ASYMPTOTIC PERFORMANCE OF A ROBUST MULTIUSER DETECTOR FOR FAST-FADING CHANNELS WITH IMPULSIVE NOISE

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ABSTRACT

This paper[1] deals with the asymptotic performance analysis of a robust multiuser detector followed by decision-directed Kalman filters recently proposed for for differentially coherent detection in fast-fading non-Gaussian channels.

1 INTRODUCTION

In many physical channels, such as urban and indoor radio channels and underwater acoustic channels, the ambient noise is known through experimental results to be decidedly non-Gaussian due to the impulsive nature of man-made electromagnetic interference and of a great deal of natural noise as well. Thus, the development of demodulation techniques for non-Gaussian multiple-access channels is of great interest.

An initial look at the problem of joint mitigation of multiple-access interference and non-Gaussian ambient noise, shows it to be a challenging one, since the reduction of MAI often relies on linear separating structures while the mitigation of impulsive noise typically relies on nonlinear detectors. Nevertheless, considerable progress has been made on this problem.

In [5] it has been shown that the performance gains afforded by maximum likelihood multiuser detection in additive impulsive noise can be substantial when compared to optimum multiuser detection based on a Gaussian noise assumption. Since the maximum-likelihood strategy is computationally intensive, in [7] a lower complexity M-estimator-based multiuser detector has been proposed for the additive noise channel.

However, CDMA transmissions are frequently made over channels that exhibit fading. Thus, in [4] an M-estimator-based multiuser detector for differential noncoherent data detection in flat-fading CDMA impulsive non-Gaussian channels, has been proposed. Due to the poor performance of this detector above a certain value of the fading rate, in [6] a multiuser detector that can counteract the degrading effects of impulsive noise in fast-fading channels, has been proposed. This multiuser detector consist of a robust decorrelator followed by decision-directed Kalman channel estimators.

In this paper the asymptotic (i.e., large processing gain) performance of this latter multiuser detector is derived and it is shown to predict the actual performance also for a relatively low value of the processing gain.

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2 SYSTEM MODEL

A synchronous CDMA channel is considered, where K active users are subject to independent, frequency-nonselective Rayleigh fading. At the receiver, a sequence $\{\boldsymbol{r}_n(i)\}$ is derived from the received waveform by (complex) basebanding, chip matched filtering, and chip rate sampling. Assuming that the fading process for each user varies at a slow enough rate that the phase and the amplitude can be taken to be constant over the duration of a bit, the received sequence in the $i-$th bit interval can be written as

$$\boldsymbol{r}_n(i) = \sum_{k=1}^{K} \sqrt{\frac{E_{b_k}}{N}} \boldsymbol{g}_k(i) b_k(i) a_n^k + \boldsymbol{w}_n(i) \;\; n = 0,..,N-1 \tag{1}$$

where N is the processing gain, E_{b_k} is the bit energy of the $k-$th user, $a_0^k, a_1^k, \ldots, a_{N-1}^k$ is a signature sequence of +1's and -1's assigned to the $k-$th user and $\boldsymbol{g}_k(i)$ is the $k-$th channel fading coefficient. The received sequence can be written in vector form as

$$\underline{\boldsymbol{r}}(i) = \sum_{k=1}^{K} \sqrt{E_{b_k}} \boldsymbol{g}_k(i) b_k(i) \underline{p}_k + \underline{\boldsymbol{w}}(i) \tag{2}$$

where
$\underline{\boldsymbol{r}}(i) \triangleq [\boldsymbol{r}_0(i), ..., \boldsymbol{r}_{N-1}(i)]^T$, $\underline{p}_k \triangleq \frac{1}{\sqrt{N}}[a_0^k, ..., a_{N-1}^k]^T$ and $\underline{\boldsymbol{w}}(i) \triangleq [\boldsymbol{w}_0(i), ..., \boldsymbol{w}_{N-1}(i)]^T$ is the channel ambient noise vector at the $i-$th symbol interval. It is assumed that the sequence of noise samples $\{\boldsymbol{w}_n(i)\}$ is a sequence of independent and identically distributed complex random variables whose in-phase and quadrature components are independent non-Gaussian random variables with a common probability density function f. Specifically, we adopt the two-term Gaussian mixture model,

