TEAGER-KAISER OPERATOR BASED FILTERING

Ridha Hamila, Markku Renfors, Taneli Haverinen
Telecommunications Laboratory,
Tampere University of Technology
P.O.Box 553, FIN-33101, Tampere, Finland

Guðni Gunnarsson
Nokia Networks,
P.O.Box 361, FIN-00045, Nokia Group Finland

ABSTRACT

In this contribution, we propose a new filtering and denoising technique for one-dimentional signals based on the nonlinear quadratic Teager-Kaiser operator. This technique is a threshold 'energy' based approach where outliers are first detected and then replaced by their estimated values. The proposed technique performs better compared to the alpha-trimmed and running mean filter, also with moderate complexity. In particular, we present an application of the proposed real-time filtering approach for postprocessing the measurements of time delay signals in a mobile positioning system highly affected by the propagation environment.

I INTRODUCTION

Various filtering methods have been proposed in the course of years. There are both linear and nonlinear alternatives, and filtering is often a trade-off between different features, such as performance in Gaussian and non-Gaussian environments, ability to adapt in case of nonstationary signals, edge preservation and computational complexity. Two most typical methods are running mean and median filters, which exploit a sliding fixed length window. Mean filters are known to remove well additive Gaussian noise, but they tend to soften edges and can not fully cope with impulsive noise. Median filters on the other hand are nonlinear, they involve sorting of the data, and are able to eliminate impulses at the expense of some streaking and edge jittering. Many modifications have been developed to find a good compromise. One of the simplest categories is trimmed mean filters, which reject some probable outliers before taking the mean. Ranked order and weighted order statistic filters represent an extension of the ordinary median filter [1].

Denoising deals basically with the same problem of estimating the underlying signal from the noisy observations, but data is now not needed to process on-line or is not time-dependent at all. Therefore, noncausal filters can be exploited, the simplest proper algorithm is gliding window smoother, which differs from the running window mean filter only in that the cursor is now in the middle of the window, not in the right-hand end. Naturally, the same procedure can be applied to the other window-based methods as well. Denoising is not restricted to the methods acting in time domain, the signal is often processed in some transform domain. Wavelet transform has recently gained research interest in several fields of signal processing, using thresholding of (orthogonal) wavelet coefficients and taking the inverse wavelet transform has been proposed to solve denoising problem. However, this is a more computationally complex approach [1].

In this paper, we propose a new filtering and denoising approach for one-dimensional signals based on a nonlinear quadratic operator called Teager-Kaiser (TK) operator. As an important potential application for these techniques, we examine improving the accuracy and certainty of mobile phone positioning. This positioning problem is an important and interesting research topic, as positioning could e.g., help to locate people in need of emergency service [2]. The location can be determined using measurement signals from multiple Base Transceiver Stations (BTS). However, due to the multipath interference, a measurement signal is highly corrupted and a proper filtering method is needed.

II TEAGER-KAISER OPERATOR

Based on Newton's law of motion, a nonlinear quadratic operator called Teager-Kaiser (TK) operator was first introduced by Teager and Kaiser [3],[4] to measure the real physical energy of a system. This nonlinear operator differs from the common way to calculate the energy of a discrete-time signal as the average sum of its squared magnitudes. The energy of a generating system of a simple oscillation signal was computed as the product of the square of the amplitude and the frequency of the signal. It was found that this nonlinear operator exhibits several attractive features such as simplicity, efficiency and ability to track instantaneouly-varying spacial patterns. Since its introduction, several applications have been derived for one-dimensional [5],[6], and two-dimensional signal processing [7].
The continuous-time TK operator of a complex-valued signal $x(t)$ is defined as follows [6]

$$E_C[x(t)] = \dot{x}(t)\dot{x}^*(t) - \frac{1}{2}\left[\ddot{x}(t)x^*(t) + x(t)\ddot{x}^*(t)\right] . \tag{1}$$

