ECG SIGNAL DENOISING USING WAVELET DOMAIN WIENER FILTERING

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ABSTRACT
A two-stage algorithm for suppression of electromyogram (EMG) artifacts from the electrocardiogram (ECG) using Wavelet Domain Wiener Filtering has been investigated. An improvement of the traditional technique is proposed by involving Time-frequency dependent threshold for calculation of the pilot estimate in the first stage. The appropriate choice of the wavelet basis functions used in each stage has been stressed. The strong relationship between the wavelet function’s support and the ECG morphology has been emphasized. The preliminary assumptions have been argued by experiments on a wide range database. They have shown that an appropriate choice of the decomposing wavelets for the two algorithm stages can considerably improve the quality of the denoised signal.

1. INTRODUCTION
The presence of parasite interference signals could cause serious problems in the registration of the ECG signals. Most common are power line interference, EMG signals, motion artifacts, and base line (drift) interference. While there are well-developed methods for power line interference and drift suppression there are still problems in EMG signal suppression, due to considerable overlapping of the frequency spectra of both types of signals. Thus, the automatic interpretation, following accurate detection of characteristic ECG points and waves and measurement of signal parameters is, become extremely difficult, sometimes virtually impossible task.

Adequate ECG denoising algorithms and procedures should:

• Improve signal-to-noise ratio (SNR) for obtaining clean and readily observable recordings, yielding the subsequent use of straightforward approaches for correct automatic detection of characteristic points in the ECG signal and recognition of its specific waves and complexes;
• Preserve the original shape of the signal and especially the sharp Q, R and S peaks, without distorting the P and T waves and the smooth transition of the ST-T segment.

Recently some new techniques based on global and local transforms have became popular in connection with signal denoising. As a first step the signal is decomposed into a transform domain where filtering procedures are applied. The noise-free signal is obtained by an inverse transform. Choosing appropriate basis functions for successful decorrelation of the signal and designing transform domain filters accommodated to the ECG signal morphology could turn this techniques into powerful means for ECG signal denoising.

A method for ECG denoising based on Wavelet Shrinkage approach [1] using Time-Frequency Dependent Threshold (TFDT) has been proposed in [2]. Generally speaking, the TFDT is high for the non-informative wavelet coefficients, and low for the informative coefficients representing the important signal features. Although giving better results in comparison with other ECG denosing methods, the latter has certain disadvantages: some oscillations may occur in the ends of the QRS complexes using long-length decomposition filters due to the poor time localisation of the basis functions; and in opposite – very short-length filters may corrupt the shapes of the “slow” P and T waves.

In the present study we aim to improve the denoising procedure by involving Empirical Wiener Filtering in Wavelet Domain [3] using the TFDT for calculation of the “pilot signal estimation”. We take advantage of this two-stage algorithm dividing our task on two subtasks:
1. Assuring good pilot estimation of the QRS areas using TFDT and wavelet basis functions with short support (short length filters), and
2. Refinement the shapes of the P and T waves using medium length filters.
2. METHOD

2.1. Wavelet Domain Wiener Filtering

Let us consider a discrete signal $s$, corresponding the ECG, mixed with EMG noise $n$: $x = s + n$, $s, n \in \mathbb{R}^N$. The procedure of de-noising contains two steps and can be described as follows:

**Stage 1.** The signal-noise mixture $x = s + n$ is decomposed in $W_1$ wavelet domain; the wavelet coefficients $y_1$ are shrinked using wavelet domain filter $H_{sh}$; the "pilot estimate" of the signal is calculated by inverse wavelet transform of the shrinked wavelet coefficients $\hat{y}_1$; and finally the coefficients estimate $\hat{y}_{21}$ in $W_2$ wavelet domain is obtained:

$$\hat{y}_{21} = W_2 W_1^{-1} H_{sh} W_1 x$$

(1)

**Stage 2.** The wavelet coefficients of the signal-noise mixture in $W_2$ domain $y_2$ and their estimate $\hat{y}_{21}$ obtained in **Stage 1** are used to design an optimal in MSE sense Wiener Filter $H_{wf}$

$$H_{wf} (j, k) = \frac{\hat{y}_{21} (j, k)}{\hat{y}_{21} (j, k)^2 + \sigma(k)^2},$$

(2)

where $j$ is the time position and $k$ is the scale position.

