PER TONE BLIND SIGNAL SEPARATION FOR A DMT-DS-CDMA SYSTEM

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ABSTRACT

In this paper, we discuss a per tone blind signal separation method for a discrete multi-tone direct-sequence code-division multiple-access (DMT-DS-CDMA) system. We show that the use of a cyclic prefix reduces the inter-chip interference (ICI). As a result, when we increase the length of the cyclic prefix, using the minimal required amount of temporal smoothing, the computational complexity of the proposed method decreases. Moreover, we demonstrate that this reduction in computational complexity even comes with a performance improvement.

1 INTRODUCTION

Blind signal separation is a challenging signal processing problem that occurs in a wireless communications context when several users are simultaneously transmitting in the same frequency band to a central base station, usually equipped with an antenna array. In this paper, we develop a per tone blind signal separation method for a discrete multi-tone direct-sequence code-division multiple-access (DMT-DS-CDMA) system [1]. An interesting feature of the DMT-DS-CDMA system is that we can make use of a cyclic prefix. We will show that this reduces the inter-chip interference (ICI). As a result, increasing the length of the cyclic prefix has some interesting consequences regarding computational complexity.

The method we propose here can be viewed as a 'blind and multi-user' extension of the (training-based and single-user) per tone equalization approach presented in [2].

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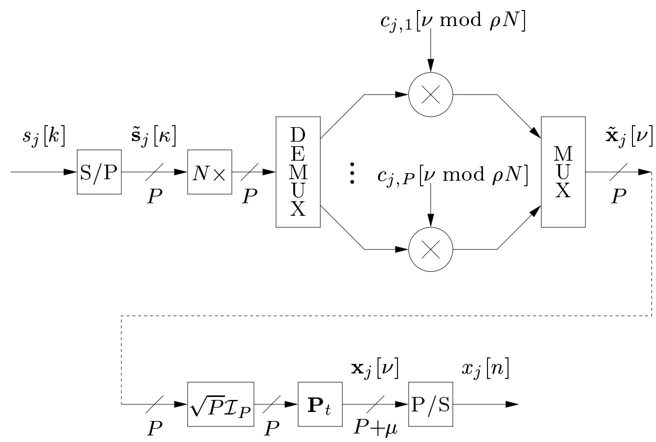

Figure 1: DMT-DS-CDMA transmitter.

2 DATA MODEL

In a DMT-DS-CDMA system [1], the jth user ($j = 1, 2, \ldots, J$) spreads his data symbol sequence $s_j[k]$ into a so-called chip sequence $x_j[n]$ in the following way (see figure 1). The data symbol sequence $s_j[k]$ is first blocked (block size P), leading to the DMT symbol sequence $\tilde{\mathbf{s}}_j[\kappa]$ (DMT symbol size P):

$$\tilde{\mathbf{s}}_j[\kappa] = \begin{bmatrix} s_j[\kappa P] & s_j[\kappa P + 1] & \cdots & s_j[(\kappa+1)P - 1] \end{bmatrix}^T.$$

Defining the DMT subsymbol sequence for the pth tone ($p = 1, 2, \ldots, P$) as

$$\tilde{s}_{j,p}[\kappa] = s_j[\kappa P + p - 1],$$

this DMT symbol sequence $\tilde{\mathbf{s}}_j[\kappa]$ can also be written as

$$\tilde{\mathbf{s}}_j[\kappa] = \begin{bmatrix} \tilde{s}_{j,1}[\kappa] & \tilde{s}_{j,2}[\kappa] & \cdots & \tilde{s}_{j,P}[\kappa] \end{bmatrix}^T.$$

The DMT subsymbol sequence $\tilde{s}_{j,p}[\kappa]$ is then spread by a factor N with the length-ρN code sequence $c_{j,p}[\nu]$ ($c_{j,p}[\nu] \neq 0$, for $\nu = 0, 1, \ldots, \rho N - 1$, and $c_{j,p}[\nu] = 0$, for

