Selection of Natural Scale in Discrete Wavelet Domain Using Eigenvalues

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Abstract

In this paper we present a novel technique for the selection of global natural scale from discrete wavelet transform. Here we define natural scale as the level associated with most prominent (dominant) eigenvalue. This technique is iterative and does not require full decomposition before finding the optimal wavelet level.

1. Introduction

Wavelet analysis finds many diverse applications including communications, computer graphics, computer vision, image processing and geophysics. It is already established that the wavelet orthonormal bases provide a useful multiresolutional signal representation and a tool for signal analysis [1].

It is well known that at lower level of the decomposition, more details are available at the expense of higher noise. On the other hand, at higher scales, more details are missed while noise is reduced considerably. The selection of the proper scale, to conduct analysis, still remains a problem. However, there is research reported on selection of natural scale by computer vision community [2-8]. The approaches discussed in [2-7] were tuned toward scale-space filtering. These approaches are not suitable for dyadic discrete wavelet domain. These techniques require complete decomposition before the natural scales could be detected. Hence, they do not provide any information as to where the decomposition should be stopped. The approach discussed in [8] is based on 2D Gabor filtering.

Here we present a simple approach to detect global natural scale in discrete wavelet domain. This technique adaptively detects optimal scale while decomposition is being carried out. The technique is based on eigenvalues and gives optimal result in mathematical sense. Moreover, this approach is not tuned to any specific application hence could be applied to any wavelet-based signal processing application.

2. Detection of Natural Scale in Discrete Wavelet Domain

Let the discrete input signal is rewritten as $D = \left(S_1 f(n)\right)_{n\in Z}$. Let us denote

$$W_{2^j}^d f = \left(W_{2^j} f(n+\omega)\right)_{n\in Z} \quad (1)$$

and

$$S_{2^j}^d f = \left(S_{2^j} f(n+\omega)\right)_{n\in Z} \quad (2)$$

where ω is the sampling shift that depends only on the wavelet $\varphi(x)$. For any coarse scale 2^J, the sequence of discrete signals,

$$\left\{S_{2^J}^d, \left(W_{2^j}^d f\right)_{1\le j\le J}\right\} \quad (3)$$

is called the *discrete dyadic wavelet transform* of $D = \left(S_1 f(n)\right)_{n\in Z}$ [9]. Where $S_{2^J}^d$ is *the last approximation* and the set of sequence $\left(W_{2^j}^d f\right)_{1\le j\le J}$ is *wavelet coefficients* or *details* at levels $1\le j\le J$.

Let *wavelet details* $\left(W_{2^j}^d f\right)_{1\le j\le J}$ be represented in matrix P with each row represents details at levels 2^j starting from level 2^1. Hence,

$$P = \begin{bmatrix} W_{2^1}^d f \\ W_{2^2}^d f \\ . \\ . \\ W_{2^J}^d f \end{bmatrix} \quad (4)$$

The matrix P is not square hence, its eigenvalues can be computed using *Singular Value Decomposition (SVD)*. Where,

$$SVD\{P\} = T.S.E \tag{5}$$

Such that,

$$P = T.S.E^T \tag{6}$$

Here '.' indicates matrix multiplication. If length of sequence D is M, where $J < M$ then matrices T, S and E have the forms,

$$T = \begin{bmatrix} T_{11} & T_{12} & . & . & T_{1M} \\ T_{21} & T_{22} & . & . & T_{2M} \\ . & . & . & . & . \\ . & . & . & . & . \\ T_{J1} & . & . & . & T_{JM} \end{bmatrix} \tag{7}$$

$$S = \begin{bmatrix} S_1 & 0 & 0 & . & . & . & 0 \\ 0 & S_2 & 0 & . & . & . & 0 \\ 0 & 0 & S_j & 0 & 0 & . & 0 \\ 0 & . & 0 & S_J & 0 & . & 0 \end{bmatrix} \tag{8}$$

$$E = \begin{bmatrix} E_{11} & E_{12} & . & . & E_{1M} \\ E_{21} & E_{22} & . & . & E_{2M} \\ . & . & . & . & . \\ . & . & . & . & . \\ E_{M1} & E_{M2} & . & . & E_{MM} \end{bmatrix} = \begin{bmatrix} E_1^* \\ E_2^* \\ . \\ . \\ E_M^* \end{bmatrix} \tag{9}$$

where $S_1 \geq S_2 ... \geq S_j \geq ... \geq S_J$.