that is

$$f = (1-\epsilon)\mathcal{N}(0,\sigma_n^2) + \epsilon\mathcal{N}(0,\sigma_I^2) \quad (3)$$

where $\mathcal{N}(0,\sigma_n^2)$ represents the nominal background noise (Gaussian noise with zero mean and variance σ_n^2), and $\mathcal{N}(0,\sigma_I^2)$ represents the impulsive component (Gaussian noise with zero mean and variance $\sigma_I^2 = \gamma^2\sigma_n^2$), with ϵ representing the probability that impulses occur. This is a commonly used and highly tractable empirical model for impulsive noise environments.

3 DETECTION STRUCTURES

In this section, we briefly describe the robust decorrelator based on M-estimators considered in [4] for noncoherent detection of DPSK signals in slow-fading channels, and its modified version for fast-fading channels proposed in [6].

To introduce the robust M-estimator-based multiuser detector, let us observe that the synchronous signal model (2) can be written in matrix notation as

$$\underline{r}(i) = \underline{H}\,\underline{\theta}(i) + \underline{w}(i) \quad (4)$$

where the real vectors $\underline{r}(i)$, $\underline{w}(i)$, and $\underline{\theta}(i)$ are obtained by stacking the real and imaginary components of the corresponding complex vectors, and

$$\underline{H} = \begin{bmatrix} \underline{A} & \underline{0} \\ \underline{0} & \underline{A} \end{bmatrix} \quad (5)$$

with $\underline{A} \triangleq [\underline{p}_1, \underline{p}_2, ..., \underline{p}_K]$ and $\underline{0}$ the $N \times K$ matrix of all zeros.

The basic idea of M-estimator-based multiuser detection is to detect the symbols in (4) by first estimating the vector $\underline{\theta}(i)$, and then extracting symbol estimates from these continuous estimates. In this case of channel fading, differential encoding must be used to overcome the resulting π-radian phase ambiguity.

The required estimates of $\underline{\theta}(i)$ are obtained by using an estimator of the class of M-estimators proposed by Huber [3]. These estimators minimize a function $\rho(\cdot)$ (called the *penalty function*) of the residuals:

$$\underline{\hat{\theta}}(i) = \arg\min_{\underline{\theta}(i)\in R^{2K}} \sum_{p=1}^{2N} \rho\left(r_p(i) - \sum_{l=1}^{2K} h_{pl}\theta_l(i)\right) \quad (6)$$

where $r_p(i)$ and $\theta_l(i)$ are the $p-$th and the $l-$th element of the vectors $\underline{r}(i)$ and $\underline{\theta}(i)$, respectively, and h_{pl} is the $p,l-$th element of the matrix $\underline{H}$. Given such an estimator, the detected symbols are given by

$$\hat{b}_k(i) = \text{sgn}\left\{\Re\left[\hat{\boldsymbol{\theta}}_k(i)\hat{\boldsymbol{\theta}}_k^*(i-1)\right]\right\} \quad (7)$$

where

$$\hat{\boldsymbol{\theta}}_k(i) \triangleq \hat{\theta}_k(i) + j\hat{\theta}_{k+K}(i). \quad (8)$$

As shown in [4] the detector based on (7) where the estimates are obtained by using the minimax Huber's penalty function is a good choice for slow-fading channels with impulse noise.

Since, above certain values of the fading rate, the performance of the detector based on (7) rapidly deteriorates, in [6] a multiuser detector that can counteract the degrading effects of impulsive noise in fast-fading channels, has been proposed. This multiuser detector is the robust decorrelator followed by decision-directed Kalman channel estimators.

