VECTOR DIRECTIONAL ORDER-STATISTICS FOR IMPULSE DETECTION

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Abstract

In this paper, a new vector directional approach for impulse noise detection and filtering in color images is provided. The novelty of this approach lies in the use of vector order-statistics achieved by the ordering according to the sum of angles between input vector samples, in other words directional information, as the base for impulse detector. Thus, the input samples are separated into two classes such as noise-free samples and corrupted samples. This simple binary decision is performed by the comparison between the operation value and the angle threshold. An importance of the vector directional order-statistics with the smallest angle distances to input samples consists in the determination of the operation value that is given by an angle between the central sample and the mean of 'smallest' vector order-statistics. After the separation, only affected samples are processed by basic vector directional filter, whereas noise-free samples are passed to a filter output without change, i.e. the system performs an identity operation. In order to achieve the optimal performance of the proposed method, there is necessary to determine the threshold angle and the number of considered vector order-statistics, too. Since, the proposed adaptive method represents directional processing, the excellent results in the term of the simultaneous impulse noise suppression and signal-details and color chromaticity preservation are desired.

INTRODUCTION

Many applications [15] such as image processing, multimedia, robot vision applications and etc. require color information that allows an effective understanding of the visual scene. Obviously, the correct recognition of the objects on the scene is expected in the case of non corrupted visual and color information. However, the interference of useful image information with the noise occurs in many applications, especially during image transmission by the influence of an atmospheric noise (satellite images) or channel error. Since, the noise affects the visual quality of the image, there is necessary to preserve a desired signal and suppress the noise, simultaneously.

Since the atmospheric noise and the channel errors are strictly non Gaussian, a popular class of linear filters exhibits worse noise attenuation characteristics. The non Gaussian noise has a short duration, i.e. an impulsive savour, and for that reason, nonlinear filters can provide the balance between the noise suppression and the signal-details preservation.

In noise filtering, a problem is often how to preserve some desired signal features while the noise elements are removed. An optimal situation would arise if the filter could be designed so that the desired features are invariant to the filtering operation and only noise would be affected. Since filters for the effective impulse noise suppression are nonlinear and the superposition is not satisfied, the optimal situation can never be fully obtained.

However, the proposed method, i.e. the connection of the impulse detector based on vector directional order-statistics and basic vector directional filter (BVDF) [14],[18] is really closely to the optimal filtering, since noise-free samples are passed on the output without change (system works as an identity filter (IF)), whereas corrupted elements are estimated by BVDF. An adaptive alternation between identity filter and BVDF is controlled by comparison of the operation value (given by the angle between the central sample of the input set and the mean of the 'smallest' vector directional order-statistics) and the threshold angle. If the operation (detector) value is greater than or equal to the threshold angle, it is the case of a noisy sample, since the central sample is very different from the 'smallest' vector directional order-statistics that are noise-free samples with a high probability. If the operation value is smaller than the threshold angle, the proposed method is equivalent to an identity filter.

Clearly, this paper focuses on the adaptive alternation between BVDF and IF, where the binary decision is given by the use of vector-directional order-statistics and the threshold angle. Thus, the performance of the method depends on the number of considered order-statistics and the value of the threshold angle.

This paper is organised as follows. In the next section, the mathematical preliminaries related to directional processing are presented. Section 'Proposed Method' focuses on a design of a new method. In 'Experimental Results', the vector definition of the impulse noise for color images is described. Three objective criteria are defined, including well-known mean absolute error, mean square error and color difference criterion for the color chromaticity preservation. The performance of the proposed method is studied according to some aspects including the threshold angle, number of order-statistics and etc. Finally, the properties of the proposed method are concluded in 'Conclusion'.

DIRECTIONAL PROCESSING

In order to suppress the impulse noise, usually the robust order-statistics theory [10] is used. In the case of color images, i.e. vector-valued image signals, the direct extension of the robust order-statistics theory is impossible [13],[17] and the observed samples are ordered according to the distance function, where both magnitude [1],[11] and direction [6],[18] of vector samples can be considered. In general, vectors' magnitude takes a measure of their brightness, whereas the direction of vector samples wreaks their chromaticity. If the distance function is created by the sum of the angles between input samples, it is the case of directional processing [9],[14],[15],[19].

