STATISTICAL ANALYSIS OF A PARAMETRIC MODEL FOR PHOTOMETRIC SIGNALS

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ABSTRACT

This communication studies a new model for photometric signals under high flux assumption. Photometric signals are modeled by Gaussian autoregressive processes having the same mean and variance denoted Constraint Gaussian AR Processes (CGARP's). This model is first derived from the data asymptotic distribution under high flux assumption. The performance of the CGARP parameter estimators is then studied by comparing their mean square errors to the Cramer Rao lower bounds (CRLB's). Asymptotic expressions are derived to approximate the CRLB's for large values of the number of samples. Computer simulations confirm the validity of these expressions. The achievable performance for CGARP parameter estimation is compared to those obtained with the unconstraint model. The purpose of this model is to derive a Neyman Pearson detector for the change-point detection problem that arises in the extrasolar planets detection problem.

1 Introduction

This paper studies a new model for photometric signals under high flux assumption. Photometric signals are modeled by Gaussian autoregressive processes having the same mean and variance denoted Constraint Gaussian AR Processes (CGARP's).

This model was used without theoretical justification in [4] in order to derive a Neyman Pearson detector for change-point detection in the mean and variance of the CGARP. This problem occurs in astronomy for the detection of extrasolar planets. Among the methods currently pursued to detect extrasolar planets, the transit method (also referred as photometric method or occultation method) may be the only one to find earth class planets in the near future [2]. The transit method is based on the detection of photometric flux variations which result from the transit of a planet in front of a star.

The first contribution of this communication is to show that the CGARP distribuition is in good agreement with the data asymptotic distribution. Afterwards, the achievable accuracy of CGARP parameter estimators is studied through the Cramér Rao lower bounds (CRLB's). A large sample size expression of CRLB's is derived and validated by a comparison with the mean square errors (MSE's) of the estimates obtained through Monte Carlo simulations. Finally, CRLB variations as functions of the model parameters are illustrated by numerical examples and discussed.

2 Signal model derivation

We assume that the signal is dominated by the photon noise *i.e.* the read-out noise and the thermal noise for the electronic are negligible. When light with stochastic attributes is incident on a photodetector, the joint probability of registering the counts $\boldsymbol{X} = (x_1, \dots, x_N)^t$, is distributed according to [6]:

$$P(\boldsymbol{X}) = \int_0^\infty \prod_{n=1}^N \frac{e^{-\lambda_n} \lambda_n^{x_n}}{x_n!} p(\boldsymbol{\Lambda}) d\lambda_1 \dots d\lambda_N, \qquad (1)$$

where $E[\lambda_n] = \lambda$ is the light intensity integrated between two successive measurements and $p(\boldsymbol{\Lambda})$ is the joint distribution of $\boldsymbol{\Lambda} = (\lambda_1, \dots, \lambda_N)^t$. Unfortunately, an a priori distribution for $\boldsymbol{\Lambda}$ is generally very difficult to choose. Moreover, an analytic expression of (1) cannot be computed for most probability density functions (pdf's) $p(\lambda_1, \dots, \lambda_N)$. For these reasons, we propose to derive a simple model for $\boldsymbol{X}$ using two realistic assumptions:

$$\mathsf{A1}: \ \mathrm{E}[\lambda_n] = \lambda \gg 1, \ \mathsf{A2}: \ \mathrm{var}[\lambda_n] \ll \lambda.$$

A1 traduces a high flux assumption and A2 conveys the fact that the variations of the integrated light intensity and the variations of the sensor are small.

Based on these assumptions, this communications proposes to model x_n as a Gaussian AR process subjected to the constraint $\mathrm{E}[x_n] = \mathrm{var}[x_n] = \lambda$, denoted *Constraint Gaussian AR Process (CGARP).* The Gaussian distribution for $\boldsymbol{X}$ is justified by the following proposition.

Proposition 1 *If the distribution of $\boldsymbol{X}$ verifies (1):*

1. *the mean and covariance matrix of $\boldsymbol{X}$ are:*

$$\mathrm{E}[\boldsymbol{X}] = \lambda \boldsymbol{u}, \quad C_{XX} = C_{\Lambda\Lambda} + \lambda I_N, \qquad (2)$$

where $\boldsymbol{u}$ is a vector of dimension N whose elements equal 1, I_N is the $N \times N$ identity matrix, λ the mean of λ_n and $C_{\Lambda\Lambda}$ the covariance matrix of $\boldsymbol{\Lambda}$.