$\nu < 0$ and $\nu \geq \rho N$), resulting into

$$\tilde{x}_{j,p}[\nu] = \tilde{s}_{j,p}[\kappa] c_{j,p}[\nu \bmod \rho N], \text{ with } \kappa = \lfloor \nu/N \rfloor. \quad (1)$$

Note that we consider constant modulus code sequences ($|c_{j,p}[\nu]| = 1/\sqrt{N}$, for $\nu = 0, 1, \ldots, \rho N - 1$). Combining the results for all P tones, we obtain

$$\tilde{\mathbf{x}}_j[\nu] = \begin{bmatrix} \tilde{x}_{j,1}[\nu] & \tilde{x}_{j,2}[\nu] & \cdots & \tilde{x}_{j,P}[\nu] \end{bmatrix}^T.$$

We then perform an inverse discrete Fourier transform (IDFT) and add a cyclic prefix of length μ:

$$\mathbf{x}_j[\nu] = \mathbf{P}_t \sqrt{P} \mathcal{I}_P \tilde{\mathbf{x}}_j[\nu],$$

where $\mathcal{I}_P$ is the $P \times P$ IDFT matrix and $\mathbf{P}_t$ is the $(P + \mu) \times P$ matrix, given by

$$\mathbf{P}_t = \left[\begin{array}{c|c} \mathbf{O} & \mathbf{I}_\mu \\ \hline \multicolumn{2}{c}{\mathbf{I}_P} \end{array} \right].$$

The desired chip sequence $x_j[n]$ is then obtained by deblocking $\mathbf{x}_j[\nu]$ (block size $P + \mu$):

$$\mathbf{x}_j[\nu] = \begin{bmatrix} x_j[\nu(P+\mu)] \\ \vdots \\ x_j[(\nu+1)(P+\mu) - 1] \end{bmatrix}.$$

All users transmit their chip sequences at the chip rate, which is $N(P+\mu)/P$ times smaller than the data symbol rate. Assume that our DMT-DS-CDMA system has M receive antennas that are sampled at chip rate. Further, let $g_j^{(m)}[n]$ denote the discrete-time channel from the jth user to the mth receive antenna. The received sequence at the mth receive antenna then is

$$y^{(m)}[n] = \sum_{j=1}^{J} \sum_{n'=-\infty}^{+\infty} g_j^{(m)}[n'] x_j[n - n'] + e^{(m)}[n],$$

where $e^{(m)}[n]$ is the discrete-time additive noise at the mth receive antenna. Stacking the received samples obtained from the M receive antennas:

$$\mathbf{y}^{st}[n] = \begin{bmatrix} y^{(1)}[n] & y^{(2)}[n] & \cdots & y^{(M)}[n] \end{bmatrix}^T,$$

we obtain

$$\mathbf{y}^{st}[n] = \sum_{j=1}^{J} \sum_{n'=-\infty}^{+\infty} \mathbf{g}_j^{st}[n'] x_j[n - n'] + \mathbf{e}^{st}[n], \quad (2)$$

where $\mathbf{e}^{st}[n]$ is similarly defined as $\mathbf{y}^{st}[n]$ and $\mathbf{g}_j^{st}[n]$ is the discrete-time $M \times 1$ vector channel from the jth user to the M receive antennas, given by

$$\mathbf{g}_j^{st}[n] = \begin{bmatrix} g_j^{(1)}[n] & g_j^{(2)}[n] & \cdots & g_j^{(M)}[n] \end{bmatrix}^T. \quad (3)$$

We model $\mathbf{g}_j^{st}[n]$ as an $M \times 1$ FIR vector filter of order L_j with delay index δ_j ($\mathbf{g}_j^{st}[n] \neq \mathbf{0}$, for $n = \delta_j$ and $n = \delta_j + L_j$, and $\mathbf{g}_j^{st}[n] = \mathbf{0}$, for $n < \delta_j$ and $n > \delta_j + L_j$). Note that the larger L_j, the more inter-chip interference (ICI) for the jth user.

