PERFORMANCE OF SPACE-FREQUENCY CODED BROADBAND OFDM UNDER REAL-WORLD PROPAGATION CONDITIONS

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ABSTRACT

In this paper, using a spatial broadband channel model taking into account transmit and receive antenna correlation, we study the influence of propagation conditions on the performance of space-frequency coded OFDM. For a given space-frequency code, we quantify the achievable diversity order and coding gain as a function of the transmit and receive correlation matrices. We find that the presence of transmit correlation results in widely varying performance losses. High-rate space-frequency codes such as spatial multiplexing are typically significantly more affected by transmit correlation than low-rate codes such as space-frequency block codes.

I. INTRODUCTION AND OUTLINE

In recent years the use of spatial (or antenna) diversity has become increasingly popular, which is mostly due to the fact that it can be provided without loss in spectral efficiency. Space-time coding [1], [2], [3] has evolved as a promising technique for improving link reliability in systems employing multiple transmit antennas.

Broadband multiple-input multiple-output (MIMO) antenna channels with delay spread offer spatial diversity as well as frequency diversity. Orthogonal frequency division multiplexing (OFDM) [4] significantly reduces receiver complexity in broadband MIMO wireless systems [5], [6]. Space-frequency coded MIMO-OFDM [7], [8] therefore seems to be a particularly promising technology for future broadband wireless systems.

Contributions. In this paper, using a physically motivated broadband MIMO channel model taking into account transmit and receive antenna correlation, we study the impact of propagation conditions on the performance of space-frequency coded OFDM. Our discussion incorporates space-frequency codes as well as OFDM-based spatial multiplexing [5], [6]. In the remainder of this paper we shall refer to both signaling techniques as space-frequency coding. Our contributions are summarized as follows.

- We extend the results reported in [7] to incorporate transmit correlation and derive the error rate behavior of space-frequency coded OFDM as a function of transmit and receive angle spread and antenna spacing.
- For a given space-frequency code, we quantify the achievable diversity order and coding gain as a function of the transmit and receive correlation matrices.
- It is shown that the presence of transmit correlation results in widely varying performance losses. In particular, we find that high-rate space-frequency codes such as spatial multiplexing are typically significantly more affected by transmit correlation than low-rate codes such as space-frequency block codes.

Organization of the paper. The rest of this paper is organized as follows. In Sec. 2, we introduce the channel model, and we briefly review space-frequency coded OFDM. In Sec. 3, we derive the error rate performance of space-frequency coded OFDM in the presence of transmit and receive antenna correlation, and we quantify the achievable diversity order and coding gain as a function of the propagation parameters. Sec. 4 contains simulation results, and Sec. 5 presents our conclusions.

II. CHANNEL MODEL AND SPACE-FREQUENCY CODING

In this section, we shall first introduce a broadband MIMO channel model taking into account transmit and receive antenna correlation, and then briefly review space-frequency coded OFDM.

II-A. The Channel Model

Our channel model is a simple extension of the model proposed in [6] to account for transmit correlation as well. In the following M_T and M_R denote the number of transmit and receive antennas, respectively. We assume that the $M_R \times M_T$ matrix-valued channel impulse response consists of L taps with transfer function given by

$$\mathbf{H}(e^{j2\pi\theta}) = \sum_{l=0}^{L-1} \mathbf{H}_l \, e^{-j2\pi l\theta}, \quad 0 \leq \theta < 1, \tag{1}$$

where the $M_R \times M_T$ random matrix $\mathbf{H}_l$ represents the l-th tap. The entries of $\mathbf{H}_l$ $(l = 0, 1, ..., L-1)$ are assumed circularly symmetric zero-mean complex Gaussian[1] (Rayleigh fading assumption). One can think of each of the taps as representing a significant scatterer cluster [6] with each of the paths emanating from within the same scatterer cluster experiencing the same delay. Each scatterer cluster has a mean angle of departure from the transmit array and a mean angle of arrival at the receive array denoted as $\bar{\theta}_{T,l}$ and $\bar{\theta}_{R,l}$, respectively, a cluster angle spread as seen by the transmitter $\delta_{T,l}$ (proportional to the scattering radius of the cluster as observed by the transmitter), and a cluster angle spread as seen by the receiver $\delta_{R,l}$ (proportional to the scattering radius of the cluster as observed by the receiver). We refer to the spread of the $\bar{\theta}_l$ as the total angle spread. Different scatterer clusters are assumed uncorrelated. Spatial fading correlation can occur both at the transmitter and the receiver and is modeled by decomposing the l-th tap according to

