AN EQUAL GAIN COMBINING DPSK MC-CDMA SYSTEM FOR FAST FADING MOBILE DOWNLINK CHANNELS

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ABSTRACT
This paper describes a multi-carrier code division multiple access downlink system which can be used in mobile or broadcast radio communication applications. It employs a novel form of differential phase shift keying, in which the phase shift keying is applied in the transmitter after all the users symbols have been combined. Theoretical performance predictions and results of simulations are presented and comparison is made with other equivalent MC-CDMA and DS-CDMA downlink systems.

1 INTRODUCTION
Multi-carrier CDMA [1, 2] is a code division multiple access scheme which spreads the signal by coding a number of orthogonal carriers. This approach allows each CDMA chip to be transmitted on a narrowband channel, with negligible inter-symbol interference. Using this approach results in simpler receiver designs when compared with an equivalent bandwidth direct sequence (DS) CDMA system due to the narrowband nature of MC-CDMA sub-channels, allowing narrowband equalisation and linear multi-user detection.

Differential binary phase shift keying (DPSK) [3] allows channel estimation without the need for transmitting pilot tones, producing a system which can exploit more of the available bandwidth. An additional advantage is that the resulting receiver has a lower computational complexity.

Therefore a system which combines these two approaches is desirable. Previous results [4, 5] have shown that if the differential phase shift keying is applied at each symbol, then MC-CDMA DPSK performs as well as DS-CDMA DPSK, but in both cases the multipath diversity is not exploited. The use of MC-CDMA DPSK applied individually to each carrier produced a large performance improvement, but only if the system was restricted to a single user.

This limitation can be overcome by a modification to the (downlink) transmitter which was first presented in [6]. Instead of applying the DPSK to each users signal independently, each users symbol is spread and then the sum of all users spread signals is computed. The differential phase shift keying is then applied before the multi-carrier modulation. The combined energy of all the users symbols is therefore used for the DPSK channel estimation. Since the amplitude of the symbol transmitted on each carrier will be unknown, it is only possible to use this method to estimate the phase shift introduced to each carrier. Consequently the resulting receiver is restricted to equal gain combining (EGC) of the sub-carrier information. The transmitter and receiver for this system are shown in figures 1 and 2.

This paper extends the results in [6] by providing a theoretical framework for estimating the probability of error for different numbers of system users. Further results comparing the system with equivalent MC-CDMA and DS-CDMA systems are also presented.

2 THEORETICAL ANALYSIS
An approximation of the probability of error can be derived for the system in additive white Gaussian noise. If the sum of $M$ users’ chips is represented by:

$$s_t(n) = \sum_{k=0}^{M-1} b_k(n)c_{k,i}$$  \hspace{1cm} (1)

Where $b_k(n)$ represents the data bit of user $k$ at time $n$ and $c_{k,i}$ represents the $i$th users code chip for the $k$th carrier.
Differential phase shift keying can then be applied to produce the narrowband signal transmitted on each sub-carrier:

\[ d_i(n) = s_i(n) \text{sgn}(d_i(n - 1)) \quad (2) \]

For an AWGN channel, the received signal on each sub-carrier is:

\[ y_i(n) = d_i(n) + \eta_i(n) \quad (3) \]

where \( \eta_i(n) \) represents the additive white Gaussian noise. To simplify the analysis, we assume that the previous and present transmitted chip sums were positive, i.e. \( d_i(n) = s_i(n), s_i(n) > 0 \) then any modification of the phase will be due to the quadrature component of the noise. This phase angle is represented by \( \theta_i \):

\[ \frac{y_i(n - 1)^*}{|y_i(n - 1)|} = \exp(-j\theta_i) \quad (4) \]

The resulting estimate of \( s_i(n) \) will be:

\[ \hat{s}_i(n) = \Re\{ (s_i(n) + \eta_i(n)) \exp(-j\theta_i) \} \quad (5) \]

where \( \Re\{x\} \) represents the real part of \( x \). By expanding out equation (5), and inserting it into equation (1), the transmitted data bit, for user \( u \), to be obtained by despreading \( \hat{s}_i(n) \):

\[
\hat{b}_u(n) = \frac{1}{L} \sum_{i=0}^{L-1} c_u i \sum_{k=0}^{M-1} c_k i b_k(n) \cos \theta_i + \Re\{ \eta_i(n) \} \cos \theta_i - \Im\{ \eta_i(n) \} \sin \theta_i \quad (6)
\]

where \( L \) is the processing gain. This can be separated into three components:

\[
\hat{b}_u(n) = \frac{1}{L} \sum_{i=0}^{L-1} c_u i c_u i \hat{b}_u(n) \cos \theta_i + \frac{1}{L} \sum_{i=0}^{L-1} c_u i \sum_{k \neq u} c_k i b_k(n) \cos \theta_i + \frac{1}{L} \sum_{i=0}^{L-1} c_u i (\Re\{ \eta_i(n) \} \cos \theta_i - \Im\{ \eta_i(n) \} \sin \theta_i) \quad (7)
\]

Which can be simplified as:

\[
\hat{b}_u(n) = \frac{b_u}{L} \sum_{i=0}^{L-1} \cos \theta_i + \text{MAI} + N \quad (8)
\]

Where \( \text{MAI} \) represents a component of multiple access interference introduced by the orthogonality loss due to the noise and \( N \) represents the direct effect of the additive Gaussian noise.