Let $y(x): Z^l \rightarrow Z^m$ represent a multichannel image, where l is an image dimension and m characterises a number of channels. If $m \geq 2$, then it is the case of m-channel image processing. In the case of standard color images $l = 2$ and $m = 3$. Let $W = \{\mathbf{x}_i \in Z^l \ ; \ i = 1,2...,N\}$ represent a filter window of a finite size N, where $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$ is a set of noised samples. Note that the position of the filter window is determined by the central sample $\mathbf{x}_{(N+1)/2}$. Each input vector $\mathbf{x}_i$ is associated with the distance measure α_i given by [9],[15]

$$\alpha_i = \sum_{j=1}^{N} \mathrm{A}(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for } i = 1,2,...,N \tag{1}$$

where

$$\mathrm{A}(\mathbf{x}_i, \mathbf{x}_j) = \cos^{-1}\left(\frac{\mathbf{x}_i . \mathbf{x}_j^T}{|\mathbf{x}_i| . |\mathbf{x}_j|} \right) \tag{2}$$

$$= \cos^{-1}\left(\frac{x_{i1}x_{j1} + x_{i2}x_{j2} + ... + x_{im}x_{jm}}{\sqrt{x_{i1}^2 + x_{i2}^2 + ... + x_{im}^2}\sqrt{x_{j1}^2 + x_{j2}^2 + ... + x_{jm}^2}} \right) \tag{3}$$

represents the angle between two m-dimensional vectors $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{im})$ and $\mathbf{x}_j = (x_{j1}, x_{j2}, ..., x_{jm})$.

In general

$$0 \leq \mathrm{A}(\mathbf{x}_i, \mathbf{x}_j) \leq \pi \tag{4}$$

is valid, whereas in the case of color images [14]

$$0 \leq \mathrm{A}(\mathbf{x}_i, \mathbf{x}_j) \leq \pi / 2 \tag{5}$$

If angle distances (1) serve as an ordering criterion, i.e.

$$\alpha_{(1)} \leq \alpha_{(2)} \leq ... \leq \alpha_{(r)} \leq ... \leq \alpha_{(N)} \tag{6}$$

it means that the same ordering is implied to the input set $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$ which results in ordered input sequence

$$\mathbf{x}^{(1)} \leq \mathbf{x}^{(2)} \leq ... \leq \mathbf{x}^{(r)} \leq ... \leq \mathbf{x}^{(N)} \tag{7}$$

Sample $\mathbf{x}^{(1)}$ associated with minimal angle distance $\alpha_{(1)}$, i.e. a sample that minimises the sum of angles with other vectors, represents an output of basic vector directional filter (BVDF) given by [18],[19]

$$\mathbf{y}_{BVDF} = \mathbf{x}^{(1)} \tag{8}$$

Since, the vector directional filters pass to a filter output a sample from a set ordered according to the sum of vector angles, these filters perform optimal filtering operation in sense of the sample direction preservation.

In the case, that a filter output can be expressed as the set of the first r terms of (7) with simultaneous valid (6), the filter is called general vector directional filter (GVDF). Mathematically, GVDF output is defined by [18],[19]

$$\mathbf{y}_{GVDF} = \left\{ \mathbf{x}^{(1)}, \mathbf{x}^{(2)}, ..., \mathbf{x}^{(r)} \right\} \tag{9}$$

GVDF output the set of r vectors whose angle α_i (for $i = 1,2,...,N$) from all other vectors is small. Usually, the output set of GVDF is used in the second level as an input for additional filter, e.g. α-trimmed average filter, multistage median filter and some morphological filters, where samples $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, ..., \mathbf{x}^{(r)}$ will be processed according to their magnitude, since these vectors have approximately equal direction in a vector space. Simply, GVDF produces a set of vectors with similar directions in color space, and thus samples with atypical directions are eliminated. It follows that GVDF differentiate the processing of color vector on directional processing and magnitude processing.

PROPOSED METHOD

Now, the basic theory related to the impulse detection (see Figure 1) is provided. In general, the decision rule of the impulse detector is given by [2],[3],[4],[8]

$$\begin{array}{llll} \text{IF} & Val \geq Tol & \text{THEN} & \mathbf{x}_{(N+1)/2} \text{ is impulse} \\ & & & \mathbf{x}_{(N+1)/2} \text{ is noise-free} \end{array} \tag{10}$$

where Val characterises a detector operation based on simple mathematical relationship between the central sample and neighbourhood samples and Tol is adaptive or fixed threshold.

In the case of $Val \geq Tol$, i.e. if the operation value Val is greater than or equal to the threshold value Tol, then the central input sample $\mathbf{x}_{(N+1)/2}$ is corrupted and processed by a smoothing filter, consecutively. If $Val < Tol$, the central sample $\mathbf{x}_{(N+1)/2}$ is noise free. Then, it is retained without change and thus, a blurring introduced by a filter influence is reduced.

