# Propagation model choice for rapid SAR measurement

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# ABSTRACT

Specific Absorption Rate (SAR) designates the electromagnetic power density deposited per unit mass of biological tissue. SAR measurements are required to assess the compliance of mobile phones with existing standards and recommendations. This paper presents a new approach where SAR calculation is based on a parametric reconstruction of the E-field distribution in the phantom to assess rapid SAR measurements. This approach allows a drastic reduction of the measurement points, from which the field distribution in the phantom can be adequately described by means of an ellipsoidal model with only 11 parameters. The estimation of these parameters is achieved from two different approaches, depending on the choice of the distribution of the measurement points in the phantom: from a reduced number of real data points exclusively, and from a combination of real data points and E-field extrapolation with a plane wave expansion model. The two reconstruction procedures are compared, and their respective application domains discussed.

#### 1 Introduction

The experimental electromagnetic dosimetry of mobile phones has much developed since the beginning. Most of the existing dosimetric facilities utilize automatic positioning systems to move an E-Field measuring probe, with the help of robotized arms [1], or three axes displacement systems in order to achieve SAR (Specific Absorption Rate) measurements. The European Standard prEN50361 [2] details the ways to measure the SAR in a head-like phantom, and the maximum value of the SAR averaged in 10 g allowed. The electromagnetic properties of the liquid filling the phantom are similar to those of the brain. According to the new European standard, a complete phone test will lasts about half a day, pointing out a new concern : the rapidity of SAR measurement. A rapid (approximately 1 minute) and non-invasive SAR measurement solution, based on a probe integrated in a spherical phantom around which the phone under test turns, has been still developped [4]. The rapid SAR measurement method that we propose here is fully compatible with popular instrumentation and, hence, can be directly implemented on most existing SAR measurement facilities using mechanical scanning of an E-field probe [5]. The number of electric field data is reduced by a factor 30 that allows dividing the measurement time by a factor 10 approximately. The physical quantity that we measure, the maximum SAR (1) averaged in 1 g and 10 g of contiguous tissues, is the volumetric integration of the quadratic electric field, weighted by the conductivity  $\sigma$  and the density  $\rho$  of the media:

$$SAR = \frac{\sigma E^2}{2\rho} \qquad (W. \, kg^{-1}) \tag{1}$$

with E (V.m<sup>-1</sup>) the E-field norm,  $\sigma$  (S.m<sup>-1</sup>) the conductivity of the medium, and  $\rho$  (kg. m<sup>3</sup>) its density. The relative simplicity of the E-field distribution in the phantom, the more often in a long spot perpendicularly to the direction of propagation, its reproducibility between the different mobile phones, allows to consider generic parametric models of the E-field able to fit the principal electromagnetic characteristics of the phones. The number of parameters of those models (between 5 and 11) is sufficiently low to reconstruct the electric field in the whole data volume, from a very small number of data, in a very reasonable computation time (on the order of one second). The acquisition time is then drastically reduced. The results presented in the last section show E-field reconstructions of commercial phones from two models involving a strict ellipsoidal approach, and a hybrid ellipsoidal-plane wave approach, this one trying to reduce the data points needed to a few points distributed in a plane. These approaches are based on a long experience in SAR measurements, and in parametric methods applied to physical measurements. Their development has been possible with the help of the hundreds SAR tests of commercial phones, and is a result of the research effort conducted to improve the accuracy and the rapidity of the Supélec dosimetric facility.

#### 2 Geometrical and physical aspects of SAR

The construction of the mathematical models representing the electric field is issued from the conclusions of systematic SAR tests on a hundred of real mobile phones (at 2 frequencies - 900 MHz and 1800 MHz).

The analysis of the E-field in function of depth in the head-like phantom, shows an exponential decay (*Fig. 1*), for more than 90 % of the tested phones. The curves corresponding to each tested phone are distributed around the line corresponding to the theoretical propagation constant  $\alpha$  of the plane wave (2).

$$E(z) = E_0 e^{-\alpha z} \tag{2}$$

For the standard materials used, and for a plane wave,  $\alpha = 28.2 \text{ m}^{-1}$  at 900 MHz, and  $\alpha = 41.0 \text{ m}^{-1}$  at 1800 MHz. The 10 % of mobile phones a bit far away from that observation concerns the phones for which measurement is too noisy to be significant. In those cases, the E-field level is at the limit of sensibility of the probe ( $\approx 1 \text{ V.m}^{-1}$ ), and is almost drowned in the noise.



