Investigation have been carried out on the iterative method proposed by Wu-Sheng [11] to design an efficient QMF banks with approximately constant group delay. Simulations show that it is quite sensitive to the choice of the initial filter. A new and important benefit of manipulating the initial filter design of QMF banks is its ability to design QMF banks with low and approximately constant group delay. By using a linear minimum phase efficient method proposed by Ivan [4] as the initial filter, improvements in the most of design specification in [11] have been obtained. The constraint of the transition band is added in the initial condition. Computer simulation shows that the proposed approach produce better results than those reported previously. However, the proposed approach requires more computational efficiency compared with existing methods.

The design problems of FIR (QMF) filter banks have received considerable attention since their introduction by Croisier. Research concerning a QMF filter bank has been carried out during the past two decades. They have been widely used in one-dimensional (1-D) and two-dimensional (2-D) signal processing [5] and have found applications in many areas such as subband coding of speech, images and telecommunication system [8]. Most of the methods for designing QMF banks employ iterative algorithms to directly minimize the associated error measure in the filter design process [1]. A particular and often used case of subband coder is the two-channel system depicted in Fig.1.

Fig.1 A Two Channel Filter Bank.

The analysis filter bank consists of $H_0(z)$ and $H_1(z)$, and the synthesis filter bank consists of $F_0(z)$ and $F_1(z)$. The signal $x(n)$ is split into two frequency subbands by two analysis filters $H_0(z)$ and $H_1(z)$ according to the energy distribution of the signal $x(n)$ in the frequency domain. The reconstruction signal $\hat{x}(n)$ differs from $x(n)$ due to three reasons: aliasing, amplitude distortion, phase distortion. The first step of the design process is to cancel aliasing effects. Amplitude distortion and/ or phase distortion can be minimized or eliminated [8,9].

In the frequency domain, many design algorithms can be found in [9] and its references therein. One particular design problem is formulated as minimization of the linear combination of the reconstruction error for the QMF and stop band error for the prototype lowpass filter, measured in quadratic norm was first proposed by Jonston [3]. It consist of selecting the filter coefficients such that $|H(w)|^2+|H(\pi-w)|^2$ is made as close to unity as possible while simultaneously minimizing the stopband energy of the transfer function $H(w)$. Several algorithms have been developed in the literature. The most notable ones is the iterative algorithm developed in [1], which have the appealing property that the minimum solution at each iterative step is easy to compute. In [1], Chen and Lee introduced an iterative method that uses a linearization of the error function, which speeds up convergence. This method needs less computation than other QMF design methods [3] and produces improved filter banks. However, the objective function involves two integrals that are evaluated by discretization. This gives rise to two problems. First, the solution obtained actually minimizes the discretized version of the objective function rather than the objective function itself, which can degrade the performance of the QMF bank designed. Second, in order to reduce the performance degradation, the density of sample points needs to be high, which leads to increase computational complexity.

To avoid the drawbacks of Chen and Lee method another iterative technique proposed by Wu-Sheng method [11] is used. This technique depends on a self-convolution to reformulate a forth-order objective function as a quadratic function whose minimization leads to the design of the two-channel QMF banks. The reformulated optimization problem can be solved by an iteration technique in which the major part of each iteration is carried out in terms of a closed-form formula. As a result, filter banks with better performance can be designed with considerably reduced
computational complexity relative to that in several existing design methods.

In some applications, QMF banks with reconstruction delays less than \(N-1\) are desired. Two-channel QMF systems are widely used for tree-structured subband coding systems. These systems usually suffer from relatively long reconstruction delays, which is a serious problem in many communication systems. Thus the design of two-channel QMF banks with low reconstruction delay is desired. However, research in the design of exactly reconstructing filter banks with minimum delay has been virtually nonexistent.

Consequently, the filter banks designed by Wu-Sheng have alternative reconstruction delay in the transition band, which are highly undesirable in some applications. From experiments, we noticed that the undesirable artifacts can occur in the amplitude responses of the analysis and synthesis filters when the reconstruction delay \(k_d\) is significantly smaller than \(N-1\). Such artifacts have been observed as well by other researchers when designing low-delay two-channel filter banks [3, 6, 7]. Artifacts can occur in the transition band of the prototype filter when the group delay is low and if there is no constraint on its transition band. These artifacts then lead to artifacts in the amplitude responses of the analysis and synthesis filters. One of solutions for this problem is to modify the objective function by including an additional term in order to control these artifacts [6]. This solution will forces the passband filter responses of the analysis and synthesis filters to be much larger than the transition responses.