2. *Define $C_{\Lambda\Lambda} = VD^2V^t$ the eigendecomposition of $C_{\Lambda\Lambda}$, $\gamma_k(\boldsymbol{\tau})$ the kth order cumulant of $(\lambda_{\tau_1}, \dots, \lambda_{\tau_k})$ where $\boldsymbol{\tau} = (\tau_1, \dots, \tau_k)$ and the standardized vector:*

$$\mathbf{Y}_\lambda = (D + \sqrt{\lambda} I_N)^{-1} V^t (\boldsymbol{X} - \lambda \boldsymbol{u}), \qquad (3)$$

($\mathrm{E}[\mathbf{Y}_\lambda] = 0$, $C_{Y_\lambda Y_\lambda} = I_N$). If $\gamma_k(\tau) = o(\lambda^{k/2})$ for $k \geq 2$, $\mathbf{Y}_\lambda$ converges in distribution to the multivariate Gaussian distribution $\mathcal{N}(0, I_N)$ when $\lambda \to +\infty$.

Proof : From (1), $\boldsymbol{X}/\boldsymbol{\Lambda}$ is distributed according to an iid Poisson distribution. The first and second order statistics of $\boldsymbol{X}$ can then be easily computed using conditional expectations :

$$\begin{aligned} \mathrm{E}[x_n] &= \mathrm{E}[\mathrm{E}[x_n/\lambda_n]] = \mathrm{E}[\lambda_n] = \lambda, \\ \mathrm{cov}[x_i, x_j] &= \mathrm{E}[\mathrm{E}[x_i x_j | \lambda_i, \lambda_j]] - \lambda^2 \\ &= \begin{cases} \mathrm{cov}[\lambda_i, \lambda_j] & \text{if} \quad i \neq j, \\ \lambda + \mathrm{var}[\lambda_i] & \text{if} \quad i = j. \end{cases} \end{aligned}$$

In order to prove the asymptotic normality of $\boldsymbol{Y}_\lambda$, we study its second characteristic function denoted $\Psi_{Y_\lambda}(\boldsymbol{\Omega}) = \log \Phi_{Y_\lambda}(\boldsymbol{\Omega})$, where $\Phi_{Y_\lambda}(\boldsymbol{\Omega}) = \mathrm{E}[e^{j\boldsymbol{\Omega}^t \boldsymbol{Y}_\lambda}]$ and $\boldsymbol{\Omega} = (\omega_1, \dots, \omega_N)^t$. Denote $\boldsymbol{\varphi}_\lambda = V(D + \sqrt{\lambda} I_N)^{-1}\boldsymbol{\Omega}$ where I_N is the $N \times N$ identity matrix. The components of the vector $\boldsymbol{\varphi}_\lambda$ are:

$$\varphi_k(\lambda) = \sum_{q=1}^{N} \frac{v_{kq}\omega_q}{d_q + \sqrt{\lambda}} \tag{4}$$

using obvious notations. Note that assumption A2 and the orthogonality of V implies that:

$$\varphi_k(\lambda) = O(\frac{1}{\sqrt{\lambda}}). \tag{5}$$

A straightforward computation yields:

$$\Psi_{Y_\lambda}(\boldsymbol{\Omega}) = -j\lambda \sum_{k=1}^{N} \varphi_k + \log \Phi_{\boldsymbol{X}}(\boldsymbol{\varphi}_\lambda). \tag{6}$$

Using conditional expectations, the first characteristic function of $\boldsymbol{X}$ can be written:

$$\begin{aligned} \Phi_{\boldsymbol{X}}(\boldsymbol{\Omega}) &= \mathrm{E}[\mathrm{E}[e^{j\boldsymbol{\Omega}^t X}|\boldsymbol{\Lambda}]] = \mathrm{E}[\prod_{k=1}^{N} e^{(e^{j\omega_k}-1)\lambda_k}] \\ &= \mathrm{E}[e^{\sum_{k=1}^{N} \lambda_k (e^{j\omega_k}-1)}]. \end{aligned} \tag{7}$$

Consequently, by denoting $\Upsilon_{\boldsymbol{\Lambda}}(\boldsymbol{S}) = \mathrm{E}[e^{\boldsymbol{\Lambda}^t \boldsymbol{S}}]$ the moment generating function of $\boldsymbol{\Lambda}$, eq. (7) reads: $\Phi_{\boldsymbol{X}}(\boldsymbol{\varphi}_\lambda) = \Upsilon_{\boldsymbol{\Lambda}}(\boldsymbol{S})$, with $\boldsymbol{S} = (s_1, \dots, s_N)^t$,$s_k = e^{j\varphi_k(\lambda)} - 1$.

