

# APPLICATION OF THE OVERCOMPLETE WAVELET TRANSFORM FOR UNDERWATER TRANSITORY SIGNAL CHARACTERIZATION

Cornel Ioana, Krystel Frezza-Boin, André Quinquis

ENSIETA, 2 rue François Verny, Brest - FRANCE

E-mail : ioanaco@ensieta.fr, frezzakr@ensieta.fr , quinquis@ensieta.fr

## ABSTRACT

In this paper we consider the problem of transitory signal characterization, extracted from the underwater environment. The main difficulty which appears in this field is due to the transient behavior of the received signals and to the noise level of the underwater environment. In fact, in the received signal structure a transient part occupies only a small zone and its capture can be done by an adaptive detection stage. Furthermore, we are interesting to characterize the extracted part in both time and frequency domains.

Consequently, we propose an adaptive time-frequency method based on the over-complete wavelet transform concept, in which case an irregular sampling procedure will be involved. This procedure uses a method based on the fourth order moment, applied for each sub-band, in order to establish the optimal weight for each sample. The obtained results for real data prove the capability of the proposed approach to accurately characterize an underwater transient signal, comparatively with the classical methods (spectrogram, for example).

## 1. INTRODUCTION

Since the last ten years, the characterization of underwater environment is a current topic very challenging, due to the richness of the potential information that can be extracted for navigation or communications, for example. One of the major method is the active tomography, which provides an environmental characterization using a man-made emitted signal. Nevertheless, it is possible to imagine the passive tomography concept which will benefit by the generated signals by the natural sources (opportunity sources). In this case, there are two major problems that can be solved. Firstly the processing system must be able to accurately detect the transient parts of the signal (figure 1). One of the most performant detection methods is based on the joint use of the wavelet techniques and high order statistical measurement. In the passive tomography context we are not only interested to detect the useful parts of the observation, but also to characterize them (the second problem). Consequently, it is necessary to use a method which could be able to extract the useful information about the processed data, knowing that the underwater environment is highly non-stationary. In this context, the use of time-frequency methods [2] can be a potential

solution. This class of methods must be able to provide a suggestive information about the signal structure. Currently, this information is provided on the time-frequency image form (see figure 1) and, the quality of this image strongly influences the performances of the following processing stages.

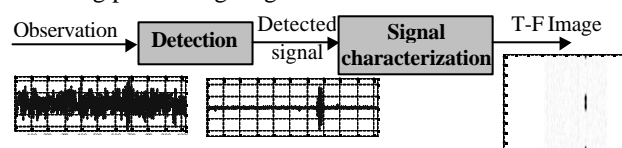


Figure 1. Processing system for underwater transitory signal

In this work we propose a method based on the Over Complete Wavelet Transform (OCWT) which leads to signal processing on interest frequency sub-bands. In each of them, an irregular sampling procedure will be used, in order to optimally detect the useful signal features. The results will be done in a time-frequency image form corresponding to the frequency content variation over time.

The organization of this paper is as follows. In section 2 we briefly present the OCWT concept. In section 3 we propose a new irregular sampling procedure, based on a *split and merge* algorithm. As we will see, the *kurtosis* will be used as a cost function. In section 4 we will study the performances of our approach, using the real underwater mammals signals. Beside, we will compare the obtained results with the ones obtained by the classical method (Spectrogram, Wigner-Ville Distribution) use. Section 5 - "Conclusion" - highlights the significance of the results and the realistic perspectives.

## 2. OVER-COMPLETE WAVELET TRANSFORM

In many applications, due to their remarkable procedures, the discrete wavelet transform (DWT) has been extensively used [3].

From a mathematical point of view, the DWT is generated by sampling, in the time-scale plane, of a corresponding continuous wavelet transform (relation 1).

$$(W_g f)(t, s) = \int_{-\infty}^{\infty} f(u) g^*(s(t-u)) du \quad (1)$$

where  $g$  is the analyzing wavelet,  $f$  is a given signal,  $\mathbf{t}_t$  is the translation operator ( $(\mathbf{t}_t f)(u) = f(u-t)$ ) and  $D_s$  is the scale operator ( $(D_s f)(u) = \frac{1}{s} f\left(\frac{u}{s}\right)$ ). Despite the fact

that there is an infinity of possible discretization of the

CWT, the term *discrete wavelet transform (DWT)* is commonly used to mean the one associated with the dyadic sampling lattice.

$$\Gamma_D \triangleq \{(2^{-n} m, 2^n)\}_{m,n \in \mathbb{Z}} \quad (2)$$

for certain analyzing wavelets that give rise to wavelet orthonormal basis.