Next, we block $y^{(m)}[n]$ (block size $P + \mu$):

$$\mathbf{y}^{(m)}[\nu; n] = \begin{bmatrix} y^{(m)}[\nu(P+\mu) + n] \\ \vdots \\ y^{(m)}[(\nu+1)(P+\mu) - 1 + n] \end{bmatrix}.$$

We then remove the cyclic prefix of length μ and perform a discrete Fourier transform (DFT):

$$\tilde{\mathbf{y}}^{(m)}[\nu; n] = 1/\sqrt{P} \mathcal{F}_P \mathbf{P}_r \mathbf{y}^{(m)}[\nu; n], \quad (4)$$

where $\mathcal{F}_P$ is the $P \times P$ DFT matrix and $\mathbf{P}_r$ is the $P \times (P + \mu)$ matrix, given by

$$\mathbf{P}_r = \left[\begin{array}{c|c} \mathbf{O} & \mathbf{I}_P \end{array} \right].$$

Introducing the notation

$$\tilde{\mathbf{y}}^{(m)}[\nu; n] = \begin{bmatrix} \tilde{y}_1^{(m)}[\nu; n] & \tilde{y}_2^{(m)}[\nu; n] & \cdots & \tilde{y}_P^{(m)}[\nu; n] \end{bmatrix},$$

and stacking the results for the pth tone obtained from the M receive antennas:

$$\tilde{\mathbf{y}}_p^{st}[\nu; n] = \begin{bmatrix} \tilde{y}_p^{(1)}[\nu; n] & \tilde{y}_p^{(2)}[\nu; n] & \cdots \tilde{y}_p^{(M)}[\nu; n] \end{bmatrix}^T,$$

we can write

$$\tilde{\mathbf{y}}_p^{st}[\nu; n] = \sum_{j=1}^{J} \sum_{n'=-\infty}^{+\infty} \mathbf{g}_j^{st}[n'] \tilde{x}_{j,p}[\nu; n - n'] + \tilde{\mathbf{e}}_p^{st}[\nu; n], \quad (5)$$

where $\tilde{\mathbf{e}}_p^{st}[\nu; n]$ is similarly defined as $\tilde{\mathbf{y}}_p^{st}[\nu; n]$ and

$$\tilde{x}_{j,p}[\nu; n] = 1/\sqrt{P} \sum_{n'=0}^{P-1} \mathcal{F}_P(p, n'+1) x_j[\nu(P+\mu) + \mu + n' + n].$$

Note that because we use a cyclic prefix of length μ, we have

$$\tilde{x}_{j,p}[\nu; -l] = e^{-i2\pi pl/P} \tilde{x}_{j,p}[\nu], \text{ for } l = 0, 1, \ldots, \mu. \quad (6)$$

For $l < 0$ and $l > \mu$, $\tilde{x}_{j,p}[\nu; -l]$ incorporates contributions from other tones than tone p at time index ν as well as from tones at other time indices than time index ν. Further note that the DFT in (4) can be viewed as a filtering operation and that the presentation in (5) is obtained by swapping this filtering operation with the vector channel $\mathbf{g}_j^{st}[n]$.

3 ICI-REDUCTION

In this section, we show how the use of a cyclic prefix reduces the ICI. For a *per tone burst length* of K ($\tilde{s}_{j,p}[\kappa] \neq 0$, for $\kappa = 0, 1, \ldots, K - 1$, and $\tilde{s}_{j,p}[\kappa] = 0$,

$$\mathcal{G}_j = \begin{bmatrix} \mathbf{g}_j^{st}[\delta_j + L_j] & \cdots & \mathbf{g}_j^{st}[\delta_j] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{g}_j^{st}[\delta_j + L_j] & \cdots & \mathbf{g}_j^{st}[\delta_j] & \cdots & \mathbf{0} \\ & & \ddots & & \ddots & \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{g}_j^{st}[\delta_j + L_j] & \cdots & \mathbf{g}_j^{st}[\delta_j] \end{bmatrix}$$