When $x(t)$ is real, Eq. (1) reduces to the continuous-time TK operator of a real-valued signal [3]

$$E_R[x(t)] = \dot{x}^2(t) - x(t)\ddot{x}(t) . \tag{2}$$

Similarly, by making certain combinations of the discretized mapped derivatives, the discrete-time TK operator for real-valued signal is given by [4]

$$E_D[x(n)] = E(n) = x^2(n) - x(n-1)x(n+1) \quad . \qquad (3)$$

III E-OTD POSITIONING METHOD

Mobile phones have gained tremendous popularity in the last few years. Many kinds of services have been designed in addition to usual voice calls. The main characteristic of a wireless phone is its mobility; it follows the user almost everywhere, and therefore one very attractive application would be a positioning service especially for emergency situations. A wireless phone cannot currently be accurately located in a GSM network [1],[8]. The identity of the serving Base Transceiver Station (BTS) is called Cell Identity (CI). CI is known and can be used for positioning but it results in poor accuracy, which is also dependent on the size of the cell. Another, preferable way for finding a solution for this positioning problem would be utilizing signal time delay or angle of arrival information [9]. In synchronous systems, the Observed Time Difference (OTD) positioning method calculates the time interval between the receptions of bursts from two different BTS. This results in a hyperbola that lies between the two BTS's. When this calculation is repeated for several neighbor BTS's, several hyperbolas are formed, and the location of the wireless phone is determined by the place where the hyperbolas intersect. Measurement errors are unfortunately unavoidable. One severe source of errors is multi-path interference, and it leads to reduction in the positioning accuracy. Therefore, post-processing of the measurement signal is obviously needed. In this paper, we used an OTD signal measured via modified software of a NOKIA 8110 mobile phone (see Fig. 2). Here, the OTD value is represented in multiple of 1/16 bit intervals which corresponds approximately to 60 meters fluctuation in distance. Notice also that the OTD signal slope varies with the MP speed.

IV TK OPERATOR IN FILERING

Teager-Kaiser filtering technique is a detection based approach in which outliers are first detected using an adaptive threshold (*Th*) based on the TK energy-like quantity, and then replaced by their estimated values. TK filter is described by the following algorithm (formulated for the OTD filtering application):

Inputs:
OTD measurement vector of length *L*
β adaptivity parameter
win length of the window

Output:
Filt_OTD filtered OTD signal of length *L*

Initializations:

$Tmp\_OTD = OTD$

$E(1) = \left|\mathrm{OTD}(2)^2 - OTD(1) \times OTD(3)\right|$

$Th = \beta \times E(1)$

TK filtering algorithm:

For $n = 2$ *to* L, *do*

$E(n) \leftarrow \left|OTD^2(n) - OTD(n-1) \times OTD(n+1)\right|$

$Th(n) \leftarrow [Th(n-1) \times (n-1) + \beta \times E(n)]/n$

if $E(n) < Th(n)$ then

Tmp_OTD(n) $\leftarrow$ OTD(n)

else

Tmp_OTD(n) $\leftarrow$ Filt_OTD($n-1$)

end if

Filt_OTD(n) $\leftarrow$ *mean*[Tmp_OTD($n - win + 1 \ldots n$)]

end do

The adaptive threshold is estimated by β times the mean of the 'energy' of the preceding samples. The energy-like quantity of the processed sample is then compared to the corresponding threshold value. If the 'energy' is greater than the corresponding threshold, the sample value of the input OTD signal is replaced by the sample value of the previous ouput. After removing impulsive interference through this process, the OTD signal is filtered by the windowed mean of the preceding samples including the processed sample.