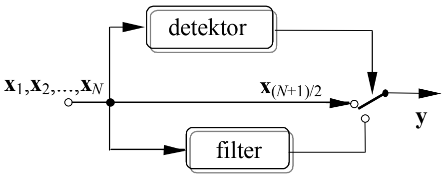

Figure 1 Architecture of the proposed method

Now, the proposed method is presented. In order to obtain information about an impulse occurrence, the following steps should be performed. According to (1), each input vector sample $\mathbf{x}_i$, $i = 1,2,...,N$, is associated with the angle distance α_i. In the next step, there is necessary to sort the set $\alpha_1, \alpha_2, ..., \alpha_N$ resulting in (6). Then, the same ordering is applied to input set $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$.

Likewise the definition of GVDF (9), let $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, ..., \mathbf{x}^{(r)}$ be a set of the r 'smallest' vector directional order statistics (7), i.e. input samples with similar directions in color space. Note that r is forced by $1 \leq r \leq N$. Then, the decision rule of vector impulse detector can be expressed by

$$\begin{array}{llll} \text{IF} & \mathrm{A}(\overline{\mathbf{x}}(r), \mathbf{x}_{(N+1)/2}) \geq Tol & \text{THEN} & \mathbf{x}_{(N+1)/2} \text{ is corrupted} \\ & & & \mathbf{x}_{(N+1)/2} \text{ is noise-free} \end{array} \tag{11}$$

where Tol is threshold angle bounded by (5) and $\mathrm{A}(\overline{\mathbf{x}}(r), \mathbf{x}_{(N+1)/2})$ characterises the angle between the central sample $\mathbf{x}_{(N+1)/2}$ and the mean $\overline{\mathbf{x}}(r)$ of

$$\overline{\mathbf{x}}(r) = \frac{1}{r} \sum_{i=1}^{r} \mathbf{x}^{(i)} \tag{12}$$

In the case of the noise detection, i.e. if the angle between $\overline{\mathbf{x}}(r)$ and $\mathbf{x}_{(N+1)/2}$ is greater than or equal to the threshold angle Tol, the noisy central sample is replaced with sample $\mathbf{x}^{(1)}$, i.e. with the BVDF output. On the other hand, if the mean $\overline{\mathbf{x}}(r)$ and the central sample $\mathbf{x}_{(N+1)/2}$ have similar direction in the vector space, then the central sample is probably noise-free and thus, it is passed to the resulting image without the change and no additional processing is performed.

The proposed method includes both identity operation with no smoothing and BVDF with maximum amount of smoothing (according to directional processing) and it reduces blurring that should be introduced by BVDF. Switching control between the identity filter and the BVDF is performed in the dependence on threshold angle Tol and parameter r corresponding to a number of considered 'smallest' vector directional order-statistics. In the next Section, the optimal parameters Tol and r are searched.

EXPERIMENTAL RESULTS

As the test image was used well-known color image Lena (Figure 1a). The noise corruption (Figure 1b) was simulated by the impulse noise that is defined by [8], [11]

$$\mathbf{x}_{i,j} = \begin{cases} \boldsymbol{\upsilon} & \text{with probability } p_{\upsilon} \\ \mathbf{o}_{i,j} & \text{with probability } 1 - p_{\upsilon} \end{cases} \qquad (13)$$

where i, j characterise the sample position in an image, $\mathbf{o}_{i,j}$ is the sample from the original image, $\mathbf{x}_{i,j}$ represents the sample from the noisy image, p_{υ} is a corruption probability and $\boldsymbol{\upsilon} = (\upsilon_R, \upsilon_G, \upsilon_B)$ is a noise vector of intensity random values. Since, single components of $\boldsymbol{\upsilon}$ are generated independently, the gray impulse, i.e. an equivalence of all components of $\boldsymbol{\upsilon}$ ($\upsilon_R = \upsilon_G = \upsilon_B$), can occur in the special case, only.

As a measure of the noise corruption and the filter performance, too, three objective criteria, namely mean absolute error (MAE), mean square error (MSE) and color difference (CD) [10], are used. In general, MAE is a mirror of the signal-details preservation, MSE evaluates the noise suppression well and CD is a measure of the color chromaticity preservation. Thus, the quality of the processed image sequences is quantified with a high accuracy related to the signal dimensionality.

Mathematically, the definitions of MAE and MSE for monochromatic images are given by

$$MAE = \frac{1}{KL} \sum_{i=1}^{K} \sum_{j=1}^{L} \left| o_{i,j} - x_{i,j} \right| \qquad (14)$$

$$MSE = \frac{1}{KL} \sum_{i=1}^{K} \sum_{j=1}^{L} \left(o_{i,j} - x_{i,j} \right)^2 \qquad (15)$$

where $\{o_{i,j}\}$ is the original image, $\{x_{i,j}\}$ is the filtered (noisy) image, i, j are indices of sample position and K, L characterise an image size. Note that in the case of color images, MAE and MSE criteria are understood as a mean over color channels.