$$\mathbf{H}_l = \mathbf{R}_l^{1/2} \mathbf{H}_{w,l} \mathbf{S}_l^{1/2}, \quad l = 0, 1, ..., L-1,$$

where $\mathbf{R}_l = \mathbf{R}_l^{1/2} \mathbf{R}_l^{1/2}$ and $\mathbf{S}_l = \mathbf{S}_l^{1/2} \mathbf{S}_l^{1/2}$ are the receive and transmit correlation matrices, respectively, and the

[1] A circularly symmetric complex Gaussian random variable is a random variable $z = (x + jy) \sim \mathcal{CN}(0, \sigma^2)$, where x and y are i.i.d. $\mathcal{N}(0, \sigma^2/2)$.

$\mathbf{H}_{w,l}$ are $M_R \times M_T$ matrices with i.i.d. $\mathcal{CN}(0, \sigma_l^2)$ entries. Note that the power delay profile σ_l^2 has been incorporated into the matrices $\mathbf{H}_{w,l}$. We assume uniform linear arrays at both the transmitter and the receiver. The relative antenna spacing is denoted as $\Delta_T = \frac{d_T}{\lambda}$ at the transmitter and $\Delta_R = \frac{d_R}{\lambda}$ at the receiver, where d_T and d_R stand for absolute antenna spacing at transmitter and receiver, respectively, and $\lambda = c/f_c$ is the wavelength of a narrowband signal with center frequency f_c.

Defining $\rho(s\Delta, \bar{\theta}, \delta)$ to be the fading correlation between two antenna elements spaced $s\Delta$ wavelengths apart, the correlation matrices $\mathbf{R}_l$ and $\mathbf{S}_l$ are given by

$$[\mathbf{R}_l]_{m,n} = \rho((n-m)\Delta_R, \bar{\theta}_{R,l}, \delta_{R,l}) \tag{2}$$

$$[\mathbf{S}_l]_{m,n} = \rho((m-n)\Delta_T, \bar{\theta}_{T,l}, \delta_{T,l}). \tag{3}$$

Assuming that for each scatterer cluster the angle of departure and the angle of arrival is Gaussian distributed around the mean angle of departure $\bar{\theta}_{T,l}$ and the mean angle of arrival $\bar{\theta}_{R,l}$, respectively, we get the following approximation valid for small angular spread [9]

$$\rho(s\Delta, \bar{\theta}_l, \delta_l) \approx e^{-j2\pi s\Delta\cos(\bar{\theta}_l)} e^{-\frac{1}{2}(2\pi s\Delta\sin(\bar{\theta}_l)\sigma_{\theta_l})^2} \tag{4}$$

with the variances $\sigma^2_{\theta_{T,l}}$ and $\sigma^2_{\theta_{R,l}}$ proportional to the angular spreads $\delta_{T,l}$ and $\delta_{R,l}$, respectively. We note that although the approximation (4) is accurate only for small angular spread, it does provide the correct trend for large angular spread, namely uncorrelated spatial fading.