Clearly the probability of a bit error is:

\[
P(\hat{b}_u(n) < 0 | b_u = 1) = P\left( \frac{\text{MAI} + N}{L \sum_{i=0}^{L-1} \cos \theta_i} < -1 \right) \quad (9)
\]

If a high number of carriers is used, the variance of \( \sum_{i=0}^{L-1} \cos \theta_i \) will be small allowing it to be replaced by the expected value of \( \cos \theta_i \). This also requires that an odd number of users is assumed, as the zeros transmitted by an even number of users would significantly increase the variance. Under these assumptions the following approximation can be made:

\[
E\left\{ \left( \frac{\text{MAI} + N}{L \sum_{i=0}^{L-1} \cos \theta_i} \right)^2 \right\} \approx E\{ (\text{MAI})^2 \} + E\{ N^2 \} \quad (10)
\]

where \( E\{ \} \) is the expectation operator. Due to the orthogonality of the codes, the MAI is dependent only on the variance of \( \cos \theta \) and therefore the noise due to \( \text{MAI} \) is [7]:

\[
E\{ (\text{MAI})^2 \} = \frac{M - 1}{L} \text{var} \{ \cos \theta_i \} \quad (11)
\]

The additive Gaussian noise component can also be shown to be:

\[
E\{ N^2 \} = \frac{\sigma^2}{L} \quad (12)
\]

Where noise variance \( \sigma^2 = E\{ \Re\{ \eta_i \}^2 \} = E\{ \Im\{ \eta_i \}^2 \} \).

If the normalised interference,

\[
(MAI + N)/(L \sum_{i=0}^{L-1} \cos \theta_i) \quad (13)
\]

is assumed to have a Gaussian distribution, then an approximation of the overall probability of error will be given by:

\[
P_e \approx \frac{1}{2} \text{erfc} \left( \sqrt{\frac{L E\{ \cos \theta \}^2}{2((M - 1) \text{var} \{ \cos \theta \} + \sigma^2)}} \right) \quad (14)
\]

The error in the previous phase estimates for each sub-carrier is defined as \( \theta \) and its cosine can be computed using:

\[
\cos \theta = \frac{s + \Re\{ \eta \}}{\sqrt{(s + \Re\{ \eta \})^2 + (\Im\{ \eta \})^2}} \quad (15)
\]

where \( s \) is the sum of all users’ chips in that sub-carrier. It has a binomial distribution and \( \eta \) is the additive white Gaussian noise.
Comparison between this theoretical performance and simulation results is shown in figure 3 for additive White Gaussian noise. The theoretical performance predicts that at low $E_b/N_0$ ratios, the improvement in channel estimation due to multiple users will provide better overall system performance than the single user case however it also indicates that high $E_b/N_0$ the channel estimation becomes less of a problem than the multi-user interference. This is confirmed by the system simulations. The system however performs significantly worse than a coherent system with perfect knowledge of the AWGN channel, and this is due to the imperfect channel estimation achieved by the DPSK employed.

3 RESULTS

To evaluate the performance of the system in a realistic environment, baseband simulations of the system have been performed using the COST 207 model for mobile channels. A Doppler spread of 300 Hz (corresponding to motion of 160 km/h with a 2 GHz carrier) was used for the simulations to demonstrate the performance of the DPSK channel estimation. The symbol rate was chosen to be 8 kbit/s, giving little time for explicit channel estimation.

Figure 4 shows the performance of the system for a variety of numbers of users, in the fast fading mobile channel. The most obvious effect is the poor performance if there is an even number of users. This is due to the high probability of the signal amplitude on any one carrier being zero when there is an even number of users, because of the binomial distribution of the sum of users chips.

When only odd numbers of users is considered, and the system is compared to the AWGN simulation results in figure 3, a significant deterioration in performance is observed, especially with the 127 user scenario. This indicates that the multi-user interference is a more severe problem in this situation. The performance improvement due to the better channel estimation with higher number of users, is still however observed at low $E_b/N_0$.

To evaluate how this system compares with other equivalent CDMA systems, performance comparisons were made with, (1) a BPSK MC-CDMA EGC system with explicit channel estimation with pilot symbol overheads of 25%, 33% and 50% and (2) a MC-CDMA DPSK system which used a single tap to represent the channel and performed estimation at the symbol level. Comparison was also made with DS-CDMA systems with perfect channel updates at 2 kHz and 5 kHz, and using a 6-tap DPSK rake receiver.

Figures 5, 7 and 7 shows the $E_b/N_0$ required to achieve a desired bit error ratio of 0.02 for a given number of users for each of the four systems, giving an indication of the degradation in performance with an increase in the number of users. The spreading factor was 128 and a single side band system was implemented using an FFT size of 256.

Figure 5 demonstrates that both the coherent EGC and
EGC-DPSK are not significantly affected by an increase in the number of users below 50% capacity. In this region, it is obvious that the coherent EGC systems with 33% and 50% pilot symbol overhead significantly outperform the EGC-DPSK system, by up to 8 dB. However due to the rapid channel variation, even a small reduction in the pilot symbol overhead reduces the performance below that of the DPSK system.

Comparing the DPSK systems in figure 6, the DS-CDMA rake receivers perform best at low numbers of users, but the performance deteriorates rapidly above 1/3 capacity. The channel estimation update rate also has a very significant effect indicating that a large proportion of the bandwidth would also have to be dedicated to tracking the channel. The DS-CDMA DPSK rake receiver also performs well (better than the MC-CDMA DPSK receivers) for a low number of users however this deteriorates rapidly above about 1/6 capacity. The performance of the symbol level DPSK (denoted DPSK-MC) is significantly worse than the other systems.

Comparison with coherent DS-CDMA systems (figure 6) show that for a large number of users, the EGC-DPSK MC-CDMA system is better, but the DS-CDMA RAKE system is better for less than 30-40 users.

4 CONCLUSION

This paper presents analysis and simulations of a novel modulation scheme combining DPSK and MC-CDMA to produce a low complexity receiver which does not require any overhead for channel estimation. This system performs comparatively well in fast fading channels and could therefore be suitable for broadcast or extra downlink capacity in such environments.

References