Sequential Monte Carlo for Mobility Management in Wireless Cellular Networks

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Abstract—We consider the application of sequential Monte Carlo (SM-C) methodology to the problem of joint mobility tracking and soft handoff detection in cellular wireless communication networks based on the pilot signal strength measurements. The dynamics of the system under consideration are described by a nonlinear state-space model. Mobility tracking involves an on-line estimation of the location and velocity of the mobile, whereas handoff detection involves an on-line prediction of the pilot signal strength at future time instant. The optimal solution to both problems is prohibitively complex due to the nonlinear nature of the system. The sequential Monte Carlo (SMC) methods are therefore employed to track the probabilistic dynamics of the system and to make the corresponding estimates and predictions.

I. INTRODUCTION

Mobility tracking and handoff detection are two important issues in mobility management for cellular networks [1], [2]. Since, both mobility tracking and handoff detection are based on the averaged signal strength measured at mobile station (MS), we therefore in this paper consider the problem of joint mobility tracking and soft handoff detection. Such a problem is essentially a problem of on-line estimation and detection in a nonlinear dynamic system.

The problem will be solved under a Bayesian framework, where on-line posterior distribution of the location and velocity will be estimated, and then used for soft handoff detection. However, for the nonlinear dynamic system considered here, an exact evaluation of this posterior distribution is analytically intractable. Therefore, we resort to the sequential Monte Carlo (SMC) technique for numerical computation. The sequential Monte Carlo (SMC) methodology [3] recently emerged in the fields of statistics and engineering, has shown a great promise in solving a wide class of nonlinear filtering problems.

To infer the location and velocity information from the noisy observation, a nonlinear state-space model is first derived for the dynamic system under consideration. A novel SMC estimator is then developed to calculate the posterior distribution of the location and velocity. We propose a novel locally optimal soft handoff scheme, which requires the prediction of future signal strengths. A SMC predictor, built on top of the SMC mobility tracker is then developed to calculate the probability distributions required by the proposed soft handoff detector. Finally, simulation results are provided to demonstrate the superiority of the proposed techniques over the existing methods for mobility tracking and soft handoff detection.

II. MOBILITY MODEL IN CELLULAR NETWORKS

A. Motion Equation

We consider the modeling of a mobile user's movement on a two-dimensional plane. Assume that observations are taken at discrete time points $t_k = t_0 + \Delta t \cdot k$. Denote x_k and y_k as the horizontal and the vertical coordinates of a mobile's random position at time t_k ; $v_{x,k}$ and $v_{y,k}$ as the corresponding velocities at time t_k . Following [1], the motion model can be expressed as



that is,

$$\boldsymbol{x}_k = \boldsymbol{A}\boldsymbol{x}_{k-1} + \boldsymbol{B}_u \boldsymbol{u}_k + \boldsymbol{B}_w \boldsymbol{w}_k, \qquad (2)$$

where $\boldsymbol{u}_k \triangleq [\boldsymbol{u}_{x,k}, \boldsymbol{u}_{y,k}]^T$ denotes the unexpected changes in acceleration; $\boldsymbol{r}_k \triangleq [\boldsymbol{r}_{x,k}, \boldsymbol{r}_{y,k}]^T$ denotes the random changes in acceleration. The unexpected acceleration \boldsymbol{u}_k can be modeled as a Markov chain with a finite number of states, S_1, S_2, \ldots, S_m , as possible discrete levels of acceleration. The transition probability $\theta_{ij} \triangleq P(\boldsymbol{u}_k = S_j | \boldsymbol{u}_{k-1} = S_i)$ can be approximated by a value p near unity for i = j, and (1-p)/(m-1) for $i \neq j$, in many tracking situations [1]. The random acceleration \boldsymbol{r}_k is modeled as a Gaussian random vector to cover the "gap" between adjacent acceleration states. To represent the correlation feature of this random acceleration, a first-order AR model is adopted, i.e., $\boldsymbol{r}_{k+1} = \alpha \boldsymbol{r}_k + \boldsymbol{w}_k$, where $\boldsymbol{w}_k \sim (0, \sigma_w^2 \boldsymbol{I})$.

