Nonlinear Order Statistic Filter Design: Methodologies and Challenges

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Abstract

Linear filtering techniques have serious limitations in dealing with signals that have been created or processed by a system exhibiting some degree of nonlinearity or, in general, situations where the relevance of information cannot be specified in frequency domain. In image processing many of these characteristics are often present and it is no wonder that image processing is the field where nonlinear filtering techniques have first shown clear superiority over linear filters. Despite this success, nonlinear filtering, excepting a few special cases, is still more of an art than an established systematic engineering discipline. Considering the immensity of the area, it can never be such but still much can be done to establish and clarify the relations of different techniques and the assumptions behind them. The present paper is a ***reflection*** *on the methodologies and challenges of nonlinear filter design with special emphasis on order statistics, polynomial and rational filter classes.*

1. Signals and Signal Properties Requiring Nonlinear Processing

In general, a signal is referred to as stationary if its statistical properties do not change over time. In many applications, stationarity in the wide sense is often assumed and it refers to processes with constant mean, finite power and having autocorrelation functions, which depend only on the time or space difference. Usually, it is also assumed that only (joint) second order statistics of the desired signal and the observed signal are known a priori. Thus, we do not have the data to determine the parameters of nonlinear estimators and the use of linear filters is most natural. Deterministically, one often uses the terms smooth or slowly varying signals. This means that when viewed in frequency domain the signal contains only low frequency components. Thus all high frequency disturbances can be removed by simple low-pass filtering. Such signals and processes preclude sharp transitions (details and edges often encountered in image and image sequences) and impulses (e.g. in heavy-tailed noise). When viewed in frequency domain these signals contain perhaps all frequencies and no frequency selective filtering is able to separate the desired signal and the noise. Optimal linear filtering will basically use more those frequencies where the relative energy of the signal is high compared to noise. In such cases and without resorting to sophisticated techniques (e.g. data-dependent tricks), linear processing will blur those transitions and will fail to remove impulses (as it tends to locally spread its energy). On the other hand, we may have some additional information available. For instance, if we knew that the signal is locally monotonic and the noise is impulsive with certain low occurrence probability, a simple median filtering may restore the signal with practically no errors.

This phenomenon is even more prevailing in multivariate signals, e.g. color images, image sequences and satellite images, where the different channels may have very nontrivial joint statistics. Such dependence is totally ignored by any marginal (or component-wise) processing.

Optimal (Wiener) linear filtering in the most widely used form assumes that the observed and the desired signals are jointly Gaussian and that the noise corrupting process is uncorrelated with the desired signal. Such assumptions are hardly valid in any practical application involving images or color images. The need to develop more suitable processing techniques to deal with such signals and applications is clearly motivated.

The theory of linear filtering is based on the theory of linear spaces. The components of the signal belong to the real or the complex field and we have well defined order or magnitude concepts available. This fits well to the case where the signal originates from a sensor with linear characteristics. On the other hand, in many image processing tasks the signal is just two-valued (black and white) and the order is more or less arbitrary. Or there are several gray levels but the scales are not linear. The relationship of sensor output to subjective sense of brightness, for instance, can be quite nontrivial. One can ask if the use of theories based on linear spaces is really the right thing to do. In biology we encounter signals where there is no clear structure in the set of "values" that the signal components take. In statistical terms we are dealing with classified data. The signal processing methods should be designed for this situation. In practice, identifying the set with some subset of integers and so imposing structure may work reasonably well but the danger is that it will induce properties that were not actually present in the original data. If we use methods that assume very little of the model of the data and the processing when there in reality is a structured model behind our signal processing task we are not fully exploiting the possibilities but we can not obtain results that are impossible in reality. On the other hand, if we are using methods that are based on assumptions hat do not hold, there is danger that the results we get are grossly wrong. Typical example would be the use of moving average to filter out impulsive noise.

Thus it is evident that we need a large variety of methods so that in each case we are able to fully utilize the structure of the data. Experienced signal processing professionals know quite well the available tools and the underlying implicit assumptions and are thus able to pick a working method for a particular task. We feel however that it would be very useful to build a classification of the methods and their underlying assumptions. Together with a comprehensive description of the behavior of the methods under non-ideal conditions, this should be helpful in choosing the filtering method for a particular task.