II-B. Space-Frequency Coding

In a MIMO-OFDM system with N tones the frequency-selective fading channel decouples into N flat-fading channels with input-output relations

$$\mathbf{r}_k = \sqrt{E_s}\,\mathbf{H}(e^{j\frac{2\pi}{N}k})\,\mathbf{c}_k + \mathbf{n}_k, \quad k = 0, 1, ..., N-1, \tag{5}$$

where $\mathbf{c}_k = [c_k^{(0)}\ c_k^{(1)}\ \dots\ c_k^{(M_T-1)}]^T$ with $c_k^{(i)}$ denoting the data symbol transmitted from the i-th antenna on the k-th tone, and $\mathbf{n}_k$ is complex-valued additive white Gaussian noise satisfying

$$\mathcal{E}\{\mathbf{n}_k\mathbf{n}_l^H\} = \sigma_n^2\mathbf{I}_{M_R}\delta[k-l] \tag{6}$$

with $\mathbf{I}_{M_R}$ denoting the identity matrix of size M_R. The data symbols $c_k^{(i)}$ are taken from a finite complex alphabet chosen such that the average energy of the constellation elements is 1, and E_s is an energy normalization factor.

The bit stream to be transmitted is encoded by the space-frequency encoder into blocks of size $M_T \times N$. The channel is assumed to be constant over at least one OFDM symbol. Assuming perfect channel state information, the maximum likelihood (ML) decoder computes $\widehat{\mathbf{C}} = [\hat{\mathbf{c}}_0\ \hat{\mathbf{c}}_1\ \dots\ \hat{\mathbf{c}}_{N-1}]$ according to

$$\widehat{\mathbf{C}} = \arg\min_{\mathbf{C}} \sum_{k=0}^{N-1} \|\mathbf{r}_k - \sqrt{E_s}\mathbf{H}(e^{j\frac{2\pi}{N}k})\,\mathbf{c}_k\|^2,$$

where $\mathbf{C} = [\mathbf{c}_0\ \mathbf{c}_1\ \dots\ \mathbf{c}_{N-1}]$ and the minimization is over all possible codeword matrices. Throughout the paper, we assume that the receiver has perfect channel state information.

III. ERROR RATE PERFORMANCE OF SPACE-FREQUENCY CODES

In this section, extending the results in [7], we shall first derive the average (with respect to the random channel) pairwise error probability for space-frequency codes taking into account the channel model introduced in the previous section. We shall then quantify the maximum achievable diversity order and coding gain for various propagation scenarios.

Let $\mathbf{C} = [\mathbf{c}_0\ \mathbf{c}_1\ \dots\ \mathbf{c}_{N-1}]$ and $\mathbf{E} = [\mathbf{e}_0\ \mathbf{e}_1\ \dots\ \mathbf{e}_{N-1}]$ be two different space-frequency codewords of size $M_T \times N$. The average (with respect to the random channel) probability that the receiver decides erroneously in favor of the signal $\mathbf{E}$ assuming that $\mathbf{C}$ was transmitted can be upper-bounded as[2]

$$P(\mathbf{C} \to \mathbf{E}) \le \prod_{i=0}^{r(\mathbf{A}(\mathbf{C},\mathbf{E}))-1} \left(1 + \lambda_i(\mathbf{A}(\mathbf{C},\mathbf{E}))\frac{E_s}{4\sigma_n^2}\right)^{-1}, \tag{7}$$

where[3]

$$\mathbf{A}(\mathbf{C},\mathbf{E}) = \sum_{l=0}^{L-1} \sigma_l^2 \left[\mathbf{D}^l(\mathbf{C}-\mathbf{E})^T\mathbf{S}_l^T(\mathbf{C}-\mathbf{E})^*\mathbf{D}^{-l}\right] \otimes \mathbf{R}_l \tag{8}$$

with $\mathbf{D} = \mathrm{diag}\{e^{-j\frac{2\pi}{N}k}\}_{k=0}^{N-1}$, and $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of the matrices $\mathbf{A}$ and $\mathbf{B}$.