B. Measurement Equation

In existing cellular systems, the distance between the mobile and a given base station (BS) can be inferred from the forward link RSSI (or received signal strength indication) signal, which is the average of the pilot signal strength received at the mobiles. It is assumed that the rapid fluctuation of multipath fading is removed by the averaging operation.

Denote $p_{k,i}$ as the RSSI signal received by a given mobile from the *i*-th BS at time t_k . To locate a mobile user on the two-

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dimensional plane, at least three independent distance measurements are needed. In this paper, we select the three largest measurements to form the observation vector, which is a nonlinear function of the state vector \boldsymbol{x}_k in (2), i.e.,

$$\boldsymbol{y}_{k} \stackrel{\Delta}{=} [p_{k,1}, p_{k,2}, p_{k,3}]^{T} = \boldsymbol{h}(\boldsymbol{x}_{k}) + \boldsymbol{v}_{k}, \qquad (3)$$

with

$$\begin{split} \boldsymbol{h}(\boldsymbol{x}_k) &= [h_1(\boldsymbol{x}_k), h_2(\boldsymbol{x}_k), h_3(\boldsymbol{x}_k)]^T, \\ \boldsymbol{v}_k &= [v_{k,1}, v_{k,2}, v_{k,3}]^T, \\ h_i(\boldsymbol{x}_k) &= p_{0,i} - 5\eta \log \left[(x_k - a_i)^2 + (y_k - b_i)^2 \right], \end{split}$$

where $p_{0,i}$ is a constant determined by the transmitted power, the wavelength, and the antenna gain of the *i*-th BS; (a_i, b_i) is the position of the *i*-th BS; η is a path-loss index; and $v_{k,i}$ is the logarithm of the shadowing component. The shadowing component $v_{k,i}$ is assumed to be uncorrelated both in time and in space, and have a Gaussian distribution, i.e., $v_{k,i} \sim \mathcal{N}(0, \sigma_d^2)$.

III. SOFT HANDOFF INITIATION IN CELLULAR NETWORKS

Handoff is the mechanism that transfers an ongoing call from one cell to another as a user moves through the coverage area of a cellular system. Handoff initiation and admission control are the two steps to make a handoff. In this paper, however, we will focus only on the problem of soft handoff initiation for mobile users.

Soft handoff is an important feature for cellular CDMA networks, which provides added diversity and improved signal quality. At any time, the group of BSs connected to a particular mobile is called the *active set* of this mobile. In the conventional soft handoff algorithm [4], handoff decision is made by comparing the pilot signal strength with some thresholds, which is simple but bears no optimality.

The optimal soft handoff algorithm should best tradeoff among the average size of active set, the number of active set updates and the SD (system degradation). In [2], the number of service failures is used as the SD metric. In what follows, we use the outage probability as the SD metric. An outage happens whenever the pilot signal strength is below some threshold. The threshold is chosen to be the expected signal power when the mobile is moving out of its BS's desired coverage.

.1 Optimization Based on Outage Probability

We consider the optimal soft handoff algorithm based on the best trade-off among the rate of active set update, the average active set size and the outage probability. Denote A_k as the active set at time k. A handoff policy over a certain trajectory can be expressed as $\Phi = \{A_1, \ldots, A_N\}$, where N is the total number of observations along the trajectory. The three performance metrics can be expressed as:

1. Rate of active set update

$$\lambda_H(\Phi) = \mathbf{E}\left[\frac{1}{N}\sum_{k=1}^N \mathbf{1} \left(A_k \neq A_{k+1}\right)\right].$$
 (4)

2. Average active set size

$$\lambda_A(\Phi) = \mathbf{E}\left[\frac{1}{N}\sum_{k=1}^N |A_k|\right].$$
(5)

3. Probability of outage

$$\lambda_{\text{out}}(\Phi) = \frac{1}{N} \sum_{k=1}^{N} \mathbf{1}\left(\bigcap_{i \in A_k} p_{k,i} < \Delta\right),\tag{6}$$

where (6) follows from the definition of an outage event: an outage occurs if all pilot signals from the BSs in the active set are below a threshold Δ . This threshold is chosen to be the expected pilot signal strength when the mobile is on the boundary of its BS's desired coverage.