Now equation (6) can be rewritten as,

$$\begin{aligned} \left(W_{2^j}^d f\right)_{1 \le j \le J} &= \sum_{m=1}^{M} T_{jM} \left(S_m E_m^*\right) \\ &= T_{j1}\left(S_1 E_1^*\right) + \sum_{m=2}^{M} \left(T_{jm}\left(S_m E_m^*\right)\right) \end{aligned} \tag{10}$$

If $S_1 >> \{S_2, S_3, ...S_j, ..., S_J\}$ then,

$$\left(W_{2^j}^d f\right)_{1 \le j \le J} = T_{j1}\left(S_1 E_1^*\right) \tag{11}$$

and hence S_1 detects dominant mode in P. In other words, S_1 detects most dominant behavior among wavelet details.

The *level* corresponding to the *dominant mode* of the $\left(W_{2^j}^d f\right)_{1 \le j \le J}$ is defined as *natural scale*. The detection of dominant scale using this approach requires wavelet decomposition be computed as all possible scales. Hence, we start with only first two levels (i.e. 2^1 and 2^2) and compute SVD adaptively while adding higher levels. Let, for any level 2^k, X_{2^k} be the matrix extracted by selecting first k rows of P. Let $\{S_1^X \quad S_2^X \quad . \quad . \quad S_k^X\}$ be the eigenvalues of X_{2^k}. We define a quantity *dominant mode difference*, D^{Xk} as,

$$D^{Xk} = S_1^X - \sum_{l=2}^{k} S_l^X \tag{12}$$

Now finding natural scale converges to find k such that D^{Xk} is maximum. Now we provide the algorithm to detect natural scale.

Algorithm

1. Compute two level wavelet details (i.e. X_{2^2}) and set $j = 2$.
2. compute D^{Xj}
3. *while* $\left(D^{X(j+1)} - D^{Xj}\right) \geq 0$
   $j = j + 1$
   compute X_{2^j} and D^{Xj}
   endwhile
4. select natural scale as $L = j$

3. Results and Discussion

Figure 1 shows the original signal (top) and the wavelet decomposition. For the sake of illustration we show decompositions up to the levels higher than the required. Figure 2 shows the dominant mode difference $\left(D^{Xk}\right)_{1 \le k \le 7}$. Here one can observe that natural scale for the given signal is 2^4. Similarly, figures 3,4,5,6,7 and 8 provide results with other signals.

It should be noted here that, the matrix P cannot be formed when dyadic decomposition is computed by sub-sampling the decompositions at each level. In other words, number of columns in matrix P will not be same. In such cases the decompositions must be resampled in order to overcome this problem. However, decomposition scheme of Mallat & Zong [9] does not have this limitation.

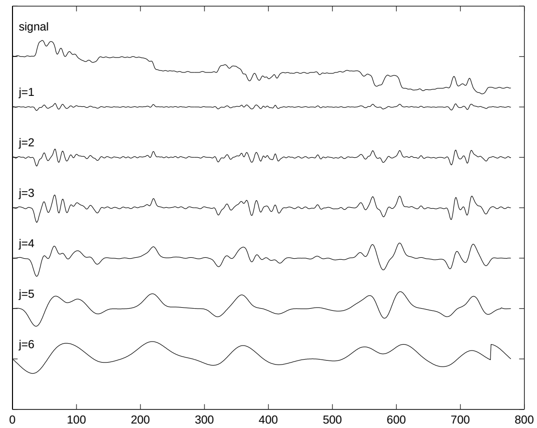

Figure 1. Input signal (top) and wavelet decompositions for levels 2^1 to 2^6

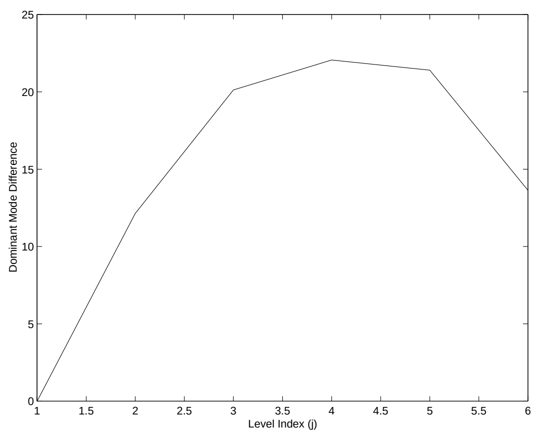

Figure 2.Dominant mode difference profile for Figure 1.

4. Conclusion

Here we have presented a novel technique for the selection of natural scale in discrete wavelet domain. The technique is based on eigenvalues and does not require full decomposition before the selection is done. The results show that eigenvalues provide very useful information about natural scale.