Receive correlation only. In this case $\mathbf{S}_l = \mathbf{I}_{M_T}$ for $l = 0, 1, ..., L-1$ and (8) reduces to

$$\mathbf{A}(\mathbf{C},\mathbf{E}) = \sum_{l=0}^{L-1} \sigma_l^2 \left[\mathbf{D}^l(\mathbf{C}-\mathbf{E})^T(\mathbf{C}-\mathbf{E})^*\mathbf{D}^{-l}\right] \otimes \mathbf{R}_l.$$

In [10] it is shown that in the presence of receive correlation only, the PEP upper bound can be expressed as

$$P(\mathbf{C} \to \mathbf{E}) \le \prod_{i=0}^{r(\mathbf{A}_u(\mathbf{C},\mathbf{E}))M_R-1} \frac{1}{1 + \frac{E_s}{4\sigma_n^2}\theta_i\lambda_i(\mathbf{A}_u(\mathbf{C},\mathbf{E}) \otimes \mathbf{I}_{M_R})}, \tag{9}$$

where $\mathbf{R}_l = \mathbf{U}_l\mathbf{\Sigma}_l\mathbf{U}_l^H$, $\bar{\mathbf{\Sigma}} = \mathrm{diag}\{\mathbf{\Sigma}_l\}_{l=0}^{L-1}$, $\lambda_{min}(\bar{\mathbf{\Sigma}}) \le \theta_i \le \lambda_{max}(\bar{\mathbf{\Sigma}})$,

$$\mathbf{A}_u(\mathbf{C},\mathbf{E}) = \sum_{l=0}^{L-1} \sigma_l^2[\mathbf{D}^l(\mathbf{C}-\mathbf{E})^T(\mathbf{C}-\mathbf{E})^*\mathbf{D}^{-l}], \tag{10}$$

and $\lambda_i(\mathbf{A}_u(\mathbf{C},\mathbf{E}) \otimes \mathbf{I}_{M_R})$ stands for the nonzero eigenvalues of $\mathbf{A}_u(\mathbf{C},\mathbf{E}) \otimes \mathbf{I}_{M_R}$. Using a standard result on Kronecker products [11], it follows furthermore that every eigenvalue $\lambda_i(\mathbf{A}_u(\mathbf{C},\mathbf{E}))$ is an eigenvalue of $\mathbf{A}_u(\mathbf{C},\mathbf{E}) \otimes \mathbf{I}_{M_R}$ with multiplicity M_R.

The results found thus far have a number of important implications which we shall discuss in the following. Assuming that all the correlation matrices $\mathbf{R}_l$ $(l = 0, 1, ..., L-1)$ have full rank and using (9) we get the following high SNR $\left(\frac{E_s}{4\sigma_n^2} \gg 1\right)$ PEP upper bound

$$P(\mathbf{C} \to \mathbf{E}) \le \left(\frac{E_s}{4\sigma_n^2}\right)^{-r(\mathbf{A}_u(\mathbf{C},\mathbf{E}))M_R} \prod_{i=0}^{r(\mathbf{A}_u(\mathbf{C},\mathbf{E}))M_R-1} \theta_i^{-1} \prod_{i=0}^{r(\mathbf{A}_u(\mathbf{C},\mathbf{E}))-1} \lambda_i(\mathbf{A}_u(\mathbf{C},\mathbf{E}))^{-M_R}.$$

Note that $\theta_i > 0$ for $i = 0, 1, ..., r(\mathbf{A}_u(\mathbf{C},\mathbf{E}))M_R - 1$ since we assumed that all the $\mathbf{R}_l$ are full rank. Since the diversity order achieved by a space-frequency code in the i.i.d. case is given by $M_R \min_{\{\mathbf{C},\mathbf{E}\}} r(\mathbf{A}_u(\mathbf{C},\mathbf{E}))$ where the minimum is taken over all codeword matrix pairs $\{\mathbf{C},\mathbf{E}\}$, we can conclude that the diversity order achieved in the presence of receive correlation only with all correlation matrices full rank is equal to the diversity order achieved by the code in

[2] $r(\mathbf{A})$ stands for the rank of the matrix $\mathbf{A}$.

[3] The superscript * stands for elementwise conjugation.

the i.i.d. case. The coding gain in the presence of receive correlation only is given by the coding gain of the space-frequency code achieved in the i.i.d. case multiplied by the factor $\prod_{i=0}^{r(\mathbf{A}_u(\mathbf{C},\mathbf{E}))M_R-1} \theta_i^{-1}$.