Fig. 1: Depth decrease of the maximum normalized E-Field, for 40 phones at 1800 MHz. The dotted line represents the decay of the theoretical plane wave in the same propagation medium. Vertical lines represent the limits in depth of the 1 g (left) and 10 g (right) averaging volume.

The characteristics of the propagation medium have an influence on the electromagnetic propagation, but also on the shape of the E-field. Statistically, the spatial distribution of the electric field, again in more than 90 % of the tested phones, is of ellipsoidal type. The Fig. 2(a)and Fig. 2(b) show examples of E-field shapes for typical commercial phones in the plane close to the bottom of the phantom (6 mm above it) (Fig. 3(a)). The E-field data have been obtained with a meshing step of 7 mm, in 70 mm size squares. The electric field shape of the Fig. 2(a) is almost of circular type, while the shape of the Fig. 2(b) is more of elliptic type. Modeling the circular case, as degenerate case of the ellipsoidal model, will constitute an important step for the estimation of the parameters of the ellipsoidal model. The 10 % of phones not in agreement with this morphological description are the same as the 10 % remarked in the previous section for their particular decay in function of depth - the two facts are evidently correlated. The presence of multiple spots is due to the very low radiation of the phone (Fig. 3(b)).



Fig. 2: Total E-Field in the head-phantom, at 1800 MHz.



Fig. 3: Total E-Field in a plane in the head-phantom, at 1800 MHz.

#### 3 Degrees of freedom of the system

The two main previous observations, elliptic spatial distribution of the E-field and propagation as an exponential function of depth, are characteristics relatively easy to model. These investigations let think that giving morphological and physical considerations on the E-field must allow introducing a priori information in the calculation SAR process [6]. The number of data points required in the standard way to compute the SAR should be decreased, by considering that a large number of those points depend from each other from electromagnetic or morphological laws. The electric field in the phantom, can then be seen in that way as a relatively simple system: the E-field generated by a spherical wave expansion, as a first approximation. A first estimation of the degree of freedom N of the distribution of the electric field in the phantom can be evaluated by the number of useful terms of its development in spherical modes [7]:  $N \approx [kr]$ , where  $k = \frac{2\pi f}{c}$  and where the square brackets indicate the largest integer smaller than or equal to kr, r is the radius of the domain of study *i.e.* the sphere of smallest radius that circumscribes the system, c the velocity of light, f the frequency in the liquid. As an example, for r = 3.5 cm we obtain  $N \approx 5$  for f = 900 MHz, and  $N \approx 10$  for f = 1800 MHz. We can

then think that about 10 parameters of a mathematical model should be enough to reconstruct the E-field in the entire volume from a reduced number of data points.

#### 4 Modeling of the electric field

#### 4.1 Ellipsoidal model

where

 $\rho_i$ 

In order to obtain a model closer to the spatial distributions of the E-field observed, we developed an ellipsoidal model (3):

$$E(r) = E_0 e^{\left[-\lambda(1-\rho_i)\left((x-x_s)^2 + (y-y_s)^2 + (z-z_s)^2\right)^{\frac{1}{2}}\right]}$$
(3)

$$= \left( \left(\frac{x'}{a}\right)^2 + \left(\frac{y'}{b}\right)^2 + \left(\frac{z'}{c}\right)^2 \right)^{-\frac{1}{2}} \quad \text{and}$$

 $\mathbf{r}'$  $\mathbf{R}_{\gamma}\mathbf{R}_{\beta}\mathbf{R}_{\alpha}\mathbf{r}$  with  $\mathbf{R}_{\alpha}$ ,  $\mathbf{R}_{\beta}$ ,  $\mathbf{R}_{\gamma}$ , the rotation \_ matrices built from the Euler rotation angles  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\mathbf{r}(\mathbf{x}, \mathbf{y}, \mathbf{z})$  the vector of positions of the E-field measurement points.  $p_e(E_0, k, x_s, y_s, z_s, a, b, c, \alpha, \beta, \gamma)$ is the vector of parameters, where  $E_0$  represents the field amplitude, k is similar to the propagation constant  $\alpha$  of the plane wave,  $x_s,\,y_s$  and  $z_s$  are the coordinates of the center of the ellipsoid, and a, b and c its semi-axes. The source of this ellipsoidal expansion is kept outside the homogeneous propagation medium (liquid), in order to ensure the exponential decay inside the phantom. In consequence,  $z_s$  is constrained to be always negative. For physical reasons, the amplitude  $E_0$  is positive, as well as k.  $x_s$  and  $y_s$  belong to  $\Re$ , and a, b and c are evidently positive. The Euler angles are taken in the range  $[-\pi;\pi]$ . The configuration represented below Fig. 4 shows an ellipsoidal envelope, cut by a measurement plane inside the semi-spherical phantom a little upper from the bottom. The part of the ellipsoid reconstructed outside the phantom shell, evidently, cannot be used to describe the real radiation. The physical sense of such a description matches with reality only inside the liquid.