Let the cost of an outage event be unit, the cost of maintaining one extra member in the active set be c_A , and the cost of one active set update be c_H . Considering a linear cost combination, the Bayesian cost over the trajectory is then

$$\mathcal{J}(\Phi) = \lambda_{\text{out}}(\Phi) + c_H \lambda_H(\Phi) + c_A \lambda_A(\Phi).$$
(7)

The optimal soft handoff algorithm minimizes this cost over all possible handoff policies Φ . Since the computation of this cost requires prior knowledge of the entire trajectory, a locally optimal (LO) handoff algorithm was suggested in [2]. The objective of the LO algorithm is to minimize the expected incremental cost at time k, which can be expressed as

$$A_{k+1} = \arg \min_{A_{k+1}} \mathcal{J}_k, \qquad (8)$$

with
$$\mathcal{J}_{k} \triangleq \mathbb{E}[J_{incr}(A_{k+1}) | \mathbf{Y}_{k}]$$

$$= \mathbb{E}_{\mathbf{Y}_{k}} \left[\mathbf{1} \left(\bigcap_{i \in A_{k+1}} p_{k+1,i} < \Delta \right) + c_{H} \mathbf{1} (A_{k+1} \neq A_{k}) + c_{A} |A_{k+1}| \right]. \quad (9)$$

Implementation of the LO algorithm involves evaluation of (9) for all possible A_{k+1} . To simplify the algorithm, the following rules help to narrow down the number of possibilities to three. 1) At each time instant, the change of the active set size can be at most one. 2) An incoming candidate BS to the active set, $B_{k,in}$, must have the strongest signal strength among the ones outside the active set. 3) An outgoing BS, $B_{k,out}$, must have the largest probability to experience an outage event at time k + 1, i.e,

$$B_{k,out} = \arg\max_{i \in A_k} P\left(p_{k+1,i} < \Delta | \boldsymbol{Y}_k\right). \tag{10}$$

Then the LO soft handoff algorithm is to minimize the incremental cost (9) among the following three possibilities, i.e., 1) $A_{k+1} = A_k$; 2) $A_{k+1} = A_k \bigcup B_{k,in}$; 3) $A_{k+1} = A_k \setminus B_{k,out}$. It is seen that the major computation involved in the implementation of the proposed handoff algorithms is to solve the following prediction problems.

$$P\left(\max_{i\in A_{k+1}}p_{k+1,i}<\Delta\mid \boldsymbol{Y}_{k}\right).$$

IV. JOINT MOBILITY TRACKING AND HARD HANDOFF DETECTION

From the previous sections, it is seen that both mobility tracking and handoff detection are based on the received pilot signal powers \boldsymbol{Y}_k . Under the nonlinear state-space model given by (2) and (3), the optimal estimates of the location and the velocity of the mobile can be made. In addition, the optimal handoff algorithms developed in Section III make use of the predicted signal strengths in future time instants, which can be obtained from the same state-space model. Therefore, it is natural to consider the problem of joint mobility tracking and handoff detection. The problem will be solved under a Bayesian framework. That is, Bayesian inference of the mobile's location and velocity will be made from the on-line observation of the pilot signal strengths, and it will be further used to implement the optimal handoff algorithms.

A. SMC Estimator for Mobility Tracking

Denote $\boldsymbol{z}_k = (\boldsymbol{x}_k, \boldsymbol{u}_k), \ \boldsymbol{X}_k = (\boldsymbol{x}_0, \dots, \boldsymbol{x}_k), \ \boldsymbol{U}_k = (\boldsymbol{u}_0, \dots, \boldsymbol{u}_k)$ and $\boldsymbol{Z}_k = (\boldsymbol{z}_0, \dots, \boldsymbol{z}_k)$. We are interested in the on-line estimation of the posterior distribution $p(\mathbf{Z}_k | \mathbf{Y}_k)$. Based on the framework of sequential Monte Carlo (SMC) technique [3], we need a set of samples $\{(\mathbf{Z}_k^{(j)}, w_k^{(j)})\}_{j=1}^M$, properly weighted w.r.t. the distribution $p(\mathbf{Z}_k | \mathbf{Y}_k)$. The MMSE estimator of the location and velocity can then be approximated by