Retaining the assumption of all the $\mathbf{R}_l$ being full rank, let us next assume that the space-frequency code was designed to achieve full space-frequency diversity, i.e., $r(\mathbf{A}_u(\mathbf{C},\mathbf{E})) = M_T L$ for all pairs of codeword matrices $\{\mathbf{C},\mathbf{E}\}$. In this case, in the high SNR regime [10]

$$P(\mathbf{C} \rightarrow \mathbf{E}) \le \left(\frac{E_s}{4\sigma_n^2}\right)^{-M_T M_R L} \prod_{i=0}^{L-1} \det(\mathbf{R}_i)^{-M_T} \prod_{i=0}^{M_T L-1} \lambda_i(\mathbf{A}_u(\mathbf{C},\mathbf{E}))^{-M_R}.$$

We thus conclude that the diversity order achieved by the space-frequency code in the presence of receive correlation only is $M_T M_R L$, i.e., the code achieves full space-frequency diversity. The coding gain is given by the coding gain achieved in the i.i.d. case multiplied by $\prod_{i=0}^{L-1} \det(\mathbf{R}_i)^{-M_T}$, which is upper-bounded by 1 assuming that the correlation matrices $\mathbf{R}_l$ are normalized according to[4] $\mathrm{Tr}(\bar{\mathbf{\Sigma}}) = M_R L$. The upper bound is achieved (i.e. no performance loss) if $\mathbf{R}_l = \mathbf{I}_{M_R}$ for $l = 0, 1, ..., L-1$ or equivalently if spatial fading is uncorrelated. The presence of receive correlation thus always leads to a performance loss with the loss in coding gain quantified by $\prod_{i=0}^{L-1} \det(\mathbf{R}_i)^{-M_T}$. We emphasize that this loss is independent of the particular space-frequency code employed.

Transmit correlation only. In the case of transmit correlation only, $\mathbf{R}_l = \mathbf{I}_{M_R}$ for $l = 0, 1, ..., L-1$, and (8) reduces to

$$\mathbf{A}(\mathbf{C},\mathbf{E}) = \underbrace{\sum_{l=0}^{L-1} \sigma_l^2 \left[\mathbf{D}^l (\mathbf{C}-\mathbf{E})^T \mathbf{S}_l^T (\mathbf{C}-\mathbf{E})^* \mathbf{D}^{-l}\right]}_{\mathbf{A}_{tc}(\mathbf{C},\mathbf{E})} \otimes \mathbf{I}_{M_R}.$$

Denoting $\alpha = r(\mathbf{A}_{tc}(\mathbf{C},\mathbf{E}))$, we get a high-SNR PEP upper bound as

$$P(\mathbf{C} \rightarrow \mathbf{E}) \le \left(\frac{E_s}{4\sigma_n^2}\right)^{-\alpha M_R} \prod_{i=0}^{\alpha-1} \lambda_i(\mathbf{A}_{tc}(\mathbf{C},\mathbf{E}))^{-M_R}. \tag{11}$$

In [10] the dependence of the average PEP on the eigenvalues of $\mathbf{S}_l$ is made explicit, assuming that all $\mathbf{S}_l$ are full rank. Denoting s as the minimum rank of $\mathbf{A}_u(\mathbf{C},\mathbf{E})$ over all pairs of codeword matrices $\{\mathbf{C},\mathbf{E}\}$ (i.e. s is the diversity gain achieved in the i.i.d. case) and focusing on a minimum rank codeword pair, we get

$$P(\mathbf{C} \rightarrow \mathbf{E}) \le \left(\frac{E_s}{4\sigma_n^2}\right)^{-s M_R} \prod_{i=0}^{s-1} (\theta_i \lambda_i(\mathbf{A}_u(\mathbf{C},\mathbf{E})))^{-M_R},$$

where $\lambda_i(\mathbf{A}_u(\mathbf{C},\mathbf{E}))$ denotes the nonzero eigenvalues of $\mathbf{A}_u(\mathbf{C},\mathbf{E})$, $\lambda_{min}(\bar{\mathbf{S}}) \le \theta_i \le \lambda_{max}(\bar{\mathbf{S}})$ with $\bar{\mathbf{S}} = \mathrm{diag}\{\mathbf{S}_l\}_{l=0}^{L-1}$, and we have used the fact that $\alpha = s$ for nonsingular $\bar{\mathbf{S}}$. For general $\bar{\mathbf{S}}$ the achievable diversity order is given by

$$d = \alpha M_R. \tag{12}$$

Using basic results on the rank of matrix products, we get [12]

$$s + r(\bar{\mathbf{S}}) - M_T L \le \alpha \le \min\{s, r(\bar{\mathbf{S}})\}. \tag{13}$$