Fig. 4: Representation of the ellipsoidal model of the electric field, inside the phantom, with the data acquisition plane. The real cut shown at the right is issued from a real measurement.

# 4.2 Hybrid plane wave - ellipsoidal model

Even if the preceding modelization technique ensure a very fast SAR measurement, we tried to go further by using the electromagnetic properties of the E-field developped in the previous section. By using E-field values deduced from the propagation constant  $\alpha$  of the plane wave, instead of the corresponding real E-Field data points, the acquisition of data requires now only one plane of 4 x 4 E-field points. The *Fig.* 5(a) and *Fig.* 5(b) shows the standard 726 E-field points distribution and the ones used for the parametric reconstruction with only 36 and 21 points. We will adapt the 21 points distribution for the hybrid plane wave-ellipsoidal model, by using a plane of 16 E-field data points and calculating the 5 points in depth in the maximum field zone from plane wave propagation constant  $\alpha$ .



Fig. 5: Standard and parametric reconstruction meshing

# 4.3 Parameters estimation

The method of the parametric reconstruction of the Efield uses a mathematical model representing the field in order to reconstruct it in the phantom, from a little set of data points, with the desired meshing. The non-linear problem to solve carries a large number of unknowns, and has few given information. From the nmeasurement points chosen, the parameters of the considered model are estimated in the sense of the least squares of the error between the model and the measurement points [14]. The estimation procedure of the ellipsoidal parameters is described below. The 5 parameters of the spherical model are first estimated. The vector of parameters  $p_s(E_0, k, xs, ys, zs)$  is then used as initialization values for those common parameters with  $p_e(E_0, k, xs, ys, zs, a, b, c, \alpha, \beta, \gamma)$ , for the estimation of the parameters of the ellipsoidal model. The other parameters of the ellipsoidal model  $(a, b, c, \alpha, \beta, \gamma)$  are initialized with the median values of their respective variation ranges. With this technique, the convergence of the estimation of the ellipsoidal parameters is optimized, avoiding that the estimation lay in a local minimum too far from a good solution. In the estimation process, the parameters are normalized, in order to eliminate the scaling factors and then avoid the prevalence of parameters. When the parameters have been determined, the calculation of every E-field point of the considered

volume is immediate, as the standard procedure of computation of the maximum SAR integrated in 10 g.

## 5 Results on real phones data

The two reconstruction process have been tested on 128 measurements corresponding to 60 different mobile phones - 4 of them have retractable antenna (2 configurations) - for 2 frequencies. The following figures present the maximum averaged SAR on 10 g, for observed and reconstructed data with 21 real data points (Fig. 6(a)) and 16 real data points plus 5 extrapolated points calculated with the plane wave propagation constant  $\alpha$  (Fig. 6(b)). It must be noticed that the quality of the reconstruction is bound to the geometrical form of the real field, but also to its intensity. The method is less powerful if the observed field strength is too low for the probe's sensitivity. The accuracy of the method (the mean error is 10% for the 21 real data points) has to be compared with the complete error made with standard dosimetric facilities, which is commonly 30 % to 40 %

. The reproducibility between two measurements for the same phone and with the same assessment is about 5 % to 10 % . The choice of an E-field propagation model based on real measurements is therefore better than the simplest description of the plane wave. This result confirm the previous observation of the relatively dispersive decreases of the E-field as a function of depth. However, the approach using the plane wave propagation constants ensuring a maximized value for the calculated SAR, could nevertheless find an application in very fast measurements, for precompliance tests for example.



Figure 6: Maximum SAR averaged on 10 g.

#### 6 Conclusion

This parametric approach employed in a novel application domain has yet proved itself with the robust estimation of other physical quantities [8]. The methods proposed in this paper, useful with standard dosimetric measurement systems, allows calculating the electric field and the SAR from a reduced number of observed data points. The local SAR is directly obtained from the knowledge of the model parameters. The maximum 10 g averaged SAR is then instantly obtained. The errors made on the 10 g averaged SAR, in the case of the full ellipsoidal propagation model, are lower than the accuracy of the complete standard measurement itself. The measurement time for a given configuration is about 1 minute (16 real points plus plane wave extrapolation), whereas the data processing time is neglectible. Furthermore, the hybrid ellipsoidal-plane wave approach, used in conjunction with an array of E-field probes, should be able to provide quasi real time SAR measurement capabilities.

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