The performance of the TK filter is very much related to the filter window length (*win*), and also to the adaptive parameter (β) which permits to select the best fitted threshold corresponding to the specific signal in question. The following table shows the MSE of the OTD filtered signal for TK filter with respect to these two parameters:

Table I

MSE	win=5	win=10	win=15	win=20	win=25	win=30	win=35	win=40
β=0.3	2,17	1,99	2,18	2,57	3,06	3,65	4,33	5,11
β=0.6	2,47	1,96	1,88	2,00	2,21	2,46	2,76	3,12
β=0.9	2,68	1,96	1,79	1,79	1,88	2,01	2,17	2,37
β=1.2	2,95	1,98	1,70	1,65	1,68	1,74	1,84	1,97
β=1.5	3,07	1,93	1,60	1,50	1,50	1,53	1,60	1,71
β=1.8	3,10	1,88	1,51	1,40	1,38	1,41	1,47	1,56
β=2.1	3,37	2,13	1,73	1,58	1,55	1,56	1,60	1,67

Notice that in case of a long window size, small values of beta result in poor performance because the filter is too slow to follow nonstationary variations of the signal. On the other hand, when *win* is small, also smaller beta is desired to achieve good performance.

Similarly, Table II provides the MSE for the alpha-trimmed mean filter [1] with respect to the window size and the alpha parameter:

Table II

MSE	win=5	win=10	win=15	win=20	win=25	win=30	win=35	win=40
α=0.00	4,27	2,60	2,02	1,78	1,65	1,58	1,57	1,58
α=0.05	4,27	2,60	2,02	1,62	1,53	1,50	1,51	1,51
α=0.1	4,27	2,60	1,81	1,62	1,48	1,46	1,47	1,50
α=0.15	4,27	2,29	1,76	1,56	1,48	1,46	1,48	1,53
α=0.20	3,85	2,24	1,73	1,56	1,48	1,46	1,49	1,53
α=0.25	3,85	2,24	1,73	1,56	1,48	1,47	1,49	1,53
α=0.30	3,85	2,27	1,75	1,58	1,49	1,47	1,49	1,53
α=0.35	3,85	2,27	1,79	1,60	1,49	1,47	1,49	1,53
α=0.40	4,38	2,43	1,87	1,62	1,51	1,49	1,51	1,55
α=0.45	4,38	2,43	1,87	1,69	1,55	1,50	1,52	1,58

Performance comparison between the three filtering algorithms - namely Teager filter, sliding window mean, and alpha-trimmed mean filter - is provided for as a function of window length in Figure 1. Only window length is varied, other parameters are fixed to the value that gave the minimum MSE. As we can see, for small window lengths the results are clearly beneficial for the Teager filter. As the length of the window increases, the differences get smaller, but running mean filter remains still slightly behind the other two algorithms.

At this point it is important to bear in mind that the computational complexity for the Teager filter and running mean filter is O(1), i.e., independent on the length of the window. Alpha trimmed mean involves sorting and is therefore more complex, at least O(*win**log(*win*)), depending on the sorting algorithm used.

The filter performance is also visualized for two specific cases. Figure 2 shows the filtering results for optimum parameters, and Figure 3 shows the filtering results for short window length of 5, with optimal α and β.

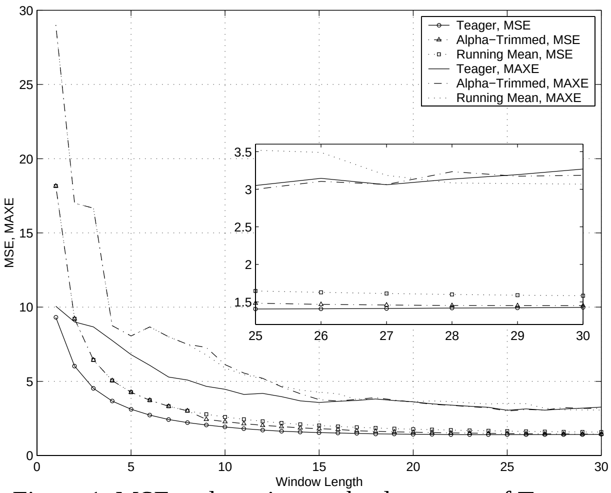

Figure 1. MSE and maximum absolute error of Teager-Kaiser, alpha-trimmed mean, and running mean filter, in the OTD filtering application.