Finally, the measure of color distortion or color chromaticity preservation is evaluated by CD that requires transformation from RGB to Luv color space [16]. For a color image, the CD is expressed as

$$\Delta E_{Luv} = \sqrt{(\Delta L)^2 + (\Delta u)^2 + (\Delta v)^2} \qquad (16)$$

where $\Delta L, \Delta u$ and Δv represent the difference between original and noisy images in L, u and v color channels. The overall value of CD is a mean value over all frames. Unlike MAE and MSE, in the case of CD was established the threshold value around 2.9 that characterises the senselessness of human eyes to color distortion.

Experimental results are summarised in Figure 2, Figure 3 and Table 1. In order to achieve the optimal threshold angle, see Figure 2. From this figure it can be seen that for $r = 5$ the optimal threshold angle was equal to 0.16. Additional increasing of the threshold angle impairs the noise attenuation characteristics of the proposed method. On the other hand, if the threshold angle is decreased, the proposed method provides the improved noise suppression, however, it declines the signal-details and color chromaticity preservation capability.

The performance of the proposed method was compared (Table 1, Figure 3) with marginal median, vector median and BVDF. These results show that the proposed method achieves the significant improvement in comparison with above-mentioned filters. Especially, MAE and CD criteria corresponding to preservation capability of the methods are really excellent.

Table 1 Performance of methods

Filter class	*MAE*	*MSE*	*CD*
Identity filter	7.312	832.0	32.717
Marginal median	3.703	56.8	17.777
Vector median	3.687	56.5	15.396
BVDF	4.099	67.6	15.343
Proposed (r = 9, Tol = 0.18)	1.366	64.1	4.796
Proposed (r = 7, Tol = 0.13)	0.985	36.7	3.168
Proposed (r = 5, Tol = 0.16)	0.936	38.6	2.815

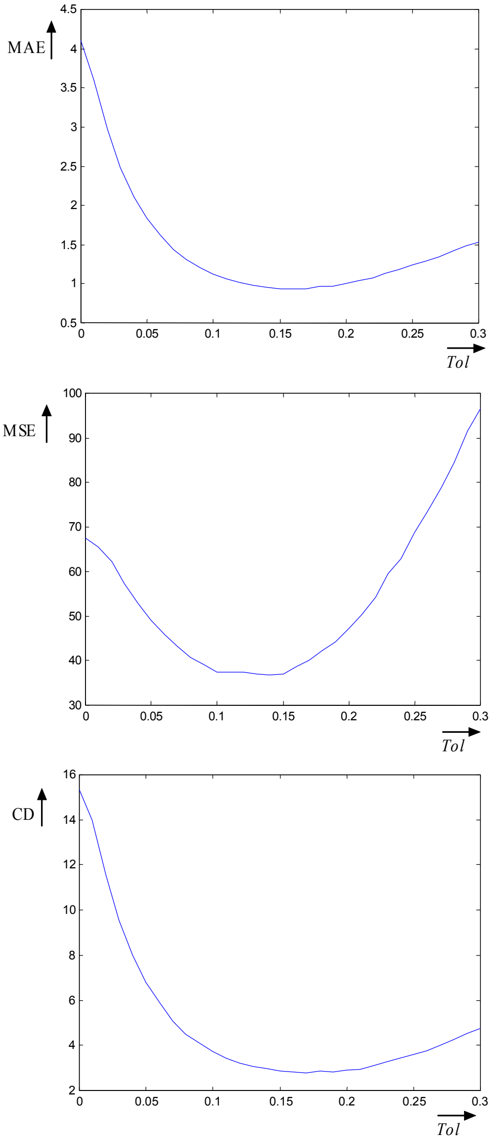

Figure 2 Searching the optimal threshold angle *Tol*

CONCLUSION

This paper has focused on the new vector approach, especially for the impulse noise suppression in color images, based on the alternate change between two smoothing levels, namely between the identity filter and the BVDF. As the switch control has served a set of the smallest vector directional order-statistics and the threshold angle. Since the proposed method processes the noisy images strictly directionally, it provides the excellent signal-details and color chromaticity preservation.

Since the proposed method utilises the fixed threshold angle, the future research should be related to the searching for the adaptive threshold angle controlled by fuzzy logic [14], neural networks [5],[12] with the use of genetic algorithms [10].


(a) (b) (c)

(d) (e) (f)

Figure 3 Achieved results
(a) Original image (b) 10% impulse noise (c) Output of BVDF
(d) Proposed method ($r = 9$, $Tol = 0.18$) (e) Proposed method ($r = 7$, $Tol = 0.13$) (f) Proposed method ($r = 5$, $Tol = 0.16$)

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