[4] $\mathrm{Tr}(\mathbf{A})$ stands for the trace of the matrix $\mathbf{A}$.

For full rank transmit correlation matrices $\mathbf{S}_l$, i.e., $r(\bar{\mathbf{S}}) = M_T L$, we get $d = s M_R$. For a space-frequency code achieving full diversity gain in the i.i.d. case, $s = M_T L$ and hence $d = r(\bar{\mathbf{S}}) M_R$. In the latter case, if the rank of $\bar{\mathbf{S}}$ is reduced by 1, we lose M_R degrees of freedom. We note that (13) shows that if $\bar{\mathbf{S}}$ is singular and the space-frequency code does not achieve full diversity gain in the i.i.d. case, i.e., $s < M_T L$, the diversity order can only be lower-bounded by $(s + r(\bar{\mathbf{S}}) - M_T L) M_R$. In this case it is difficult to make statements on the exact diversity order since the geometry of $\bar{\mathbf{S}}$ and the geometry of the space-frequency code, i.e., the geometry of the code difference matrices play an important role in assessing the exact diversity order. Loosely speaking, in this case it is important that the space-frequency code excites the range space of $\bar{\mathbf{S}}$ in order to obtain good performance in terms of error rate. We conclude by noting that as opposed to the case of receive correlation only, where the performance loss due to spatial fading correlation is independent of the space-frequency code employed, in the presence of transmit correlation this loss in general depends significantly on the specific space-frequency code used. In the next section this observation will be corroborated through simulation results.

IV. SIMULATION RESULTS

Unless specified otherwise, we simulated a MIMO-OFDM system with $M_T = M_R = 2$, $L = 2$, $\Delta_T = \Delta_R = 1$, and $\sigma_0^2 = \sigma_1^2 = 1/2$. The signal-to-noise ratio (SNR) is defined as $\mathrm{SNR} = 10 \log_{10}\left(\frac{2E_s}{\sigma_n^2}\right)$. In all simulations ML decoding was used. We employed the space-frequency coding scheme proposed in [13] where an arbitrary (inner) space-frequency code is used to transmit data in the first $\frac{N}{2}$ tones followed by a repetition of this block. This construction ensures that the additionally available frequency diversity is fully exploited.

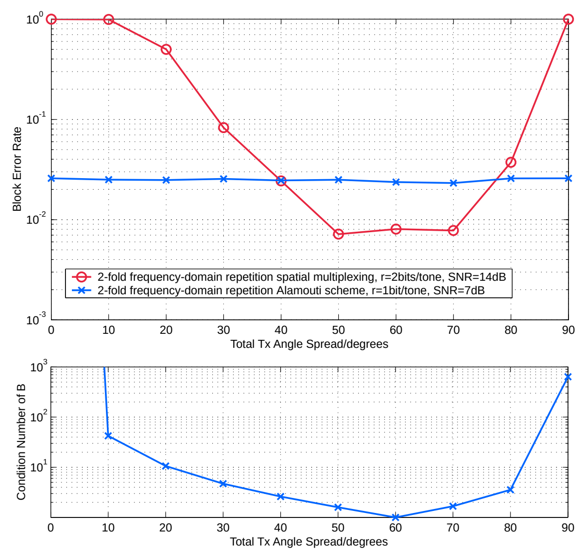

Fig. 1. Impact of total transmit angle spread on performance of space-frequency codes.