To use a Kalman filter as a channel estimator, the fading processes are approximated in [6] by lightly damped second order Auto-Regressive (AR) processes. Specifically, as in [8], the $k-$th channel fading process is described by the state-space equation,

$$\underline{\boldsymbol{X}}_k(i+1) = \underline{C}\,\underline{\boldsymbol{X}}_k(i) + \underline{G}\,\boldsymbol{d}_k(i) \quad (9)$$

where the driving noise process $\boldsymbol{d}_k(i)$ is a zero-mean white Gaussian process, $\underline{\boldsymbol{X}}_k(i) = [\boldsymbol{g}_k(i), \boldsymbol{g}_k(i-1)]^T$ is the state variable containing the sequence of channel coefficients of the $k-$th user,

$$\underline{C} = \begin{bmatrix} -\beta_1 & -\beta_2 \\ 1 & 0 \end{bmatrix} \quad (10)$$

and $\underline{G} = [1,0]^T$ are the model parameter matrices.

The measurement or the observation used to estimate the $k-$th channel coefficient is $\hat{\boldsymbol{\theta}}_k(i)$, that is, the $k-$th output of the robust decorrelator. Since, under certain regularity conditions, the M-estimators defined by (6) are consistent and asymptotically (i.e., for large processing gains) normal [2], in the $i-$th symbol interval, the complex random variable $\hat{\boldsymbol{\theta}}_k(i)$ can be approximately expressed as

$$\hat{\boldsymbol{\theta}}_k(i) = \sqrt{E_{b_k}}b_k(i)\boldsymbol{g}_k(i) + \boldsymbol{v}_k(i) = \underline{\boldsymbol{H}}_k^T(i)\underline{\boldsymbol{X}}_k(i) + \boldsymbol{v}_k(i), \quad (11)$$

where $\underline{\boldsymbol{H}}_k(i) = \sqrt{E_{b_k}}b_k(i)[1,0]^T$, and $\boldsymbol{v}_k(i)$ is a complex Gaussian random variable with variance

$$E\left[|\,\boldsymbol{v}_k(i)\,|^2\right] = \nu^2\left[\underline{R}^{*^{-1}}\right]_{kk}. \quad (12)$$

In the previus equation $\underline{R}^*$ is the cross-correlation matrix of the random infinite-length signature waveforms of the K users, $[\underline{B}]_{ll}$ denotes the $l,l-$th element of the matrix $\underline{B}$ and

$$\nu^2 \triangleq \frac{\int_{-\infty}^{\infty}\dot{\rho}(x)^2 f(x)dx}{\left[\int_{-\infty}^{\infty}\ddot{\rho}(x)f(x)dx\right]^2} \quad (13)$$

where the dot denotes derivative.

Thus, given the fading model (9) and the measurement (11) the channel estimates $\hat{\boldsymbol{g}}_k(i)$ are generated by the Kalman filters and the detected symbols are given by

$$\hat{b}_k(i) = \text{sgn}\left\{\Re\left[\hat{\boldsymbol{\theta}}_k(i)\hat{\boldsymbol{g}}_k^*(i)\right]\right\} \quad (14)$$

Note that if $\rho(x) = x^2/2\zeta$, i.e., if $\hat{\boldsymbol{\theta}}_k(i)$ is the least-squares estimate, the resulting multiuser detector is the linear decorrelating detector with Kalman filtering proposed in [9].

4 ASYMPTOTIC PERFORMANCE OF THE ROBUST MULTIUSER DETECTOR WITH KALMAN FILTERING

In this section the asymptotic (i.e., large processing gain) error probability of the robust multiuser detector with Kalman filtering in the presence of Rayleigh fading, is derived and analyzed.

Let us observe that taking into account the decision rule (14) the error probability for the $k-$th user can be written as

$$P_k^{BPSK}(e) = P\left(\mid \boldsymbol{X_1} \mid^2 - \mid \boldsymbol{X_2} \mid^2 < 0 \mid b_k(i) = 1\right) \quad (15)$$

where

$$\boldsymbol{X_1} \triangleq \frac{\hat{\boldsymbol{\theta}}_k(i) + \hat{\boldsymbol{g}}_k(i)}{2} \quad (16)$$

and

$$\boldsymbol{X_2} \triangleq \frac{\hat{\boldsymbol{\theta}}_k(i) - \hat{\boldsymbol{g}}_k(i)}{2}. \quad (17)$$