Assuming that $\boldsymbol{\Lambda}$ satisfies some regularity conditions (see [8, p. 198]) and denoting $\gamma_k(\boldsymbol{\tau})$ the kth order cumulant of $(\lambda_{\tau_1}, \dots, \lambda_{\tau_k})$ where $\boldsymbol{\tau} = (\tau_1, \dots, \tau_k)$, the second moment generating function of $\boldsymbol{\Lambda}$ can be expanded as:

$$\begin{aligned} \log \Upsilon_{\boldsymbol{\Lambda}}(\boldsymbol{S}) = \lambda \sum_{\tau_1=1}^{N} s_{\tau_1} + \frac{1}{2} \sum_{\tau_1,\tau_2=1}^{N} \gamma_2(\tau_1, \tau_2) s_{\tau_1} s_{\tau_2} + \\ \frac{1}{3!} \sum_{\tau_1,\tau_2,\tau_3=1}^{N} \gamma_3(\tau_1, \tau_2, \tau_3) s_{\tau_1} s_{\tau_2} s_{\tau_3} + \dots \end{aligned} \tag{8}$$

for $\lambda > \rho$ ($\rho > 0$ is defined such that the series in eq. (8) represents a function which is regular for $\lambda > \rho$ [8, p. 198]). The second characteristic function of Y_λ can then be computed by replacing (8) in (6):

$$\begin{aligned} \Psi_{Y_\lambda}(\boldsymbol{\Omega}) = -j\lambda \sum_{\tau_1=1}^{N} \varphi_{\tau_1}(\lambda) + \lambda \sum_{\tau_1=1}^{N} (e^{j\varphi_{\tau_1}(\lambda)} - 1) + \\ \frac{1}{2} \sum_{\tau_1,\tau_2=1}^{N} \gamma_2(\tau_1, \tau_2)(e^{j\varphi_{\tau_1}(\lambda)} - 1)(e^{j\varphi_{\tau_2}(\lambda)} - 1) + \dots \end{aligned}$$

Consequently, by substituting $e^{j\varphi_k(\lambda)}$ by its power series expansion by using eq. (5) and the hypothesis $\gamma_k(\boldsymbol{\tau}) = o(\lambda^{k/2})$, $k \geq 2$, we obtain:

$$\Psi_{Y_\lambda}(\boldsymbol{\Omega}) = -\frac{\lambda}{2} \sum_{\tau_1=1}^{N} \varphi_{\tau_1}(\lambda)^2 + \mathrm{o}(1). \tag{9}$$

Finally, the limit of $\Psi_{Y_\lambda}(\boldsymbol{\Omega})$ can be computed noticing that the sum in the first term is the norm of $\boldsymbol{\varphi}_\lambda$:

$$\begin{aligned} \lim_{\lambda \to +\infty} \Psi_{Y_\lambda}(\boldsymbol{\Omega}) &= \lim_{\lambda \to +\infty} -\frac{\lambda}{2} \sum_{q=1}^{N} \frac{\omega_q^2}{(d_q + \sqrt{\lambda})^2} \\ &= -\frac{1}{2} \sum_{q=1}^{N} \omega_q^2. \end{aligned} \tag{10}$$

Eq. (10) shows that the second characteristic function of Y_λ converges to the second characteristic function of the multivariate Gaussian distribution $\mathcal{N}(0, I_N)$ when $\lambda \to \infty$. Consequently, Y_λ converges in distribution to the multivariate Gaussian distribution $\mathcal{N}(0, I_N)$. ■

The vector $\boldsymbol{Y}_\lambda$ converges in distribution to the multivariate Gaussian distribution $\mathcal{N}(0, I_N)$. Consequently, the distribution of $\boldsymbol{Y}_\lambda$ can be well approximated for large λ by its asymptotic distribution (see for instance [1, p. 204]). Hence, eq. (3) and assumption A1, A2 imply that the distribution of $\boldsymbol{X}$ can be approximated by the multivariate Gaussian distribution $\mathcal{N}(\lambda \boldsymbol{u}, C_{\Lambda\Lambda} + \lambda I_N)$. Eq. (2) shows that second order stationarity for $\boldsymbol{\Lambda}$ implies second order stationnarity for $\boldsymbol{X}$. Moreover, assumption A2 implies that the variance of x_n, which equals $\lambda + \mathrm{var}[\lambda_n]$, can be approximated by λ.