Simulation 1. In this simulation example, we study the impact of total transmit angle spread $\Delta\bar{\theta} = \bar{\theta}_1 - \bar{\theta}_0$ in the presence of high transmit correlation (i.e. small transmit antenna spacing) on the performance of space-frequency codes. As inner codes we used QPSK-based spatial multiplexing [14], [15] (which amounts to transmitting independent data symbols on each tone and each antenna) and the Alamouti scheme [3] based on QPSK.

The transmit cluster angle spreads satisfy $\sigma_{\theta_0} = \sigma_{\theta_1} = 0$ so that $\mathbf{S}_l = \mathbf{a}(\bar{\theta}_l)\mathbf{a}^H(\bar{\theta}_l)\,(l = 0, 1)$ with $\mathbf{a}(\theta) = \left[1 \;\; e^{-j2\pi\Delta\cos(\theta)} \;\; \ldots \;\; e^{-j2\pi(M_T-1)\Delta\cos(\theta)}\right]^T$. The top diagram in Fig. 1 shows the block error rate as a function of $\Delta\bar{\theta}$ for spatial multiplexing at an SNR of 14dB and for the Alamouti scheme at an SNR of 7dB, respectively. We can clearly see that spatial multiplexing is very sensitive to total transmit angle spread and that the performance improves significantly for the case where the array response vectors of the two taps $\mathbf{a}(\bar{\theta}_0)$ and $\mathbf{a}(\bar{\theta}_1)$ are close to orthogonal to each other. The bottom diagram in Fig. 1 displaying the condition number of the 2×2 matrix $\mathbf{B} = [\mathbf{a}(\bar{\theta}_0) \;\; \mathbf{a}(\bar{\theta}_1)]$ corroborates this statement. The lower rate Alamouti scheme is virtually unaffected by the total transmit angle spread, which can be explained as follows. The Alamouti scheme orthogonalizes the channel irrespectively of the channel realization; its performance is therefore independent of the rank properties of the channel. For spatial multiplexing the situation is different since the performance depends significantly on the rank of the channel realizations. To be more specific we note the following. Each transmit correlation matrix $\mathbf{S}_l$ spans a one-dimensional subspace. The angle between these one-dimensional subspaces varies with varying total angle spread. If the total angle spread is small, the two subspaces tend to be aligned so that certain transmit signal vectors which happen to lie in the orthogonal complement of this subspace will result in high error probability. If the total angle spread is large, the $\mathbf{S}_l$ tend to span different subspaces and hence if a transmit vector excites the null-space of $\mathbf{S}_0$ it will lie in the range space of $\mathbf{S}_1$ and vice versa. This leads to good PEP performance as none of the transmit vectors will be attenuated by the channel (on average where the average is with respect to the random channel).

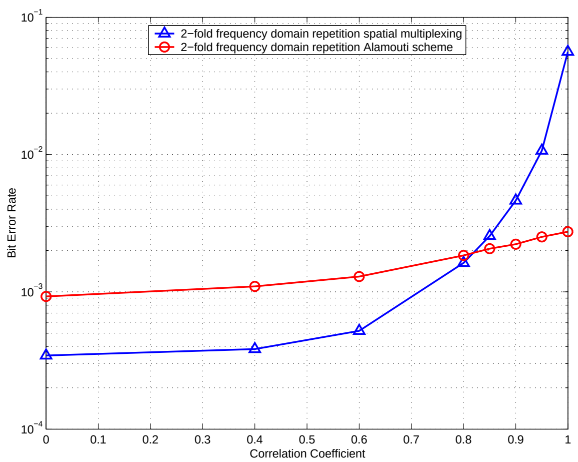

Fig. 2. Impact of varying transmit antenna correlation on performance of space-frequency codes.