In practice, it was observed, that the use of orthonormal representation is not necessarily well suited for a given signal processing problem [4]. For example, by regular sampling, used to compute the MRA, we can loose the signal characteristics, represented by its maxima. Intuitively, we can see this phenomenon in the next figure.

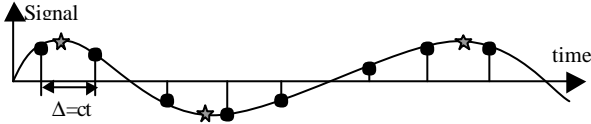


Figure 2. Regular sampling grid (DWT case)

Consequently, the orthonormal representation drawbacks are due to the dyadic grid. In order to eliminate them, the key point is the use of a non-dyadic sampling structure, which is the case of the OCWT [5]. This method is composed by two stages :

**I.** Firstly, we decompose the signal with the linear filter bank structure. The impulse responses of the filter bank are determined by the analyzing wavelet  $g$  and the scale samples  $s_m$ . The filtering stage result is presented in the next equation.

$$W_g f(t, s_m) = (f * D_{s_m} g^*)(t) \quad (3)$$

Here  $s$  is a scaling index which controls the filter bandwidth and the central frequency of each filter. In addition, we can control the overlapping between the filter transfer functions (3.b). For  $s=2$ , we obtain the filter bank structure used for the DWT computation; a filter bank example is shown in 4.a., using the Morlet wavelet as the analyzing function, which has the following analytical expression :

$$g_{Morlet}(t) = \frac{1}{\sqrt{p g_b}} e^{j 2 p g_c t - (t^2 / g_b)} \quad (4)$$

where  $g_b$  and  $g_c$  are the bandwidth and the frequency center of the  $g$  Fourier transform.

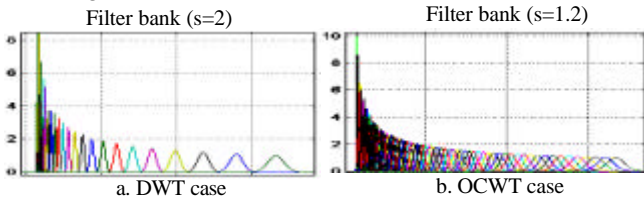


Figure 3. Comparison between filter banks at different scales

**II.** In the second stage we will sample the signal issue at the filter bank output. We take into account the samples at discrete times given by  $\{t_{m,n}\}$ . The full algorithm to compute the OCWT is shown in the next figure.

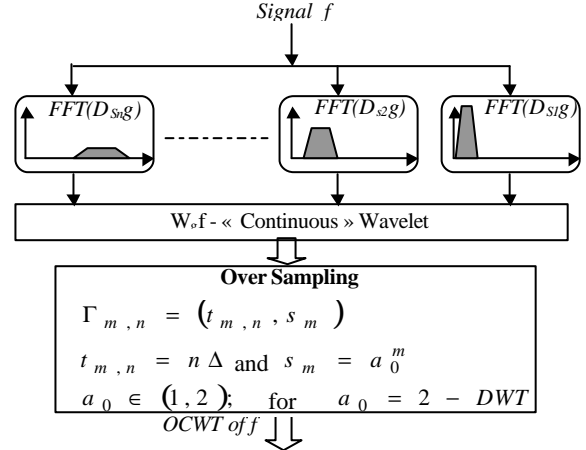


Figure 4. OCWT algorithm

Mathematically, OCWT may be interpreted as the CWT sampled version of the signal by a non-dyadic structure. Usually, we use the semi-logarithmic regular sampling, given by the next definition [4].

$$\Gamma(\Delta, a_0) = \{n\Delta\} \times \{a_0^m\}, 2 > a_0 > 1, \Delta > 0 \quad (5)$$

Consequently, the  $a_0$  parameter controls the filter overlapping, and, implicitly, the redundancy degree. If  $a_0$  is 2, the redundancy will be null : the wavelet basis will generate an orthonormal reconstruction error null, but the extraction of the signal characteristics is not guaranteed. If  $a_0 < 2$ , the wavelet function set will be a frame : the reconstruction is not perfect but we can adapt our distribution to the signal time-frequency structure.