In this paper, we are able to use the iterative method of Wu-Sheng to design an efficient QMF banks having low reconstruction delay. Employing the method of Ivan [4] to add the constraints of the transition-band not in the objective function but in the initial condition in order to achieve a constant group delay in the design of FIR QMF banks with good peak reconstruction error and stopband attenuation properties. The problem is formulated to design an approximately constant low delay QMF bank, which is highly desired in some applications. Our proposed method improved the method of WU-Sheng in obtaining a constant group delay; it is based on manipulating the initial filter impulse response of Wu-Sheng using Ivan method [4].

The outline of this paper is as follows. The basic equations of how to design QMF banks are given in Section II. Section III presents the proposed design procedure. One simulation example and the performance of the QMF bank designed by the proposed technique for illustration and comparison is presented in section IV. Finally, we conclude this paper in section V.

## II. Formulation of QMF filter banks design

The reconstruction signal in the two-channel QMF system of Fig. 1 is related to the input signal by

\[
\hat{X}(z) = T(z)X(z) + A(z)\hat{X}(-z)
\]

where

\[
T(z) = 0.5[H_0(z) F_0(z) + H_1(z) F_1(z)]
\]

and

\[
A(z) = 0.5[H_0(-z) F_0(z) + H_1(-z) F_1(z)]
\]

Where \(T(z)\) represents a linear shift-invariant system response that is desired signal translation from \(x(n)\) to \(\hat{x}(n)\). \(A(z)\) represents the aliasing error due to the change of the sampling rate in the filter bank. The general objective function for the design of several types of QMF banks can be formulated as

\[
E = E_1 + \alpha E_2
\]

\[
E_1 = \int_{\omega = 0}^{\pi/2} \left[ |H_0(e^{j\omega})|^2 + |H_0(e^{j(\omega + \pi)} + e^{jkdw})|^2 \right] dw
\]

\[
E_2 = \int_{\omega = \pi}^{\pi} |H_0(e^{j\omega})|^2 d\omega
\]

The first term denotes the reconstruction error and the second term denotes the stopband error. The weight \(\alpha\) in (2a) is a relative positive between the reconstruction error and the stopband error that can be used to control stopband attenuation for the filter \(H_0\) and \(\omega_s\) is the frequency of the stopband edge. Where \(k_d\) is the system delay. Note that if filter \(H_0\) has a linear phase response and \(k_d = N-1\) is the order of \(H_0(z)\), then the filter bank is a conventional QMF bank; if \(k_d < N-1\) and one does not assume that the filter \(H_0\) has a linear phase response, then the filter bank has a low reconstruction delay. In (2) depending on the numerical value of \(k_d\) and whether or not \(H_0\) has a linear phase response one can represent both the conventional and low delay QMF banks [9].

## III. Proposed Design Procedure

In the paper we proposed to use the method of [4] for initiating the design process. The reason for this is that we are able to have the constraint in the transition band of the initial filter design. The coefficients of the new impulse response are applied as an initial condition for the iterative process. Our proposed approach is based on manipulate the initial filter impulse response in order to improve the design specifications. The resulting QMF bank has a lower reconstruction delay furthermore lower peak reconstruction error (PRE) compared to the method [11]. Nevertheless, the examples demonstrate that our approach can be used to
design QMF banks with a reconstruction delay that is small enough for many applications. In effect, we have been able to obtain an improved two-channel low reconstruction delay QMF banks while using our proposed approach. At each iteration, the vector of Lagrange multipliers $\mu$ is calculated and the vector of the known filter parameter $a$ is evaluated. If the minimum value of Lagrange multiplier is below a certain predefined value (min $\mu < 0$) the algorithm is continue. Otherwise, it goes back and does the calculations again. On each iteration, the constraint set is updated so that at convergence, the only frequency points at which equality constraints are imposed are those where $A(w)$ touches the constraint [2]. The equality constrained problem is solved with Lagrange multipliers. According to the Kuhn-Tucker conditions, the solution to the equality-constrained problem solves the corresponding inequality constrained if all the Lagrange multipliers are non-negative [10].