The paper is organized as follows. Next, we briefly discuss the most common families of nonlinear filter classes and mention some of their application areas. Section 3 is devoted to important challenges in nonlinear filter design. Discussion and some conclusions are drawn in Section 4. The reference list given at the end of the paper is by no means complete, as the paper is not intended to be a tutorial in nonlinear filtering.

2. Families of Common Nonlinear Filters

Boolean, stack, OS and morphological filters. Many nonlinear signal processing methods have their origin in statistics. In fact, the median filter was first introduced in statistics for smoothing economical time series [24]. It soon became evident that the median filter performs very well especially in image processing applications where sharp transitions are common. Especially in urban or other "man-made" scenes we almost always have sharp edges and these edges usually are the most important information in the image. Attempts to retain sharp edges in linear filtering lead to "ringing" effects that are often more disturbing than noise.

In applications involving images, image sequences and color images, order statistics and their close kin morphological filters have by far been the most prominent and successful classes of nonlinear techniques, see [1], [3], [5], [6], [17], and [21].

One of the greatest limitations of order statistics filters is the fact that they are "smoothers". Without additional processing or combinations, their use remains limited to restoration applications, in which they excel especially in the presence of heavy-tailed noise (to be removed) and important signal details (to be preserved). General Boolean filters and morphological filters with non-flat structuring elements do not suffer from such a shortcomings; however, they do not benefit from the stacking property which unifies all subclasses of stack filters; ranked order, median and weighted median and weighted order statistics filters. The stacking property says that the Boolean function that defines the filter is positive (or increasing as is the standard term in mathematical morphology). There is usually no underlying physical model that would demand the filter to be increasing. The power of increasing filters comes from the fact that concept narrows the filter class in a way that fits well to design processes. For instance, if we have information of the possible desired signal form expressed in Boolean vectors in such a form that it does not conflict the positivity of the defining Boolean function, designing optimal increasing filter becomes straightforward, see [4], [11], [13], [26], and [28]. In some problems, notably in document image processing, noise is loosely speaking signal dependent binary union and intersection noise and increasing filters turn out to perform quite badly [14]. Here one must give up positivity and the penalty is that the large number of parameters and non-robust behavior of an unconstrained Boolean function makes the design of filters with large window sizes impossible. Recently there have emerged new ways to constrain the function leading to much better performance for large window sizes [20]. We shall return to the unification concept later.

Volterra, polynomial and quadratic filters. Volterra systems are mainly used in communications and other widely different applications, such as control, hydrology, turbulence, motion of vessels and population studies, see [15]. Volterra systems are conceptually easier to assimilate, as they are straightforward extensions (to higher kernel order) of linear filters. One can directly optimize a polynomial filter in the mean square sense if one has information of the joint higher order moments of the desired and observed signal. Usually there is no underlying model that would provide this information and direct estimation from data has proved to be unreliable.

Rational and OS- rational hybrid filters. Rational filters are direct generalizations of polynomial filters and thus many similar considerations apply. As the class is much wider and from approximation theory we know that the approximation power of rational functions is much higher than that of polynomial functions, it also follows that the design is more difficult and robustness problems are more pronounced.

Rational filters were used by Leung and Haykin [12] based on the work of Walsh [25] for signal detection and estimation, and were later applied in image filtering and enhancement by Ramponi [18, 19]. Cheikh et. al [2] extended these filters to vector rational filters and applied them to color image interpolation. Median-rational and vector median-rational hybrid filters were developed in [9] and [10], respectively, for the purpose restoring images and color images corrupted by Gaussian and impulsive type noises.

Homomorphic filters. Restoration applications involving multiplicative corrupting noise rely on homomorphic processing which first transforms the multiplicative process into an additive one, then filters the unwanted components, e.g. using often linear or even nonlinear filtering.