### 3. IRREGULAR SAMPLING PROCEDURE

Generally speaking, there are some advantages to adopting an irregular sampling strategy in a representation. Many of these advantages are inherited from the ability of an irregular sampling to be sensitive to a signal time-frequency behaviors. This thing is illustrated on the figure 2 : by using an irregular sampling grid (marked by a star), we are able to extract the local maxima. The theoretical frame of the irregular sampling strategies is presented in [5] and some applications (for noise suppression, digital communication, compression, etc.) are presented in [4]. In this section we introduce a new irregular sampling technique, well adapted for transient signal detection, in a noisy environment. This technique will be applied to the corresponding waveform, provided by OCWT for each frequency channel.

It is well known [6] that the non-gaussian wavelet coefficients provide a large value of a fourth order statistic moment (*kurtosis*). On the other hand, the noise coefficients, which currently have a gaussian probability density function, provide a small value of the *kurtosis*. The kurtosis allows us to discriminate between the useful (transitory) and useless (noise) parts of the signal. This principle will be applied to detect an optimal sampling grid for each signal's representation, issued from OCWT.

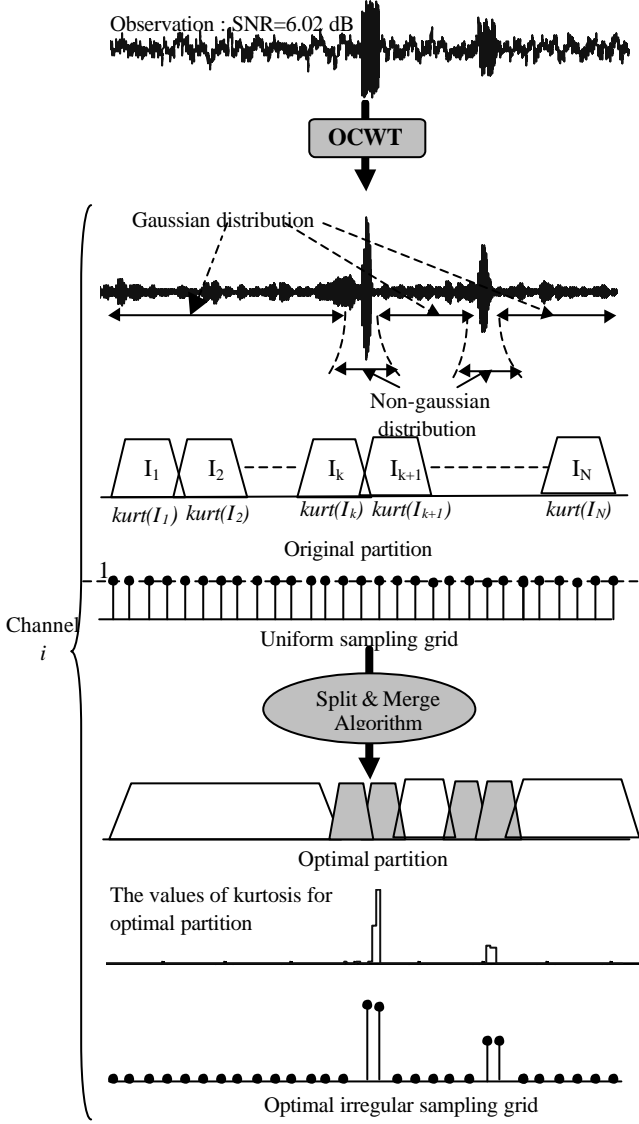


Figure 5. The Irregular sampling procedure for channel  $i$

In the above figure we present the principle of the irregular sampling procedure. First, the waveform issued from  $i$ -channel of the OCWT filter bank (figure 3.b.) is uniformly partitioned in equal length intervals. For each of them the value of kurtosis is estimated, using the following relation [6]:

$$kurt(I_k) = N \left( \frac{\sum_{i=1}^N x_i^4}{\sum_{i=1}^N x_i^2} \right)^2 ; \{x_i\} \in I_k \quad (6)$$

Using these values, we apply an iterative split & merge algorithm in order to establish the optimal partition.