The denoised signal is obtained by inverse wavelet transform of the filtered by $H_{wf}$ wavelet coefficients $\hat{y}_2$:

$$\hat{s} = W_2^{-1} H_{wf} W_2 x.$$  

(3)

Here $H_{wf}$ is a diagonal matrix containing $H_{wf} (j, k)$ in the main diagonal and $H_{sh}$ is a diagonal matrix containing the TFDST [2]. The noise variance for each scale $\sigma(k)$ is estimated using the wavelet coefficients in $W_2$ domain representing the areas outside the QRS complexes.

Several analytical conclusions on the above algorithm in the case a hard shrinkage threshold used for **Stage 1** have been done in [4]. It has been shown that the change of the basis functions from $W_1$ to $W_2$ leads to compensation of the MSE due to the model mish-mashes.

However, it is not clear how to choose $W_1$ and $W_2$ for a given application. Together with the TFDST as first step in the ECG denoising procedure in the present work we have also investigated appropriate couples of wavelet bases $W_1$ and $W_2$.

2.2. Choice of filter bank for $W_1$

Let us consider wavelet decomposition at dyadic scales. If we assume a sampling rate of 200 Hz, the energy of the ECG in the first three high frequency scales of the wavelet decomposition is concentrated in the QRS area. This will produce high value coefficients through these scales and, as has been argued in [4], even if their estimation in the **Stage 1** is not very good, this would not affect significantly the final result of the de-noising. However, using long-length filters some coefficients caring information for both high frequency QRS complex and low frequency PQ and ST areas may occur, due to the poor time localization of the basis functions. These coefficients are more influenced by the noise and after their shrinkage some oscillations in the pilot estimate may occur in the transient PQ/ST areas. Such oscillations are very annoying for ECG and cannot be compensated in the **Stage 2** due to the small values of the wavelet coefficients representing them. Hence, we propose the pilot estimate to be obtained using wavelet basis with good time localization i.e. with short length corresponding filter aiming concentration of the QRS energy for each scale in as less numbers of coefficients as possible.

The usage of short filters in **Stage 1** leads to corruption of the shapes of the P and T waves due to the poor frequency localization of the corresponding wavelet basis functions. Although all the wavelet coefficients representing PT areas in the first three high frequency scales are suppressed by the TFDST and all the coefficients in the rest of the scales are left unchanged, some high frequency components are kept and some low frequency components are lost after the signal restoration. This is due to the fact that the frequency responses of the filters are not perfect high-pass/low-pass. This effect will be stronger when the filter has shorter length and softer increasing the filter length. However, the model mishmash would compensate this error. We can expect that the usage of a filter having longer length and hence providing better frequency localization in the **Stage 2** will concentrate the energy of the P and T waves in the low frequency scales producing high value wavelet coefficients – i.e. the error caused from the poor estimation will be lower.

2.3. Choice of filter bank for $W_2$

Achieving good signal estimation in the transient PQ and ST areas we need to improve the shapes of the P and T waves in **Stage 2**. For this purpose we have to use filters with good frequency localization. According to the discussion in the previous subsection, we have to keep away the very long-length filters, which may produce some oscillations in the transient PQ and ST areas. Hence we propose the filters for $W_2$ decomposition to be with medium length in order to improve the P and T shapes and in the same time to preserve the PQ and ST areas.

3. RESULTS

We have used an ECG database containing 192 8-channels normal and pathological signals from different patients. Each of the signals has been mixed with EMG noise achieving SNR=14dB.

We have performed a number of experiments searching the most suitable bases for $W_1$ and $W_2$. In Experi
ment 1 and Experiment 2 we have treated the Stage 1 and Stage 2 separately. The appropriate couples of W1 and W2 are investigated in Experiment 3.

3.1. Experiment 1

The signals have been de-noised using Wavelet Shrinkage and TFDT. The residual signals (the difference between the noise free signal and the de-noised signal) have been obtained for the whole set of bases of Daubechies’ wavelets, symmlets, coiflets, biorthogonal spline wavelets, and Villasenor’s wavelets. The corresponding SNRs have been averaged over all signals and channels. Figure 1a shows the relationship between the averaged SNR and the length of the filters when orthogonal wavelets – Daubechies’ wavelets, symmlets, and coiflets have been used. It can be seen from the figure that though the maximum is achieved for wavelets which corresponding filter is with medium length, the short length filters gives good results in SNR sense.

![Figure 1](image)

Figure 1. SNRs versus filter length; a) for Stage 1 only; b) for Stage 2 only

A typical signal from the ECG database is shown in Fig. 3a. The same signal mixed with EMG noise is shown in Fig. 3b. The denoising results using Coiflet4 (C4) and Daubechies’2 (D2) wavelets are shown in Fig. 3c and 3d, respectively. In the C4 case (filter length 24) the achieved SNR is about 1 dB higher than for the D2 case (filter length 4). Nevertheless we prefer to use D2 in Stage 1 of the algorithm since the oscillations produced by the longer filters cannot be compensated in Stage 2. This fact is emphasized in the C4 case before and after each QRS complex.