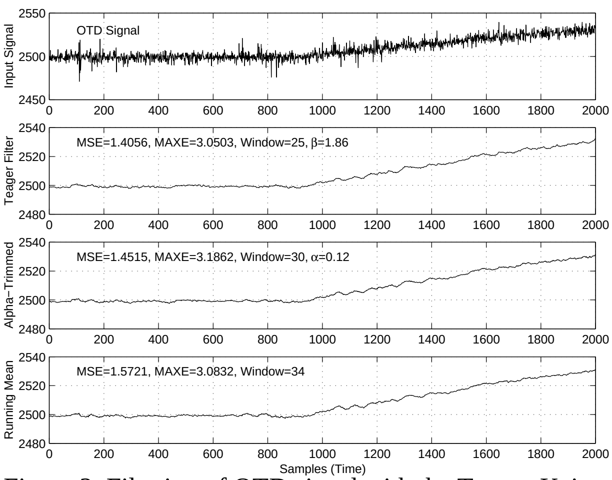

Figure 2. Filtering of OTD signal with the Teager-Kaiser filter, alpha-trimmed mean, and running mean filter, using optimum parameters.

The proposed filtering technique is used in on-line applications, which require causal signal processing methods. On the contrary, denoising is not time-dependent process.

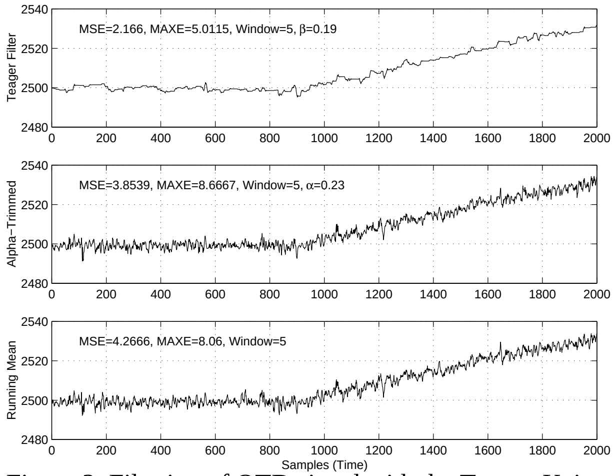

Figure 3. Filtering of OTD signal with the Teager-Kaiser filter, alpha-trimmed mean, and running mean filter, using optimal α and β parameters. The window length is equal to 5.

V TK OPERATOR IN DENOISING

The TK operator based denoising algorithm consists of three steps:

1. Calculate the energy-like (*E*) and threshold (*Th*) for each sample, and save indices of the samples for which $E < Th$ as in Section IV.
2. Use linear interpolation to replace the sample-values between indexed samples.
3. Use a sliding window mean to smoothen the output.

The algorithm used in the simulations is as follows:

Inputs:

OTD measurement vector of length L

β adaptivity parameter

win length of the window

Ouput:

$Filt\_OTD$ Filtered OTD signal of length L

Initializations:

$Tmp\_OTD = OTD$

$E(1) = \left|\mathrm{OTD}(2)^2 - OTD(1)\times OTD(3)\right|$

$Th = \beta \times E(1)$

$ind = 1$

$N = 0$

TK denoising algorithm:

for $n = 2$ to L, do

$\quad \mathrm{E(n)} \leftarrow \left|\mathrm{OTD(n)}^2 - OTD(n-1)\times OTD(n+1)\right|$