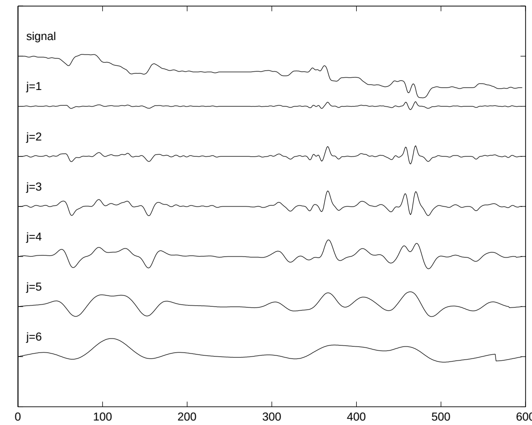

Figure 3.Input signal (top) and wavelet decompositions for levels 2^1 to 2^6

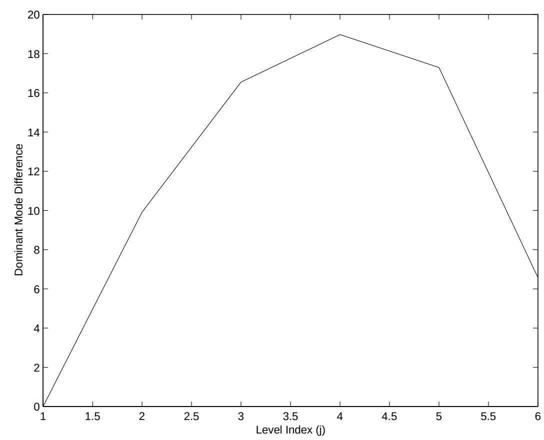

Figure 4. Dominant mode difference profile for Figure 3

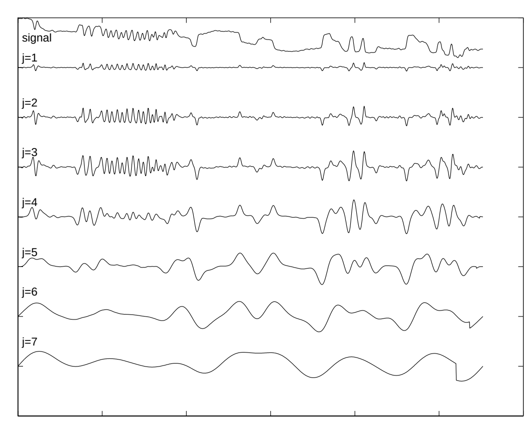

Figure 5. Input signal (top) and wavelet decompositions for levels 2^1 to 2^7

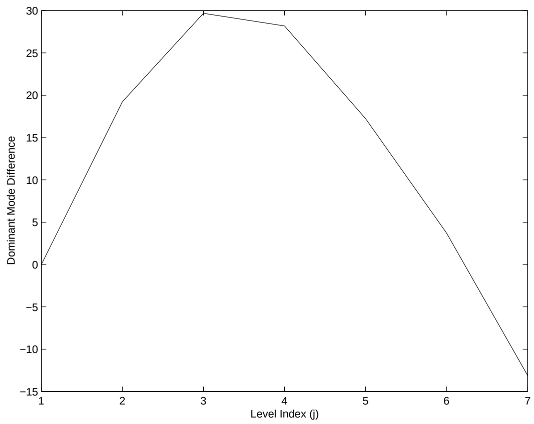

Figure 6. Dominant mode difference profile for Figure 5

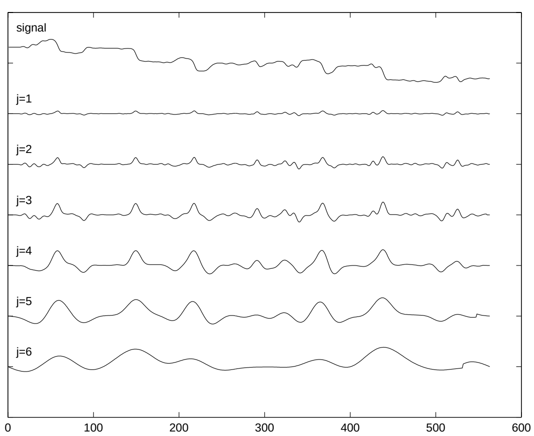

Figure 7. Input signal (top) and wavelet decompositions for levels 2^1 to 2^6

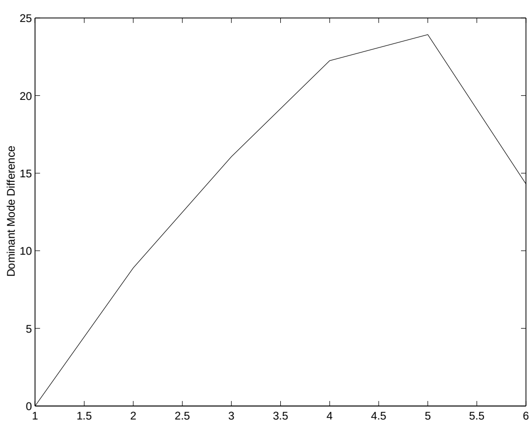

Figure 8. Dominant mode difference profile for Figure 7

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