For each two adjacent intervals  $I_k$  et  $I_{k+1}$ , we test the following condition :

$$\begin{aligned} H_0 : & \text{ if } kurt(I_j) \leq m_s \text{ \& } kurt(I_{j+1}) \leq m_s \Rightarrow I'_j = I_j \cup I_{j+1} \\ & \Rightarrow kurt(I'_j) = \max[kurt(I_j), kurt(I_{j+1})] \\ H_1 : & \text{ if } kurt(I_j) > m_s \text{ or } kurt(I_{j+1}) > m_s \Rightarrow \text{the intervals will} \\ & \text{ be conserved} \end{aligned}$$

The  $H_0$  hypothesis states that there is no useful part in the considered intervals, so, these ones will be *merged* (fusion). Alternatively, The  $H_1$  hypothesis states that one or both intervals are subject to the useful parts of signals and will be *conserved*. The algorithm runs until no fusion is possible.

The involved threshold  $m_s$  is computed for each channel using the following formula [1,6] :

$$m_s = \frac{1}{\sqrt{1-a}} \sqrt{a_0^s \frac{24}{N}} \quad (7)$$

where  $a$  - is a confidence degree [6],  $a_0$  is the overlapped degree (see the previous section),  $N$  is a sequence length and  $s$  is the channel index ( $s=1: \text{Number\_of\_channels}$ ).

Finally, we obtain an optimal partition (figure 5) and the values of the kurtosis for the optimal partition. The obtained curve weights the samples of the supposed waveform, ensuring an irregular sampling of this one : the samples associated to transient parts of signal will be "highlighted", whereas the ones associated to noise will be almost precluded. This effect is illustrated in the figure 5. We consider two chirps atoms (both on 128 samples), mixed with real oceanic noise (SNR=6.02 dB). After the OCWT (the number of the channels is 128) we apply the method to the extracted waveform from the channel 120. The values of kurtosis for the optimal partition provide an optimal sampling grid which improves the representation quality. Repeating the same algorithm for all OCWT channels, we obtain a two-dimensional irregular sampling grid which lead to an optimal time-frequency representation. On the other hand, by unifying the kurtosis curves of the all sub-bands we obtain *the detection curve*, which gives an information about temporal localization of the transient parts of the signals. For the considered test signal the detection curve is shown in the next figure.

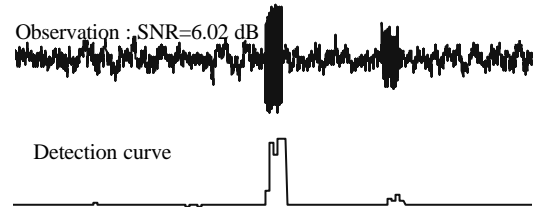


Figure 6. Detection curve, obtained via OCWT

#### 4. RESULTS

We have tested our approach with real data corresponding to the signal emitted by a long-finned pilot whale (*Globicephala melas*). The sampling frequency is 44.1 kHz and we have taken into account an observation of 5.92 seconds. The test signal is presented in the figure 7.a.

Ones of the most employed methods for the underwater signal processing signals are the spectrogram and the Wigner-Ville Distribution [2]. In this case, the obtained results are presented in the figure 7.b, c. Finally, the obtained result using our approach is depicted in the figure 7.d.