$$E(\boldsymbol{x}_k \mid \boldsymbol{Y}_k) \cong \frac{1}{W_k} \sum_{j=1}^M \boldsymbol{x}_k^{(j)} \boldsymbol{w}_k^{(j)}, \qquad (11)$$

with $W_k \stackrel{\Delta}{=} \sum_j w_k^{(j)}$. Following the framework of the sequential important sampling method, first, we choose the trial distribution $q(\cdot)$ to be

$$q(\boldsymbol{z}_{k}^{(j)} \mid \boldsymbol{Z}_{k-1}^{(j)}, \boldsymbol{Y}_{k}) = p(\boldsymbol{z}_{k}^{(j)} \mid \boldsymbol{X}_{k-1}^{(j)}, \boldsymbol{Y}_{k-1})$$

= $p(\boldsymbol{x}_{k}^{(j)} \mid \boldsymbol{x}_{k-1}^{(j)}, \boldsymbol{u}_{k}^{(j)}) \cdot P(\boldsymbol{u}_{k}^{(j)} \mid \boldsymbol{u}_{k-1}^{(j)}),$ (12)

The important weight is updated according to

$$w_{k}^{(j)} = w_{k-1}^{(j)} \cdot \frac{p(\boldsymbol{Z}_{k}^{(j)} | \boldsymbol{Y}_{k})}{p(\boldsymbol{Z}_{k-1}^{(j)} | \boldsymbol{Y}_{k-1}) q(\boldsymbol{z}_{k} | \boldsymbol{Z}_{k-1}, \boldsymbol{Y}_{k})} \\ \propto w_{k-1}^{(j)} \cdot p(\boldsymbol{y}_{k} | \boldsymbol{x}_{k}^{(j)}),$$
(13)

More specifically, we have

$$p(\boldsymbol{x}_{k} \mid \boldsymbol{x}_{k-1}^{(j)}, \boldsymbol{u}_{k}^{(j)}) \sim \mathcal{N}(\boldsymbol{x}_{k} - \boldsymbol{A}\boldsymbol{x}_{k-1}^{(j)} - \boldsymbol{B}_{u}\boldsymbol{u}_{k}^{(j)}, \sigma_{w}^{2}\boldsymbol{B}_{w}\boldsymbol{B}_{w}^{T}), \quad (14)$$

$$p(\boldsymbol{y}_k \mid \boldsymbol{x}_k^{(j)}) \sim \mathcal{N}(\boldsymbol{h}(\boldsymbol{x}_k^{(j)}), \sigma_d^2 \boldsymbol{I}).$$
(15)

We next summarize the SMC algorithm for mobility tracking as follows. At the k-th recursion,

G1. For j = 1, ..., M: • Drawn a sample $\boldsymbol{u}_{k}^{(j)}$ from $P(\boldsymbol{u}_{k}|\boldsymbol{u}_{k-1}^{(j)})$ given a priori;

- Drawn a sample $\boldsymbol{x}_{k}^{(j)}$ from $p(\boldsymbol{x}_{k}|\boldsymbol{x}_{k-1}^{(j)},\boldsymbol{u}_{k}^{(j)})$ given by (14); • Form $Z_k^{(j)} = \{Z_{k-1}^{(j)}, z_k^{(j)}\};$
- Update the weight $w_k^{(j)} = w_{k-1}^{(j)} \cdot p(\boldsymbol{y}_k | \boldsymbol{x}_k^{(j)})$ by (15); G2. Implement the resampling procedure [5] if the effective sampling size $\bar{M}_k \stackrel{\Delta}{=} \frac{M}{1+v_k^2} \leq M/10$, where v_t^2 is the coefficient

of variation of the important weights.