Finally, *parametric AR modelling* for $\boldsymbol{X}$ is motivated in the stationary case by the fact that for any continuous spectral density $S(f)$, an AR process can be found with a spectral density arbitrary close to $S(f)$ ([1, p. 132]). Classical justifications for this model in the stationary Gaussian context such as the Wold decomposition can also be found in [9].

Based on the previous comments, this paper proposes to approximate $\boldsymbol{X}$ by a pth order CGARP defined by:

$$x_n = -\sum_{k=1}^{p} a_k x_{n-k} + \lambda \sum_{k=0}^{p} a_k + e_n, \tag{11}$$

where $a_0 = 1$ and e_n is an i.i.d. zero mean Gaussian sequence. The variance of e_n in model (11) is such that $\mathrm{var}[x_n] = \lambda$. It is denoted $\sigma_e^2(\boldsymbol{a}, \lambda)$ in order to take into account its dependence toward the a_k and λ. A formal expression of $\sigma_e^2(\boldsymbol{a}, \lambda)$ as a function of the a_k and λ is:

$$\sigma_e^2(\boldsymbol{a}, \lambda) = \frac{\lambda}{\boldsymbol{e}_1^t (A_1 + A_2)^{-1} \boldsymbol{e}_1} = \lambda \sigma_e^2(\boldsymbol{a}, 1), \tag{12}$$

where the matrices A_1 and A_2, which are function af the a_k, are given in [4] and $\boldsymbol{e}_1 = (1, 0, \dots, 0)^t$.

3 Cramér Rao lower bounds

CRLB's are convenient tools for determining the achievable accuracy of estimators. This section derives the asymptotic CRLB's for the parameters $\boldsymbol{a}$ and λ of the model (11).

Proposition 2 *The asymptotic Cramér Rao Lower Bounds* *for the parameters of a CGARP defined in eq. (11) satisfy:*

$$\lim_{N\to+\infty} N.\mathrm{CRLB}(\lambda) = \Bigg[\frac{2\lambda^2}{\sigma_e^2(\boldsymbol{a},\lambda)} + \nabla_{\boldsymbol{a}}(\lambda)^t C_p^{-1}\nabla_{\boldsymbol{a}}(\lambda) - \mu\left(\frac{2\lambda^2}{\sigma_e^2(\boldsymbol{a},\lambda)} + \nabla_{\boldsymbol{a}}(\lambda)^t C_p^{-1}\nabla_{\boldsymbol{a}}(\lambda)\right)^2\Bigg]\sigma_e^2(\boldsymbol{a},\lambda),$$

$$\lim_{N\to+\infty} N.\mathrm{CRLB}(\boldsymbol{a}) = \left[C_p^{-1} - \mu C_p^{-1}\nabla_{\boldsymbol{a}}(\lambda)\nabla_{\boldsymbol{a}}(\lambda)^t C_p^{-1}\right]\sigma_e^2(\boldsymbol{a},\lambda),$$

where μ is given by:

$$\mu = \frac{(\sum_{k=0}^{p} a_k)^2}{\left(1 + (\sum_{k=0}^{p} a_k)^2\left(\frac{2\lambda^2}{\sigma_e^2} + \nabla_{\boldsymbol{a}}(\lambda)^t C_p^{-1}\nabla_{\boldsymbol{a}}(\lambda)\right)\right)}.$$

The components of $\nabla_{\boldsymbol{a}}(\lambda)$ equal:

$$\frac{\partial\lambda}{\partial a_k} = -\sigma_e^2\boldsymbol{e}_1^t(A_1+A_2)^{-1}\frac{\partial(A_1+A_2)}{\partial a_k}(A_1+A_2)^{-1}\boldsymbol{e}_1$$

and C_p^{-1}, the inverse of the pth order process covariance matrix, is computed using the Gohberg-Semencul formula.