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for $\kappa < 0$ and $\kappa \geq K$), we introduce the following $(Q+1)M \times KN$ output matrix (not a Hankel structure) for the pth tone:

$$\tilde{\mathbf{Y}}_{p;a} = \begin{bmatrix} \tilde{\mathbf{y}}_p^{st}[0;a] & \cdots & \tilde{\mathbf{y}}_p^{st}[KN-1;a] \\ \vdots & & \vdots \\ \tilde{\mathbf{y}}_p^{st}[0;a+Q] & \cdots & \tilde{\mathbf{y}}_p^{st}[KN-1;a+Q] \end{bmatrix},$$

where Q determines the amount of *temporal smoothing* and a represents the *processing delay*. This output matrix for the pth tone can be written as

$$\tilde{\mathbf{Y}}_{p;a} = \sum_{j=1}^{J} \mathcal{G}_j \tilde{\mathbf{X}}_{j,p;a} + \tilde{\mathbf{E}}_{p;a}, \tag{7}$$

where $\tilde{\mathbf{E}}_{p;a}$ is similarly defined as $\tilde{\mathbf{Y}}_{p;a}$, $\mathcal{G}_j$ is the $(Q+1)M \times r_j$ ($r_j = Q+1+L_j$) channel matrix (with Toeplitz structure) for the jth user, given at the top of this page, and $\tilde{\mathbf{X}}_{j,p;a}$ is the following $r_j \times KN$ input matrix (not a Hankel structure) for the pth tone of the jth user:

$$\tilde{\mathbf{X}}_{j,p;a} = \begin{bmatrix} \tilde{\mathbf{x}}_{j,p;a-\delta_j-L_j}^T & \cdots & \tilde{\mathbf{x}}_{j,p;a-\delta_j+Q}^T \end{bmatrix}^T,$$

with

$$\tilde{\mathbf{x}}_{j,p;n} = \begin{bmatrix} \tilde{x}_{j,p}[0;n] & \tilde{x}_{j,p}[1;n] & \cdots & \tilde{x}_{j,p}[KN-1;n] \end{bmatrix}.$$

Because of (6), which is equivalent to

$$\tilde{\mathbf{x}}_{j,p;-l} = e^{-i2\pi pl/P} \tilde{\mathbf{x}}_{j,p}, \quad \text{for } l = 0, 1, \ldots, \mu, \tag{8}$$

with

$$\tilde{\mathbf{x}}_{j,p} = \begin{bmatrix} \tilde{x}_{j,p}[0] & \tilde{x}_{j,p}[1] & \cdots & \tilde{x}_{j,p}[KN-1] \end{bmatrix}, \tag{9}$$

we can rewrite model (7) as

$$\tilde{\mathbf{Y}}_{p;a} = \sum_{j=1}^{J} \mathcal{G}_{j,p;a}^{red} \tilde{\mathbf{X}}_{j,p;a}^{red} + \tilde{\mathbf{E}}_{p;a}, \tag{10}$$

where $\tilde{\mathbf{X}}_{j,p;a}^{red}$ is the $r_{j;a}^{red} \times KN$ input matrix obtained from $\tilde{\mathbf{X}}_{j,p;a}$ as follows. If $\tilde{\mathbf{X}}_{j,p;a}$ has no vectors from the set $\{\tilde{\mathbf{x}}_{j,p;-l}\}_{l=0}^{\mu}$ as a row or only the vector $\tilde{\mathbf{x}}_{j,p;0} = \tilde{\mathbf{x}}_{j,p}$ as a row, then $\tilde{\mathbf{X}}_{j,p;a}^{red} = \tilde{\mathbf{X}}_{j,p;a}$. Else, we remove from $\tilde{\mathbf{X}}_{j,p;a}$ all the rows that correspond to vectors from the set $\{\tilde{\mathbf{x}}_{j,p;-l}\}_{l=0}^{\mu}$ and insert the vector $\tilde{\mathbf{x}}_{j,p}$ at the place where we have removed them. The corresponding $(Q+1)M \times r_{j;a}^{red}$ channel matrix $\mathcal{G}_{j,p;a}^{red}$ is then easily obtained using (8). Note that (10) can also be written as