Since, under certain regularity conditions, the M-estimators defined by (6) are consistent and asymptotically (i.e., for large processing gains) normal [2], the desired error probability is given by (see e.g., [1])

$$P_k^{BPSK}(e) = \frac{1}{2}(1 - \mu) \quad (18)$$

where

$$\mu = \frac{R_{X_1X_1} - R_{X_2X_2}}{\sqrt{(R_{X_1X_1} + R_{X_2X_2})^2 - 4R_{X_1X_2}R_{X_2X_1}}} \quad (19)$$

with

$$R_{X_iX_j} \triangleq E\left[\boldsymbol{X_i}\boldsymbol{X_j}^* \mid b_k(i) = 1\right]. \quad (20)$$

Thus, from (11),(12) and (16)-(20) it follows that

$$P_k^{BPSK}(e) = \frac{1}{2}\left(1 - \sqrt{\frac{1 - \Gamma_k}{1 + \frac{\nu^2\left[\underline{R}^{*^{-1}}\right]_{kk}}{\sigma^2 SNR_k}}}\right) \quad (21)$$

where

$$\Gamma_k \triangleq \frac{E\left[\mid \boldsymbol{e}_k(i) \mid^2\right]}{E[\mid \boldsymbol{g}_k(i) \mid^2]} \quad (22)$$

is the normalized estimation error variance and

$$SNR_k \triangleq \frac{E_{b_k}E[\mid \boldsymbol{g}_k(i) \mid^2]}{\sigma^2} \quad (23)$$

with

$$\sigma^2 = (1 - \epsilon)\sigma_n^2 + \epsilon\sigma_I^2 \quad (24)$$

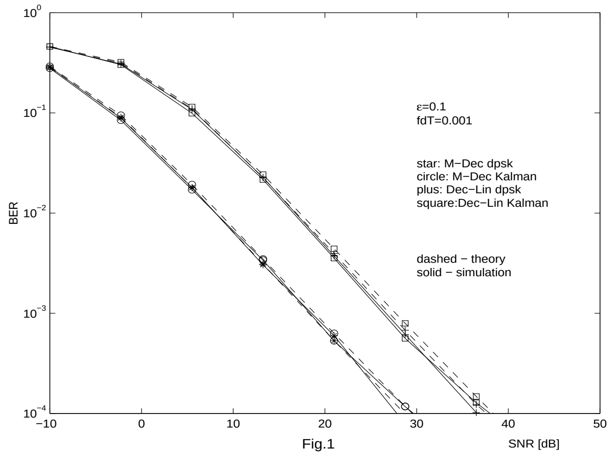

Figure 1: Probability of error versus signal-to-noise ratio for user 1 for the considered detectors, in a synchronous CDMA channel with highly impulsive noise. N=127, K=6, $f_dT = 0.001$, $\epsilon = 0.1$ and $\gamma^2 = 100$.

Since, in decision-directed mode the channel estimate and the data estimate are correlated and result in bursts of opposite decisions due to the deep fades, differential encoding is used. In such a case for sufficiently high SNR, the asymptotic error rate for the $k-$th user can be written as

$$P_k^{DC-PSK}(e) = 2\left[1 - P_k^{BPSK}(e)\right]P_k^{BPSK}(e) \quad (25)$$

Note that in a Gaussian ambient noise environment and for $\rho(x) = x^2/2\zeta$ (21) reduces to the error rate previously derived in [9] for linear coherent multiuser detection of BPSK signals in the presence of fading.

From (21) it follows immediately that the asymptotic error rate of the linear decorrelating detector with Kalman filtering depends on the noise distribution only through its variance (as one would expect). Moreover, for sufficiently high values of SNR, (21) suggests that the performance of the proposed robust coherent multiuser detector presents an error floor that depend mainly on the steady-state value of the normalized estimation error variance.

5 NUMERICAL RESULTS

In this section the performance of the robust multiuser detector with Kalman filtering is assessed via Monte Carlo simulations and is compared with the asymptotic error rate predicted by (21) and (25).

