \sqrt{P}$, we obtain [6]

$$r = \mathbf{f}_j^T \mathbf{R} \mathbf{f}_j + \frac{|\mathbf{f}_j|^2}{\overline{\gamma}_h}. \quad (4)$$

Finally, assuming the much-used *Jakes spectrum* [1] for the fading process, the MAP-optimal filter coefficient vector on a Rayleigh fading channel can be deduced from [5, p. 742, Eq. (14.36)] as

$$\mathbf{f}_{j,\mathrm{MAP}}^T = \mathbf{r}_j^T \left(\mathbf{R} + \frac{1}{\overline{\gamma}_h} \mathbf{I} \right)^{-1}. \quad (5)$$

where $\mathbf{r}_j = \frac{2}{\Omega}[R_I(jL), \ldots, R_I((j+K-1)L)]^T$, with $R_I(\tau) = \frac{\Omega}{2} J_0(2\pi f_D \tau)$. Here, $J_0(x)$ is the 0th order Bessel function of the first kind, and f_D [Hz] is the Doppler frequency shift due to terminal movement.

3 BER ANALYSIS

Following [4], the average BER is given as

$$\overline{\mathrm{BER}} = \frac{\sum_{n=0}^{N-1} R_n \cdot \overline{\mathrm{BER}}_n}{\sum_{n=0}^{N-1} R_n P_n}, \quad (6)$$

where R_n is the rate of code n [information bits/symbol], P_n is the probability that code n is used, and $\overline{\mathrm{BER}}_n$ is the average BER experienced when code n is used. We have (see e.g. [3])

$$P_n = Q\left(H, \frac{H\gamma_n}{r\overline{\gamma}}\right) - Q\left(H, \frac{H\gamma_{n+1}}{r\overline{\gamma}}\right) \quad (7)$$

where $Q(x, y)$ is the *normalized complementary incomplete Gamma function* [7]. Furthermore [4],

$$\overline{\mathrm{BER}}_n = \int_{\gamma_n}^{\gamma_{n+1}} \int_0^\infty \mathrm{BER}_n(\gamma \mid \widehat{\gamma}) p(\gamma, \widehat{\gamma}) d\gamma d\widehat{\gamma}, \quad (8)$$

where $\mathrm{BER}_n(\gamma \mid \widehat{\gamma})$ is the BER experienced when applying code n, where the choice of n is based on the belief that the CSNR is $\widehat{\gamma}$, while it actually is γ. That is, n should be viewed as dependent on $\widehat{\gamma}$ in the expressions to follow. Furthermore, $p(\gamma, \widehat{\gamma})$ is the joint distribution of the actual and the predicted CSNR; in our case a *bivariate gamma distribution*.

To analyze (6) further we must approximate the BER–CSNR relationship for code n by an analytical expression which will make (8) solvable. In [4] it was shown that the BER–CSNR relationship for *trellis codes* on AWGN channels is well modelled as

$$\mathrm{BER}_n(\gamma|\widehat{\gamma}) = \begin{cases} a_n \cdot \exp\left(-\frac{b_n \gamma}{M_n}\right), \gamma \geq \gamma_n^l \\ \frac{1}{2}, \gamma < \gamma_n^l \end{cases} \quad (9)$$

where a_n and b_n are code-dependent constants found by curve fitting to simulated BER–CSNR data on AWGN channels. M_n is the size of the symbol constellation used by the trellis code, and $\gamma_n^l = \ln(2a_n)M_n/b_n$. We assume that such trellis codes are used as component codes in our system, so Equation (9) is valid. For a $2 \cdot J$-dimensional trellis code, meaning that a sequence of J complex (i.e., 2-dimensional) channel symbols results from $J \cdot R_n$ input information bits, the relation between code rate and symbol constellation size is [3] $R_n = \log_2(M_n) - 1/J$. This expression will be used in Equation (6) when evaluating the average BER.

Applying (9) and inserting the bivariate gamma distribution, it is possible to derive a general closed-form expression for Eq. (8). The result is a sum of three terms, $\overline{\mathrm{BER}}_n = \mathcal{I}1(n) - (\mathcal{I}21(n) - \mathcal{I}22(n))$, where $\mathcal{I}1(n)$, $\mathcal{I}21(n)$, and $\mathcal{I}22(n)$ are integrals over the range $[\gamma_n, \gamma_{n+1}\rangle$ for $\widehat{\gamma}$ of

$$\mathcal{J}1(n,\widehat{\gamma}) = \int_0^\infty a_n \exp\left(-\frac{b_n\gamma}{M_n}\right) p_{\gamma,\widehat{\gamma}}(\gamma,\widehat{\gamma})\, d\gamma,$$

$$\mathcal{J}21(n,\widehat{\gamma}) = \int_0^{\gamma_n^l} a_n \exp\left(-\frac{b_n\gamma}{M_n}\right) p_{\gamma,\widehat{\gamma}}(\gamma,\widehat{\gamma})\, d\gamma, \text{ and}$$

$$\mathcal{J}22(n,\widehat{\gamma}) = \frac{1}{2}\int_0^{\gamma_n^l} p_{\gamma,\widehat{\gamma}}(\gamma,\widehat{\gamma})\, d\gamma \quad (10)$$

respectively. Manipulations yield closed form expressions, which unfortunately are too long-winded to reproduce here. The system parameters involved are r, H, and $\overline{\gamma}_h$; γ_n, γ_n^l, a_n, b_n, and M_n for $n \in \{1, \ldots, N\}$; and finally the *normalized correlation coefficient* between true and predicted CSNR,

$$\rho = \frac{\mathrm{Cov}(\gamma,\widehat{\gamma})}{\sqrt{\mathrm{Var}(\gamma)\,\mathrm{Var}(\widehat{\gamma})}} = \frac{E[\gamma\widehat{\gamma}^*]}{\sqrt{\mathrm{Var}(\gamma)\,\mathrm{Var}(\widehat{\gamma})}}. \quad (11)$$