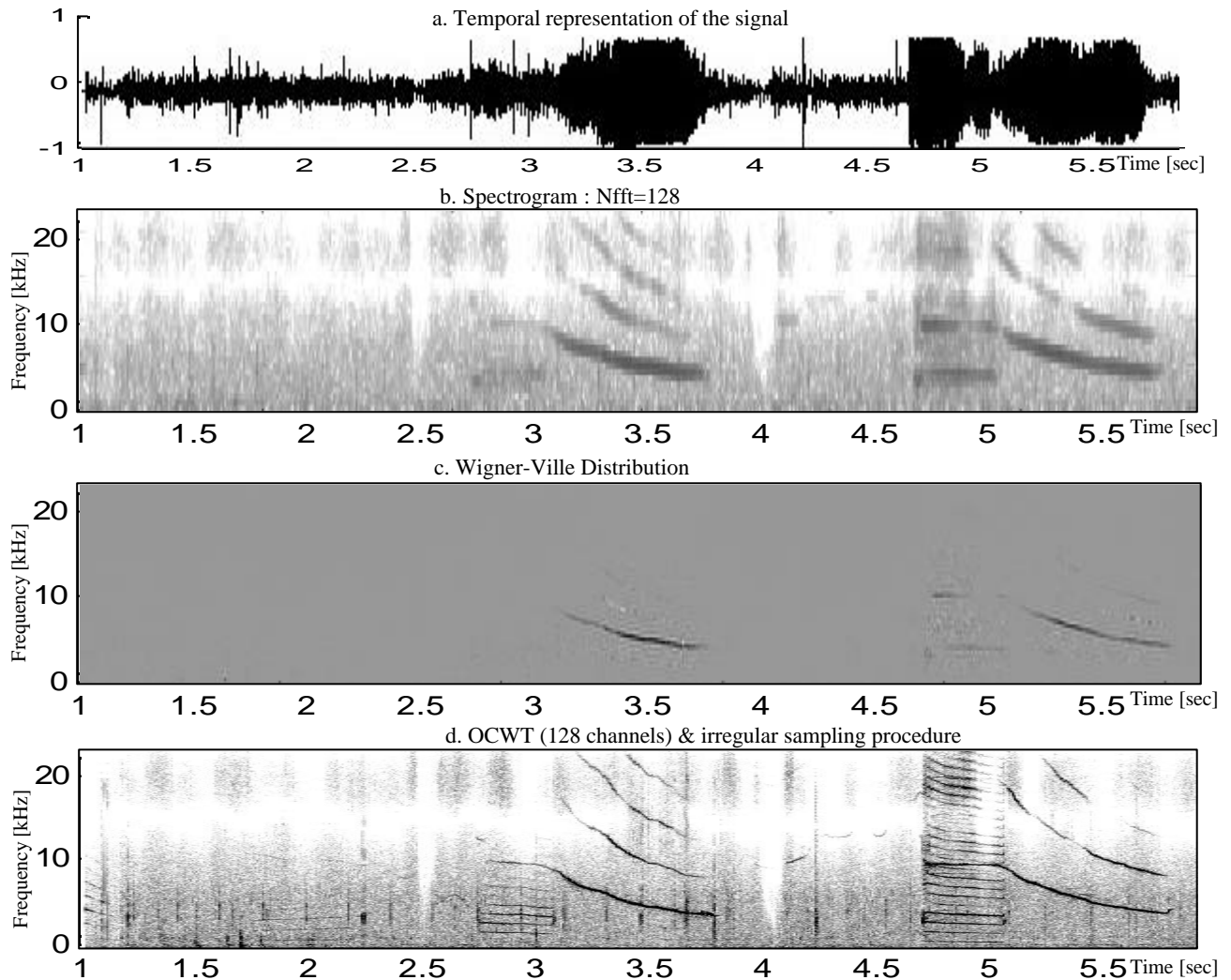


Figure 7. Comparative results for real analysis data

## 5. CONCLUSIONS

The results presented in the above figure highlight the superiority of the proposed method over the classical time-frequency ones. The obtained time-frequency image (7.d.) provides a complete and satisfactory information about time-frequency behaviors of the considered signals. Consequently, due to its good readability, it may be successfully used for a further feature extraction algorithm.

On the other hand, the classical methods fall, for specifically reasons : in the spectrogram case, there is a trade off between time and frequency resolutions which affects the signal feature representation. In the Wigner-Ville distribution case, the interference term creates wrong features, no correlated to the physical process.

So, we have experimentally proved the method based on the OCWT and the irregular sampling procedure allows an substantial improvement of the time-frequency information. In further works, we intend to use this

algorithm as a feature extraction method in the context of underwater transient signal classification.

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