IV. Design Example and Comparisons

In this section, we illustrate the effectiveness of the initial design filter on the performance of the QMF bank regarding the group delay. One computer simulation of designing QMF was carried out on a Pentium/100 personal computer by running MATLAB programs code. The performances of the filter banks designed are examined in terms of computational efficiency, reconstruction errors and CPU time considerations. The performance criteria are as follows: the number of iterations (NOI); number of millions of floating point operations (MFLOP); minimum stop-band attenuation ($A_s$); maximum pass-band ripple $A_p$; maximum Reconstruction error (PRE); signal-to-reconstruction ratio SNR and CPU time for the two methods.

Comparisons are carried out in terms of the performance and group delay characteristic of the filter banks obtained. The performance evaluations were made as in [11]. The reconstruction performance of the example system is examined by computing the signal-to-reconstruction noise SNR for two different signals. SNR is computed value of SNR for a step input signal 1024 samples long and SNR is the value of the SNR for 1024 sample random input signal. All the SNR are in decibels units. The constant $\alpha$ which is a relative weight between the reconstruction error and the stopband error that can be used to control the stopband attenuation for the filter $H_0$ is chosen 0.3 as in [11]. The prescribed $\varepsilon$ that is used for terminating the design process is $10^{-4}$. The iterations continue until $\|h_0 - h_1\|$ less than $\varepsilon$ is satisfied. Computer simulations show that the proposed approach produced better results than those reported in [11]. For comparative purpose, the magnitude response of the analysis and synthesis filters is presented.

In this design example, the frequency of the stopband edge $w_p = 0.2$, the frequency of the passband edge $w_p = 0.3$ and the reconstruction delay $k_p = 15$ were selected as in [11, Example 2]. The initial $h_0$ was obtained by first designing a linear phase filter using [4] and truncating its impulse response to the length $N/2 + \text{floor}(k_p/2) + 1$ and padding it with length $N/2\text{-floor}(k_p/2) - 1$ zeros. The significant design results of the resultant QMF banks are presented in Table 1. A comparison is made between our proposed approach and [11]. However the passband ripple of the filter obtained in the method in [11] is slightly better than our proposed as can be deduced from Table 1. To aid the user, the amplitude response of the analysis filters for the resulting QMF are depicted in Fig. 1. The inset shows the scaled reconstruction error and the scaled group delay characteristic. To compared with [11, Fig. 5, 6, 7] we noticed that there is an improvement in the group delay in the transition band and also an improvement in reconstruction error. The designed filter has approximately linear phase in the passband as shown in the inset. Clearly, the proposed designed approach produces an approximately constant group delay in the transition band. Also, we can deduced that there is an improvement in the performance of the QMF designed by the proposed approach such as the passband attenuation of about 2dB, a saving in the PRE of about 23.3% and the undesirable artifacts are avoided.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Initial Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Window</td>
<td>Proposed</td>
</tr>
<tr>
<td>NOI</td>
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</tr>
<tr>
<td>MFLOPs</td>
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<tr>
<td>$A_s$ (dB)</td>
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<tr>
<td>$A_p$ (dB)</td>
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</tr>
<tr>
<td>SNR, (dB)</td>
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<td>SNR, (dB)</td>
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<tr>
<td>PRE (dB)</td>
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</tr>
<tr>
<td>CPU time(s)</td>
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</tr>
</tbody>
</table>

However the passband ripple of the filter obtained in the method [11] is slightly better than our proposed approach as can be deduced from Table 1. The price paid for these improvements is the increase in computation, CPU time and the number of iterations required compared with [11].

V. Conclusions

The purpose of this paper to propose a method for designing an FIR QMF banks with low reconstruction delay. The iterative method proposed by Wu-Sheng was examined. The proposed method discussed in this paper depends on Ivan method [4], which has been applied as an
initial condition for the design of two-channel PR FIR QMF with low reconstruction delay. Ivan method [4] is simple, efficient, and flexible. A design example was given which demonstrates that the quality of the reconstruction is good. A new and important benefit of manipulating the initial filter design of QMF banks is its ability to design QMF banks with low or minimum delay. System delay has always been regarded as a crucial issue in speech coding. However, research in the design of exactly reconstructing filter banks with minimum delay has been virtually nonexistent. The experimental results show that the signal-to-reconstruction noise ratio of above 70 dB can easily be achieved, which is sufficient for most practical systems. A particularly fascinating result is the ability of the design procedure to design low delay analysis/reconstruction systems with group delay approximately constant in the passband and transition band. This allows the designer to impose any possible reconstruction delay on the system.

References