B. SMC Predictors for Handoff Detection

For a general prediction problem, suppose we are interested in computing the posterior distribution $p(\mathbf{Z}_{k+1}|\mathbf{Y}_k)$. For the new target distribution $p(\mathbf{Z}_{k+1}|\mathbf{Y}_k)$, choose the new trial distribution $\widetilde{q}(\cdot)$ to be

$$\widetilde{q}(\widetilde{\boldsymbol{Z}}_{k+1}^{(j)} \mid \boldsymbol{Y}_{k}) \stackrel{\Delta}{=} q(\widetilde{\boldsymbol{Z}}_{k}^{(j)} \mid \boldsymbol{Y}_{k}) \cdot p(\widetilde{\boldsymbol{Z}}_{k+1}^{(j)} \mid \widetilde{\boldsymbol{Z}}_{k}^{(j)}, \boldsymbol{Y}_{k}) \quad (16)$$

where $\{(\widetilde{Z}_k^{(j)}, \widetilde{w}_k^{(j)})\}_{j=1}^M$ are sampled from the mobility tracker and properly weighted w.r.t. the distribution $p(Z_k | Y_k)$. Then, the important weight is given by

$$\widetilde{w}_{k+1}^{(j)} \triangleq \frac{p(\widetilde{\boldsymbol{Z}}_{k+1}^{(j)} \mid \boldsymbol{Y}_k)}{\widetilde{q}(\widetilde{\boldsymbol{Z}}_{k+1}^{(j)} \mid \boldsymbol{Y}_k)} = \widetilde{w}_k^{(j)}.$$
(17)

The trial distribution given in (16) implies the following steps to obtain the samples $\{\widetilde{Z}_{k+1}^{(j)}\}_{j=1}^{M}$. At time k,

H1. Duplicate a set $\{(\boldsymbol{Z}_k^{(j)}, w_k^{(j)})\}_{j=1}^M$, (obtained from the mobility tracker [G1-G2]), to form the set $\{(\widetilde{\boldsymbol{Z}}_{k}^{(j)}, \widetilde{w}_{k}^{(j)})\}_{i=1}^{M}$;

- H2. For j = 1, ..., M, do
- Drawn a sample $\widetilde{u}_{k+1}^{(j)}$ from $P(\widetilde{u}_{k+1} | \widetilde{u}_{k}^{(j)})$; Drawn a sample $\widetilde{x}_{k+1}^{(j)}$ from $p(\widetilde{x}_{k+1} | \widetilde{x}_{k}^{(j)}, \widetilde{u}_{k+1}^{(j)})$;
- Form $\widetilde{Z}_{k+1}^{(j)} = \{\widetilde{Z}_{k}^{(j)}, \widetilde{z}_{k+1}^{(j)}\};\$

Based on (17), the samples $\{(\widetilde{\boldsymbol{Z}}_{k+1}^{(j)}, \widetilde{\boldsymbol{w}}_{k}^{(j)})\}_{j=1}^{M}$ are properly weighted with respect to the target distribution $p(\boldsymbol{Z}_{k+1} | \boldsymbol{Y}_{k})$.

Following the sampling procedure [H1-H2], we can obtain a set of samples $\{(\widetilde{\boldsymbol{Z}}_{k+1}^{(j)}, \widetilde{\boldsymbol{w}}_{k}^{(j)})\}_{j=1}^{M}$, properly weighted with respect to the distribution $p(\boldsymbol{Z}_{k+1} | \boldsymbol{Y}_{k})$. With these weighted samples, the objective predictive probability (11) can then be approximated by

$$P\left(\bigcap_{i \in A_{k+1}} p_{k+1,i} < \Delta \mid \mathbf{Y}_{k}\right)$$
$$\cong \quad \frac{1}{\widetilde{W}_{k}} \sum_{j=1}^{M} \left[\prod_{i \in A_{k+1}} \mathcal{Q}\left(\frac{h_{i}(\widetilde{\boldsymbol{x}}_{k+1}^{(j)}) - \Delta}{\sigma_{d}}\right)\right] \widetilde{w}_{k}^{(j)}.$$
(18)

Equation (18) follows from the observation model (3) and the assumption that the shadowing is a Gaussian random variable uncorrelated in space.

V. SIMULATION RESULTS

To examine the performance of the proposed mobility tracking and handoff schemes, simulations are performed for the conventional hexagon cellular network system. Simulation parameters are summarized in Table I. In order to cover the range of dynamic acceleration $[-10m/s^2, 10m/s^2]$, five levels $(0, \pm 4, \pm 8)m/s^2$ are selected as the states of the driving command. We consider a simulation environment where the mobile trajectory is generated randomly and then fixed for the rest of the simulation; the pilot signals are generated randomly for 50 simulation realization.