Simulation Example 2. This example compares the performance of orthogonal and nonorthogonal space-frequency codes in the presence of transmit correlation. In Sec. 3, we concluded that the impact of transmit antenna correlation is highly dependent on how the code excites the channel (i.e. the code geometry). Given that the transmitter does not know the channel, an orthogonal scheme such as the Alamouti scheme, which excites all spatial directions uniformly, should exhibit maximum robustness with respect to this kind of channel impairment. Nonorthogonal schemes such as spatial multiplexing do not excite all spatial directions uniformly and hence suffer from widely varying performance loss in the presence of transmit correlation. In order to demonstrate this effect, we compared a 16-QAM based Alamouti scheme as a simple orthogonal inner code to spatial multiplexing based on QPSK (nonorthogonal inner code). The total angle spread was set to 0, i.e., $\bar{\theta}_0 = \bar{\theta}_1$. The SNR was kept constant at 12 dB while the absolute value of the transmit antenna correlation coefficient was varied from 0 to 1. Fig. 2 shows the bit error rates assuming Gray encoding. It is clearly seen that whilst with uncorrelated transmit antennas the Alamouti scheme performs inferior compared to spatial multiplexing, performance degradation with increasing correlation is much worse for spatial multiplexing such that at full correlation the Alamouti scheme performs significantly better.

These performance differences along with the results of Simulation Example 1 lead us to the conclusion that space-frequency block codes exhibit superior robustness with respect to varying propagation conditions when compared to high-rate schemes such as spatial multiplexing.

V. CONCLUSION

We studied the impact of the propagation environment on the performance of space-frequency coded broadband OFDM systems. In particular, we derived the error-rate performance of space-frequency codes in the presence of transmit and receive correlation, and we quantified the maximum achievable diversity order and coding gain as a function of the propagation parameters. We found that space-frequency block codes such as the Alamouti scheme exhibit superior robustness with respect to varying propagation conditions when compared to high-rate schemes such as spatial multiplexing. Finally, we provided simulation results.

VI. REFERENCES

[1] J. Guey, M. Fitz, M. Bell, and W. Kuo, "Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels," in *Proc. IEEE VTC*, pp. 136–140, 1996.

[2] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, pp. 744–765, March 1998.

[3] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Comm.*, vol. 16, pp. 1451–1458, Oct. 1998.

[4] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *Proc. IEEE ICASSP-80*, (Denver, CO), pp. 964–967, 1980.

[5] G. G. Raleigh and J. M. Cioffi, "Spatio-temporal coding for wireless communication," *IEEE Trans. Comm.*, vol. 46, no. 3, pp. 357–366, 1998.

[6] H. Bölcskei, D. Gesbert, and A. J. Paulraj, "On the capacity of OFDM-based spatial multiplexing systems," *IEEE Trans. Comm.*, vol. 50, pp. 225–234, Feb. 2002.

[7] H. Bölcskei and A. J. Paulraj, "Space-frequency coded broadband OFDM systems," in *Proc. IEEE WCNC-2000*, vol. 1, (Chicago, IL), pp. 1–6, Sept. 2000.

[8] H. Bölcskei and A. J. Paulraj, "Space-frequency codes for broadband fading channels," in *IEEE International Symposium on Information Theory (ISIT)*, (Washington, D.C.), p. 219, June 2001.

[9] D. Asztély, "On antenna arrays in mobile communication systems: Fast fading and GSM base station receiver algorithms," Tech. Rep. IR-S3-SB-9611, Royal Institute of Technology, Stockholm, Sweden, March 1996.

[10] H. Bölcskei, M. Borgmann, and A. J. Paulraj, "Impact of the propagation environment on the performance of space-frequency coded broadband OFDM systems," *IEEE J. Sel. Areas Comm.*, 2002. to be submitted.

[11] R. A. Horn and C. R. Johnson, *Topics in matrix analysis*. New York: Cambridge Press, 1991.

[12] R. A. Horn and C. R. Johnson, *Matrix analysis*. New York: Cambridge Press, 1985.

[13] H. Bölcskei, M. Borgmann, and A. J. Paulraj, "Space-frequency coded broadband OFDM with variable diversity-multiplexing tradeoff," *IEEE Trans. Comm.*, 2002. to be submitted.

[14] A. J. Paulraj and T. Kailath, "Increasing capacity in wireless broadcast systems using distributed transmission/directional reception," *U. S. Patent*, no. 5,345,599, 1994.

[15] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, pp. 41–59, Autumn 1996.