Proof : The unknown parameter vector for the CGARP defined by (11) is $\boldsymbol{\alpha} = (\lambda, \boldsymbol{a})$. Since there is a one-to-one transformation between $\boldsymbol{\alpha} = (\lambda, \boldsymbol{a})$ and $\boldsymbol{\theta} = (\sigma_e^2, \boldsymbol{a})$, the CRLB's for $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$ are linked by the following relation [7]:

$$\mathrm{CRLB}(\boldsymbol{\alpha}) = \frac{\partial g(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\mathrm{CRLB}(\boldsymbol{\theta})\frac{\partial g(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}^t, \qquad (13)$$

where $\partial g(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$ is the Hessian of the transformation:

$$\frac{\partial g(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} = \begin{pmatrix} \lambda/\sigma_e^2 & \nabla_{\boldsymbol{a}}(\lambda)^t \\ 0 & I_p \end{pmatrix}. \qquad (14)$$

In this expression $\lambda/\sigma_e^2 = 1/\sigma_e^2(\boldsymbol{a}, 1)$ is the partial derivation of λ with respect to σ_e^2 (according to (12)) and $\nabla_{\boldsymbol{a}}(\lambda)$ is the gradient of λ with respect to $\boldsymbol{a}$.

The coefficients of the Fisher Information Matrix (FIM) for a Gaussian process are the sum of a term that only depends on the covariances of the process and a term that takes into account its mean [10]. The first term of the asymptotic FIM for $\boldsymbol{\theta}$ is the well-known asymptotic form given for example in [5]. The mean of the process being $\lambda\mathbf{u}$, $\mathbf{u} = (1, \ldots, 1)^t$, the coefficients of the second term are:

$$\left(\frac{\partial\lambda}{\partial\theta_k}\boldsymbol{u}^t\right)C_N^{-1}\left(\frac{\partial\lambda}{\partial\theta_l}\boldsymbol{u}\right) = \boldsymbol{u}^t C_N^{-1}\boldsymbol{u}\frac{\partial\lambda}{\partial\theta_k}\frac{\partial\lambda}{\partial\theta_l}.$$

Consequently:

$$\mathrm{FIM}(\boldsymbol{\theta}) = N\begin{pmatrix} (2\sigma_e^4)^{-1} & 0 \\ 0 & (\sigma_e^2)^{-1}C_p \end{pmatrix} + \boldsymbol{u}^t C_N^{-1}\boldsymbol{u}\nabla_{\boldsymbol{\theta}}(\lambda)\nabla_{\boldsymbol{\theta}}(\lambda)^t, \qquad (15)$$

where:

$$\nabla_{\boldsymbol{\theta}}(\lambda)^t = (\frac{\lambda}{\sigma_e^2}, \nabla_{\boldsymbol{a}}(\lambda)^t). \qquad (16)$$

The inversion of (15) using the inversion lemma and products of bloc matrices yields:

$$\mathrm{CRLB}(\boldsymbol{\theta}) = \frac{\sigma_e^2}{N}\begin{pmatrix} 2\sigma_e^2 & 0 \\ 0 & C_p^{-1} \end{pmatrix} - \frac{\sigma_e^2}{N}\mu_N\begin{pmatrix} 2\lambda \\ C_p^{-1}\nabla_{\boldsymbol{a}}(\lambda) \end{pmatrix}\begin{pmatrix} 2\lambda & \nabla_{\boldsymbol{a}}(\lambda)^t C_p^{-1} \end{pmatrix}, \qquad (17)$$

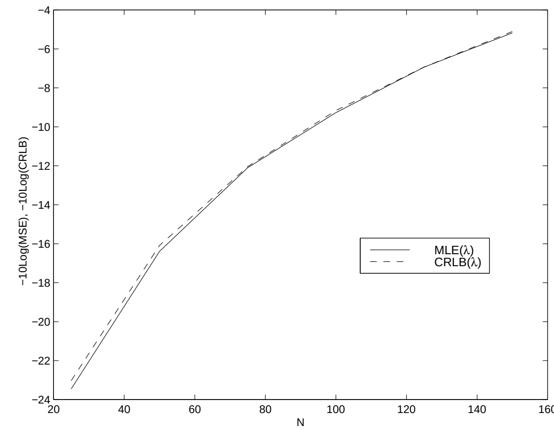

Figure 1: CRLB of λ and estimated MSE of the maximum likelihood estimator of $\lambda = 1000$ for $p = 1$, $a_1 = 0.8$.