TABLE I SIMULATION PARAMETERS

Parameters	Comments
$\Delta t = 0.5 \mathrm{s}$	Sampling interval
$\alpha = 0.6$	Correlation coefficient defined in (2)
$\sigma_{w}^{2} = 0.72$	Variance of \boldsymbol{w}_k defined in (2)
p = 0.9	Transition probability $\theta_{i,i}$
$\sigma_d = 5 dB$	variance of the lognormal shadowing
$p_{0,i} = 90$	Base station transmission power
$\eta = 3$	Path-loss index
$\Delta = -1$ dB	Threshold of outage event

A. Results of Mobility Tracking

In Table II, the performance of the SMC mobility estimator (11) is compared with that of the MEKF, with different number of Monte Carlo samples employed by the SMC algorithm. The performance is evaluated in terms of the normalized mean square error (NMSE) of the mobile position. It is seen that with reasonable number of samples (e.g., M = 250), the SMC estimator is about 5 - 6dB better than the MEKF estimator.

TABLE II NMSE OF THE SMC AND THE MEKF TRACKERS.

	MEKF			
M = 100	250	500	800	(dB)
-27.0	-32.0	-33.3	-32.6	-27.4

B. Results of Soft Handoff

In this simulation, the following two soft handoff algorithms are implemented.

• The static threshold soft handoff strategy in [4] is implemented with the add threshold chosen from $-5dB \le P_{add} \le 5dB$, the relative drop threshold chosen from -10dB $\leq P_{drop} - P_{add} \leq$ -1dB, and a fixed timer set to be $T_d = 5$.

• The locally optimal (LO) soft handoff strategy based on the outage probability [cf. Section III-.1] is implemented with the handoff cost chosen from $c_H \in \{0.01, 0.03, 0.06, 0.1, 0.2\}$ and a fixed relative cost $c_A/c_H = 0.5$.

The objective of the soft handoff is to best tradeoff among the average size of active set λ_A , the number of active set updates $N\lambda_H$, and the outage probability λ_{out} . In Figure 1, the trade-off among the three metrics $(N\lambda_H, \lambda_A, \lambda_{out})$ are demonstrated for both strategies for various threshold and cost parameters. It is seen that comparing with the LO algorithm, the static threshold handoff algorithm achieves the same system quality (in terms of

outage probability) at the penalty of a larger handoff rate and a larger number of active set size. Furthermore, Bayesian cost given by (7) of different static handoff scheme are calculated and depicted in Figure 2. It is seen that the minimum cost is 18.7930. On the other hand, the LO soft handoff scheme is unique with the selected parameters $\{c_A, c_H\}$, and has a Bayesian cost of 15.9203. Evidently, the LO algorithm incurs a smaller cost compared with the static soft handoff algorithm, and therefore makes a better trade-off between service quality and system resource utilization.







Fig. 2. Bayesian cost ($c_A = 0.01, c_H = 0.005$) of the static threshold soft handoff algorithms over different sets of thresholds, where $P_{\rm add}$ denotes the add threshold; $P_{drop} - P_{add}$ denotes the difference between the add threshold and the drop threshold.

REFERENCES

- [1] T. Liu, P. Bahl, and I. Chlamtac, "Mobility modeling, location tracking, and trajectory prediction in wireless ATM networks," IEEE J. Sel. Areas in Com*mun.*, vol. 16, no. 6, pp. 922–936, Aug. 1998. R. Prakash and V. V. Veeravalli, "Locally optimal soft handoff algorithm,"
- [2] Proc. VTC 00, pp. 1450-1454, Fall, 2000.
- J. Liu and R. Chen, "Sequential Monte Carlo methods for dynamic systems," J. Amer. Stat. Assoc., vol. 93, pp. 1032-1044, 1998.
- TIA/EIA/IS-95, Mobile Station-Based Station Compatibility Standard for Dual-Mode Wideband Spread Spectrum Cellular System. Telecommunication Industry Association, July, 1993.
- [5] J. Liu and R. Chen, "Blind deconvolution via sequential imputations," J. Amer. Stat. Assoc., vol. 90, pp. 567-576, 1995.