Following [6] and using (4),

$$\rho = \frac{\overline{\gamma}_h \left(\mathbf{f}_j^T \mathbf{r}_j\right)^2}{\overline{\gamma}_h \mathbf{f}_j^T \mathbf{R} \mathbf{f}_j + \|\mathbf{f}_j\|^2}. \quad (12)$$

By inserting $\mathbf{f}_{j,\mathrm{MAP}}$ in this expression, we may evaluate the BER in the case of optimal MAP prediction.

4 EXPERIMENTS

We have performed experiments on an example system described in [3], designed for $\overline{\mathrm{BER}} = 10^{-4}$. The system utilizes $N = 8$ 4-dimensional trellis codes based on M-QAM constellations, with rates from 1.5 to 8.5 bits per symbol. Values for $\{a_n\}$, $\{b_n\}$, $\{M_n\}$, and $\{\gamma_n\}$ are given in [3, 4].

Figure 2 shows how $\overline{\mathrm{BER}}$ varies with $\overline{\gamma}_h$ and the normalized (w.r.t. Doppler period) feedback delay $f_D\tau T_s$, for $H = 2$. The black plane in the figure represents BER_0, so the "ridge" that rises above

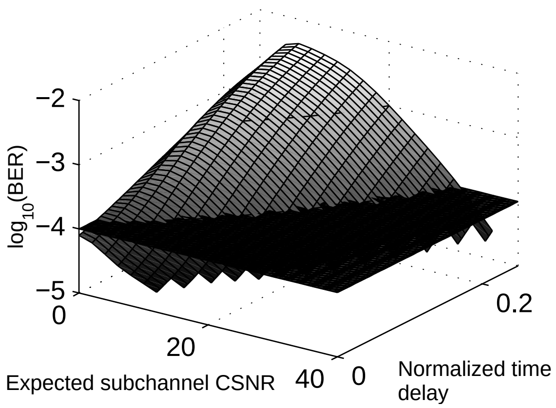

Figure 2: $\overline{\text{BER}}$ as function of normalized delay and $\overline{\gamma}_h$ [dB] for example system. $H = 2$, $\text{BER}_0 = 10^{-4}$.

this plane represents the domain where the system does *not* satisfy the BER requirements. The CSNR range where the system can be expected to operate properly shrinks as the normalized feedback delay increases (corresponding to increased terminal velocity or increased delay in seconds). Figure 3 shows contours at $\overline{\text{BER}} = \text{BER}_0$, extracted from similar 3-D plots, for varying L and H. Each contour divides the $\overline{\gamma}_h$–$f_D \tau T_s$–domain into a left half-plane where the system satisfies the BER requirement, and a right half-plane where it does not. For example, if the normalized delay is 0.05, $L = 15$, and $H = 2$, the system needs at least an average subchannel CSNR of 10 dB to operate properly. System robustness increases as L decreases (which, unfortunately, also decreases ASE), and as H increases.

5 CONCLUSIONS

An ACM system utilizing trellis codes and designed to operate under the idealized assumption of perfect channel state information may still fulfill the BER requirements in a realistic setting, unless the channel exhibits rapid fading due to very fast terminal movements, or has an extremely low average CSNR. The use of multiple receive antennas increases the system robustness considerably, while the choice of pilot symbol period has a significant impact on the system robustness. For our example system, 10-20 % of the channel bandwidth should be utilized for pilot information in order to ensure satisfactory performance. In principle we may also adapt L with respect to $\overline{\gamma}_h$, as faster channel sampling is typically needed to maintain performance at low CSNR.

We reemphasize that our simulation results hold for MAP-optimal prediction, i.e., we assume perfectly known, time-invariant channel model parameters, and predictor filter complexity has not been taken into account. It is also known that practical radio channels may be harder to predict than a channel with Jakes-like Doppler spectrum. Thus, the results are probably still best viewed as providing upper bounds on ACM performance and robustness. More work is needed to evaluate system performance when the model parameters are also time-variant estimates rather than time-invariant and perfectly known, and when predictor complexity must be limited due to processing delays.

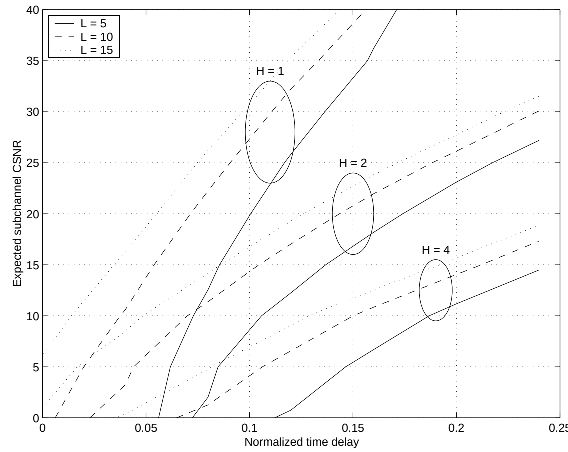

Figure 3: Contours at $\overline{\text{BER}} = \text{BER}_0$ as function of normalized delay and $\overline{\gamma}_h$ [dB] for varying L, H.

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