The fact that all of these filters are ***nonlinear*** does not unify them under a common umbrella with any useful structure. There have been attempts to build a formalism classifying nonlinear filters. A quite descriptive classification [27] is based on viewing filters as estimators and relying on different estimator classes in statistics. In the absence of such a unified theory, the user must pre-select a filter class for a specific application at hand. What makes a good class of nonlinear filters? The answer is more indicative than concrete. The following suggestions might help in such a process:

- A class that is capable of modeling the underlying system in the application at hand. Because we never know the correct model parameters, but they have to be estimated from data, among the classes of filters that are capable of modeling the system one should choose one that is robust with respect to errors in the model. Especially when the filter parameters are found by a training procedure, the more there are parameters the more data is needed. In principle, there are methods based on stochastic complexity to analyze these concepts but no ready-made guidelines are yet available.
- A class that is simple to represent, easy to analyze and design. This is also related to the robustness aspect above in the sense that these properties enable us to verify the desired behavior.

- A class that is easy to implement. A simple example is low order polynomial filter that can be implemented very much along the lines of linear filters. Also there are many efficient algorithms for implementing median type filters.

Practical considerations:

- The ultimate goal of filtering: preprocessing for a detection or an estimation system, enhancement or restoration for human processing
- Amount and form of data available, is there model, training material
- Selection of the filter class
- Optimization

3. Challenges in Filter Design

A unified and efficient framework for nonlinear filter design remains one of the most challenging tasks in this field. Even though we can not hope to obtain a framework as powerful as the techniques for designing linear filters we should be able to build a methodology that would tie together the conditions and assumptions of the problem, the major nonlinear filter classes, relevant cost functions and accessible optimization algorithms. It is clear that the methodology must be able to deal with both statistical and deterministic aspects of the problem and filters. This framework cannot be obtained by one step (leap) but it will emerge as the result of incremental steps from the joint efforts of the signal processing community. However it is good to keep the ultimate goal in mind while solving problems for more immediate demands. Few attempts have been made to this end, see for instance [8] and [29]. Here we consider some problems whose solutions will clearly take us forward on this path.

We all agree that it would be important to be able to devise a feasible optimization procedure with a suitable cost function, even for a specific application, e.g. image restoration.

The unification of two or more existing filter classes will un-doubtfully increase the modeling power of the framework. Therefore, it would of great interest to determine the class of problems (signals) that can be solved (represented) by the new framework.

A related problem is that of the filter structure, or more specifically, the filter size. An often asked question is how large should the filter size be. Most of the answers have been "try and see" type. In [22], we proposed a solution to this problem, in which we combined both the optimization and the filter structure in a recursive manner.

Another equally important challenge to the nonlinear signal and image processing community is to develop new and attracting applications. Next to a mature theory (still developing), interesting applications would be the driving force to open up new frontiers in the field. Most of the current applications remain in the areas of signal (1- and M-D) restoration, enhancement, edge detection and interpolation. Recently, stack and Boolean filters were successfully used as predictors in a DPCM lossless image compression scheme, [16]. More such endeavors are needed in other areas such as speech analysis and processing, telecommunications and data analysis and communication.

4. Discussion and Conclusions

The past two decades have witnessed the development of a number of successful nonlinear classes equipped with strong analytical tools, allowing the designer to solve a number of interesting problems, particularly in the area of image processing. Among these, one can cite the class of stack filters, [4], [13], and [23]; weighted median filters [26] and [28]; morphological filters [3] and [21]; polynomial filters [15]; and order statistics filters [17].

On the other hand, it is evident from the previous discussion that a number of important open problems and challenges remain to be solved. Among these, we mentioned a unifying framework involving two or more existing classes of nonlinear filters. It would be important to identify useful classes of signals and systems that can be modeled by this new and unifying framework. The hybrid approach, where two or more classes are combined, has proven to be useful in a number of applications involving conflicting constraints (e.g. removal of different types of noises and preservation of details) see e.g. [7], [9], and [10]. Although this is not what is meant by unification, it is nevertheless an indication of the additional capability resulting from combining different filter classes.

Finally, the applications areas must be further developed and diversified in order to attract users as well as young researchers to devote more efforts in the field.

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