A synchronous CDMA system with $K = 6$ users in which the spreading sequence of each user is a shifted version of an $m-$sequence of length $N = 127$, is considered. As regards the Kalman filters, 100 reference bits have been used for obtaining the steady-state error covariance. Moreover, in the simulations the variance of each of the in-phase and quadrature components of ambient noise has been held constant and equal to

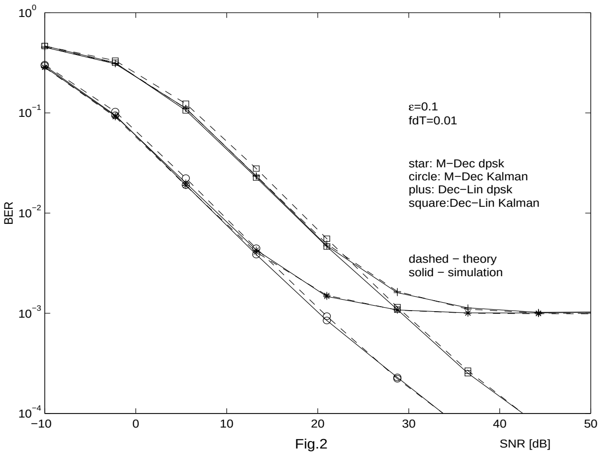

Figure 2: Probability of error versus signal-to-noise ratio for user 1 for the considered detectors, in a synchronous CDMA channel with highly impulsive noise. N=127, K=6, $f_dT = 0.01$, $\epsilon = 0.1$ and $\gamma^2 = 100$.

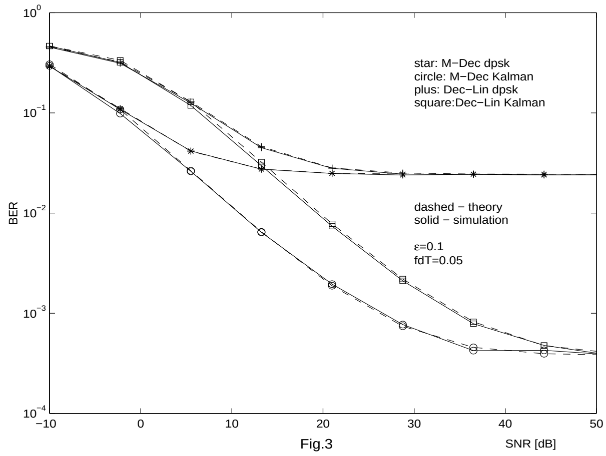

Figure 3: Probability of error versus signal-to-noise ratio for user 1 for the considered detectors, in a synchronous CDMA channel with highly impulsive noise. N=31, K=6, $f_dT = 0.05$, $\epsilon = 0.1$ and $\gamma^2 = 100$.

1, whereas the powers of all users has been varied to achieve the desired values of the signal-to-noise ratio.

Figure 1 shows the performance of the considered detectors as a function of SNR, in a noise with a relatively high fraction of impulses ($\epsilon = 0.1$, $\gamma^2 = 100$). Specifically, the bit error rate for user 1 is reported, for the case where the received users' powers are all equal (i.e., in the case of tight power control). The normalized fading rate has been fixed at $f_dT = 0.001$. The results show that the asymptotic error rate (dashed curves) can be exploited to predict the actual performance (solid curves) also for this relatively low value of the processing gain.

Figure 2 report the performance of the considered detector in fading channel with the normalized fading rate fixed at $f_dT = 0.01$. In a channel characterized by this higher value of the fading rate both of the coherent structures, that is the coherent robust multiuser detector with Kalman filtering (labeled as M-Dec Kalman) and the linear decorrelating detector with Kalman filtering (labeled as Dec-Lin Kalman) outperform, above a certain values of SNR, their noncoherent counterparts, the robust noncoherent detector based on the decision statistic (7) (labeled as M-Ddec dpsk), and the differential form of the linear decorrelator (labeled as Dec-Lin dpsk). The range of values where this improvement is observed increases as the fading rate increases (see Fig.3).

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