with:

$$\mu_N = \frac{\sigma_e^2\boldsymbol{u}^t C_N^{-1}\boldsymbol{u}}{N\left(1 + \frac{\sigma_e^2}{N}\boldsymbol{u}^t C_N^{-1}\boldsymbol{u}\left(\frac{2\lambda^2}{\sigma_e^2} + \nabla_{\boldsymbol{a}}(\lambda)^t C_p^{-1}\nabla_{\boldsymbol{a}}(\lambda)\right)\right)}.$$

The inverse of C_N is computed using the Gohberg-Semencul formula [10] which yields:

$$\boldsymbol{u}^t C_N^{-1}\boldsymbol{u} = \frac{1}{\sigma_e^2}\left((N-p)(\sum_{k=0}^{p} a_k)^2 + \sum_{q=1}^{p}(\sum_{k=0}^{p-q} a_k)^2 - \sum_{q=1}^{p}(\sum_{k=q}^{p} a_k)^2\right).$$

Hence:

$$\lim_{N\to+\infty}\frac{\boldsymbol{u}^t C_N^{-1}\boldsymbol{u}}{N} = \frac{1}{\sigma_e^2}(\sum_{k=0}^{p} a_k)^2$$

Replacing this expression in μ_N yields: $\mu = \lim_{N\to+\infty}\mu_N$.

The substitution of eq.'s (14), (17) in (13) terminates the proof. Note that $\nabla_{\boldsymbol{a}}(\lambda)$ can be computed from (12) as follows:

$$\frac{\partial\lambda}{\partial a_k} = \sigma_e^2\frac{\partial(\sigma_e^2(\boldsymbol{a},1))^{-1}}{\partial a_k} = \sigma_e^2\frac{\partial\boldsymbol{e}_1^t(A_1+A_2)^{-1}\boldsymbol{e}_1}{\partial a_k} = -\sigma_e^2\boldsymbol{e}_1^t(A_1+A_2)^{-1}\frac{\partial(A_1+A_2)}{\partial a_k}(A_1+A_2)^{-1}\boldsymbol{e}_1.$$

■

The theoretical expressions of the CRLB's have been compared to the MSE's of the parameter MLE's. For this purpose, 1000 independent realizations of the signal (11) with $p = 1$, $a_1 = 0.8$, $\lambda = 1000$ have been generated for different values of N. The parameter MLE's have been determined for each realization and the corresponding MSE's have been computed. A comparison between the estimated MSE's and the CRLB's is depicted in fig. 1 and 2 . This result shows the perfect adequation between the estimated CRLB and the MSE.

Next simulations compare the CRLB's for the CGARP parameters corresponding to model (11) with the CRLB's for a standard unconstraint Gaussian AR Process (GARP) with mean λ, AR parameter vector $\boldsymbol{a}$ and driving noise

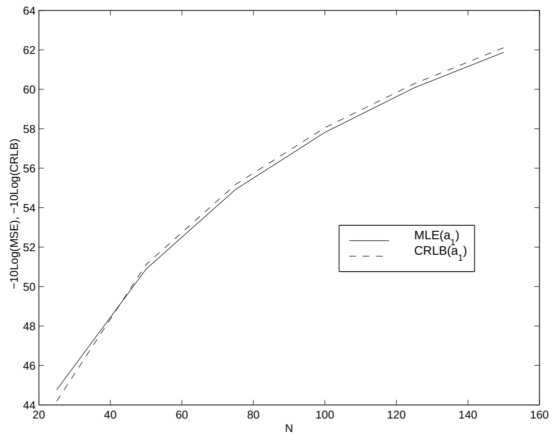

Figure 2: CRLB of a_1 and estimated MSE of the maximum likelihood estimator of $a_1 = 0.8$ for $p = 1$ and $\lambda = 1000$.