$\quad Th(n) \leftarrow \left[Th(n-1)\times(n-1) + \beta \times E(n)\right]/n$

$\quad$ if $E(n) < Th(n)$ then

$\qquad ind \leftarrow [ind \quad n]$

$\qquad N \leftarrow N + 1$

$\quad$ end if

end do

$ind \leftarrow [ind \quad L]$

for $k = 1$ to N, do

$slope \leftarrow [Tmp\_OTD(ind(k+1)) - Tmp\_OTD(ind(k))] / [ind(k+1) - ind(k)]$

for $i = ind(k)+1$ to $ind(k+1)-1$, do

$Tmp\_OTD(n) \leftarrow Tmp\_OTD(n-1) + slope$

end do

end do

$Pad\_1 \leftarrow [Tmp\_OTD(1) \ \cdots \ Tmp\_OTD((win-1)/2)]$

$Pad\_2 \leftarrow [Tmp\_OTD(L+1-(win-1)/2) \ \cdots \ Tmp\_OTD(L)]$

$Tmp\_OTD \leftarrow [Pad\_1 \ \ Tmp\_OTD \ \ Pad\_2]$

$sum \leftarrow [Tmp\_OTD(1) \ \cdots \ Tmp\_OTD(win)] \times [1 \ \cdots \ 1]^T$

for $t = 1$ to L, do

$Filt\_OTD(t) \leftarrow sum/win$

$sum \leftarrow sum + Tmp\_OTD(t+win+1) - Tmp\_OTD(t)$

end do

As figure 4 shows, the performance of the TK denoising approach is superior to the two filter types with respect to the MSE and maximum absolute error. Figure 5 shows the denoising results for an OTD signal.

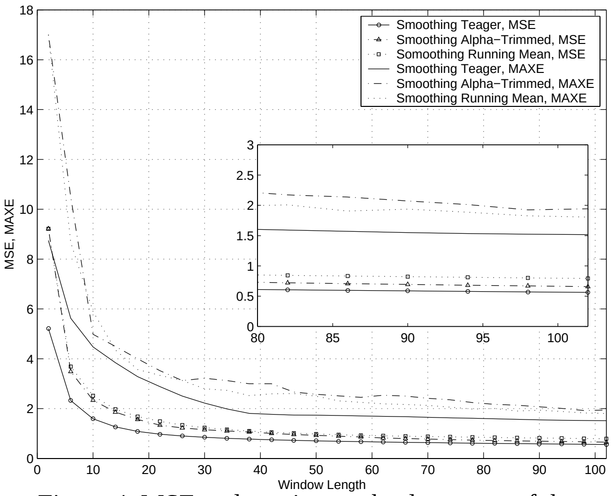

Figure 4. MSE and maximum absolute error of the smoothing Teager-Kaiser, alpha-trimmed, and running mean filter.

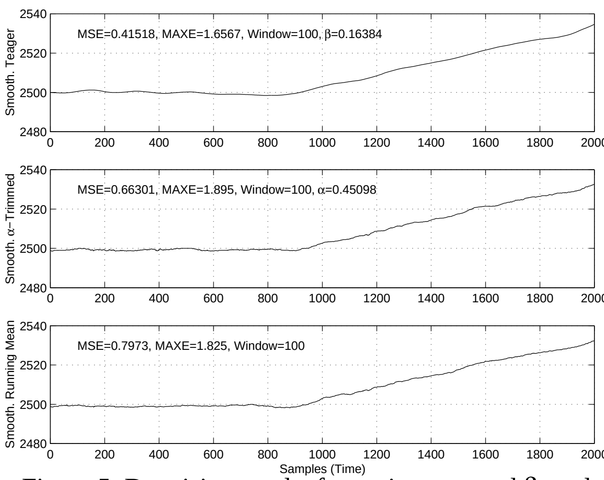

Figure 5. Denoising results for optimum α and β, and window length 100.

VI CONCLUSIONS

In this paper we proposed a new approach for filtering and denoising one-dimensional signals based on the energy-like Teager operator. In particular, the filtering approach is used for real-time postprocessing of OTD signals heavily affected by multipath interference. Consequently, by minimizing the variations in the OTD signals, we improve the positioning accuracy. The Teager filter gives clearly better performance than the other approaches when short window length is used. In other words, the Teager filter would allow to use shorter window length for a given MSE performance, and clearly faster changes in the desired signal component could be tolerated than with the other approaches.

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