3.2. Experiment 2

The signals have been de-noised using Wavelet Domain Wiener Filtering with \( \hat{y}_{21} = W_{12} s \), i.e. we used the original signal as a pilot estimate. The averaged SNRs are obtained for the whole set of wavelet bases used in Experiment 1 and the relationships between the averaged SNRs and the length of the filters when orthogonal wavelets have been used are shown in Fig. 1b. It can be concluded from the figure that there is no significant difference in SNR sense which wavelet should be used. However, as we argued in Experiment 1 the filters having longer lengths produce oscillations in the transient PQ and ST areas. This can be seen in Fig. 3e and 3f where the signals after denoising using Symmlet10 (filter length 20) and biorthogonal Spline23 (filter length 3 and 5) are shown.

3.3. Experiment 3

The signals have been de-noised with the proposed algorithm using all the combinations of W1 and W2 of the wavelets used in Experiment 1 and Experiment 2. Fig. 2 shows the relationship between the averaged SNR and the length of the filters for W1 and W2 when Daubechies’ wavelets have been used. Our experiments argued that best results are obtained using basis function with short corresponding filter for calculation of the pilot estimate of the signal (Stage 1) and a filter with a medium length for Stage 2 of the algorithm.

![Figure 2](image)

Figure 2. SNR as a function of filter lengths

Figure 3g shows the signal after denoising using Daubechies2 and biorthogonal Spline23 wavelets for Stage 1 and Stage 2. It can be seen from the figure that the shape of the denoised signal is very close to this in the ideal case, when the original signal is used as a pilot estimate. The amplitudes and the shapes of the very sharp R and S waves have been preserved and the noise has been removed from the important areas before and after the QRS complex. The ST area is denoised in a way the parameters of the T wave – the duration and amplitude – to be easy to be extracted. The QRS onset and offset and the T wave onset and offset are easy observable. In addition a SNR over 20dB is achieved (only 0.3dB below the ideal case).

Table 1 shows the denoising result averaged over the whole database. The D2 and biorthogonal Spline23 wavelets for Stage 1 and Stage 2, respectively, have been used. First two columns present the SNR before and after denoising, respectively. RSmean is the relative R and S reduction, averaged over all QRS complexes in the signal. RSmax is the maximal relative R or S reduction. It could be seen from the table that the SNR has been improved with about 6 dB for all levels of the noise energy. RSmean and RSmax depend very slightly on the noise energy. Furthermore, their values are very small – far below the standard requirement of 0.1mV for 1mV signal peak-to-peak amplitude [5].

As a conclusion the experimental results have showed that the proposed two-stage denoising approach leads to very effective suppression of the parasite EMG
and in the same time provides preservation of the shapes and amplitudes of the important waves of the ECG.

<table>
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<th>SNR in, [dB]</th>
<th>SNR out, [dB]</th>
<th>Rsmean, [%]</th>
<th>Rsmax, [%]</th>
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<td>1.2</td>
<td>2.4</td>
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<tr>
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<td>1.1</td>
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<td>0.9</td>
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Table 1. Denoising results with the clean signal as a reference signal

4. CONCLUSIONS

We have applied Wavelet Domain Wiener Filtering for the electrocardiogram (ECG) denoising. We have proposed an improvement of the algorithm by involving Time-frequency dependent threshold for calculation of the pilot signal estimate. Our assumption was based strongly on the ECG features. We took advantage of the two-stage algorithm dividing the ECG denoising task into two subtasks: assuring good pilot estimate especially of the QRS areas using wavelet basis functions with short-length corresponding filters in the Stage 1, and refining the shapes of the P and T waves using wavelet functions with medium-length corresponding filters in Stage 2. By experiments on a wide range database, we have qualified the applicability of the Empirical Wiener filtering in wavelet domain for the particular case of ECG denoising. Assuming a decorrelating transform close to the optimal, we have shown how important is the choice of the decomposing basic functions. Further practical improvements should be achieved by exploration of possible dependencies of the transform domain coefficients.

5. REFERENCES


Figure 3. a) noise-free ECG; b) the same mixed with EMG; denoised with: c) C4 (W1 only); d) D2 (W1 only); e) Symlet10 (W2 only); f) Spline23 (W2 only); g) D2 (W1) and Spline23 (W2).