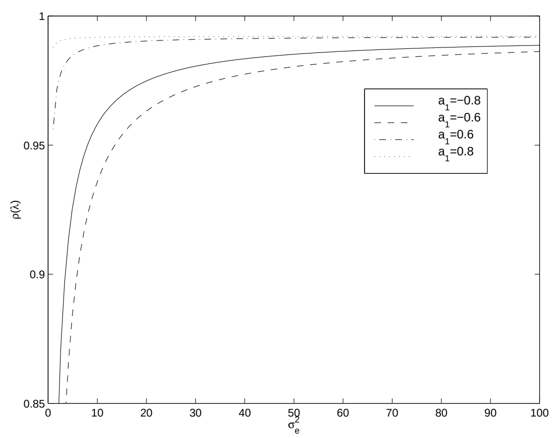

Figure 3: Comparison between the CRLB of the mean for the CGARP and the GARP.

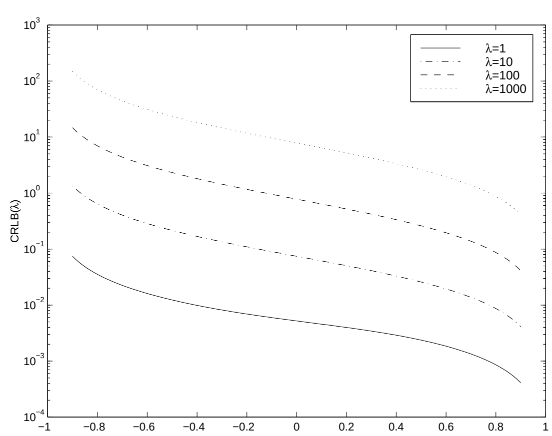

Figure 4: CRLB of λ for the first order model ($N = 128$).

variance σ_e^2 [5]. The latter are denoted $\text{CRLB}_u(\boldsymbol{\theta})$. The CRLB's are compared for a first order model by means of $\rho(\lambda) = \text{CRLB}(\lambda)/\text{CRLB}_u(\lambda)$, where $\text{CRLB}_u(\lambda) = \sigma_e^2/(N(\sum_{k=0}^{p} a_k)^2)$.

Fig. 3 shows $\rho(\lambda)$ for different values of a_1. The results are plotted as functions of σ_e^2 to guarantee that the two processes have the same variance. Since $\rho(\lambda) < 1$ for any λ, these results prove the better identifiability of CGARP's with respect to unconstraint GARP's.

Finally, fig. 4 depicts the behavior of $\text{CRLB}(\lambda)$ as a function of a_1 for different values of λ. It shows that, similarly to the unconstraint case, the bounds increases as λ increases.

4 Conclusions

This communication studied a new model for the high flux photometric signal denoted constraint Gaussian AR model. This model was justified by deriving the asymptotic distribution of the photometric signal. The Cramer Rao bounds for the CGARP parameters were derived. These bounds were then compared to the maximum likelihood estimator mean square errors computed from Monte Carlo simulations. The CGARP parameter CRLB's were also compared to the bounds obtained for the usual unconstraint Gaussian AR processes. The influence of light intensity was also analyzed.

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References

[1] P. J. Brockwell and R. Davis. *Time Series: Theory and Methods.* Springer Verlag, 2nd edition, 1990.

[2] A. Dempster, N. Laird, and D. Rubin. Photometric detection of extrasolar planets by the transit method. *ASP conf series*, pages 216–223, 1997.

[3] A. Ferrari, J. Tourneret, and G. Alengrin. Parametric modeling of photometric signals. *IEEE Transactions on Signal Processing*, 1999. submitted.

[4] A. Ferrari, J. Tourneret, and F.-X. Schmider. Detection of extra solar planets using parametric modeling. In *IEEE International Conference on Acoustics, Speech and Signal Processing*, pages 1369–1372, 1998.

[5] B. Friedlander and B. Porat. The Exact Cramer-Rao Bound for Gaussian Autoregressive Processes. *IEEE Transactions on Aerospace and Electronic Systems*, 25(1):3–8, Jan. 1989.

[6] J. Goodman. *Statistical Optics.* Wiley interscience, 1985.

[7] S. M. Kay. *Fundamentals of Statistical Signal Processing : Estimation Theory.* Signal Processing Series. Prentice Hall, 1993.

[8] E. Lukacs. *Characteristic Functions.* Griffin London, 2nd edition, 1970.

[9] M. B. Priestley. *Spectral Analysis and Time Series.* Monographs and Textbooks. Academic Press, 8th edition, 1994.

[10] P. Stoica and R. Moses. *Introduction to Spectral Analysis.* Prentice Hall, 1997.