$$\tilde{\mathbf{Y}}_{p;a} = \mathcal{G}_{p;a}^{red} \tilde{\mathbf{X}}_{p;a}^{red} + \tilde{\mathbf{E}}_{p;a}, \tag{11}$$

where $\mathcal{G}_{p;a}^{red}$ is the $(Q+1)M \times r_a^{red}$ ($r_a^{red} = \sum_{j=1}^{J} r_{j;a}^{red}$) channel matrix, given by

$$\mathcal{G}_{p;a}^{red} = \begin{bmatrix} \mathcal{G}_{1,p;a}^{red} & \cdots & \mathcal{G}_{J,p;a}^{red} \end{bmatrix},$$

and $\tilde{\mathbf{X}}_{p;a}^{red}$ is the following $r_a^{red} \times KN$ input matrix:

$$\tilde{\mathbf{X}}_{p;a}^{red} = \begin{bmatrix} \tilde{\mathbf{X}}_{1,p;a}^{redT} & \tilde{\mathbf{X}}_{2,p;a}^{redT} & \cdots & \tilde{\mathbf{X}}_{J,p;a}^{redT} \end{bmatrix}^T.$$

Note that we have $r_j - \mu \leq r_{j;a}^{red} \leq r_j$ and $r - J\mu \leq r_a^{red} \leq r$.

Example 1. *Assume that $\delta_j = 0$, for $j = 1, 2, \ldots, J$ (synchronous users). Further, assume that $L_j \leq \mu$, for $j = 1, 2, \ldots, J'$, and $L_j > \mu$, for $j = J'+1, J'+2, \ldots, J$. For $j = 1, 2, \ldots, J'$, we then know that the vectors $\{\tilde{\mathbf{x}}_{j,p;-l}\}_{l=0}^{L_j}$ are all rows of the $r_j \times KN$ matrix $\tilde{\mathbf{X}}_{j,p;0}$ and hence $\tilde{\mathbf{X}}_{j,p;0}^{red}$ will contain $r_j - L_j$ rows: $r_{j;0}^{red} = r_j - L_j$. For $j = J'+1, J'+2, \ldots, J$, on the other hand, we then know that the vectors $\{\tilde{\mathbf{x}}_{j,p;-l}\}_{l=0}^{\mu}$ are all rows of the $r_j \times KN$ matrix $\tilde{\mathbf{X}}_{j,p;0}$ and hence $\tilde{\mathbf{X}}_{j,p;0}^{red}$ will contain $r_j - \mu$ rows: $r_{j;0}^{red} = r_j - \mu$. It is therefore clear that $r_0^{red} = r - (J - J')\mu - \sum_{j=1}^{J'} L_j$. Hence, in this case, it seems as if the channel order of the jth user is reduced to 0 (if $L_j \leq \mu$) or reduced with μ (if $L_j > \mu$).*

4 BLIND SIGNAL SEPARATION

In this section, we assume that the pth tone of user 1 is desired and that δ_1 is known at the receiver. We further assume without loss of generality that $\delta_1 = 0$. The desired data vector is

$$\tilde{\mathbf{s}}_{1,p} = \begin{bmatrix} \tilde{s}_{1,p}[0] & \tilde{s}_{1,p}[1] & \cdots & \tilde{s}_{1,p}[K-1] \end{bmatrix}. \tag{12}$$

Using (1) and (9), it is clear that this vector is 'contained' in $\tilde{\mathbf{x}}_{1,p}$:

$$\begin{aligned} \tilde{\mathbf{x}}_{1,p} &= \begin{bmatrix} \tilde{s}_{1,p}[0]\mathbf{c}_{1,p}[0] & \cdots & \tilde{s}_{1,p}[K-1]\mathbf{c}_{1,p}[K-1] \end{bmatrix} \\ &= \tilde{\mathbf{s}}_{1,p} \mathbf{C}_{1,p}, \end{aligned} \tag{13}$$

where $\mathbf{c}_{1,p}[\kappa]$ is the code vector used to spread the DMT subsymbol $\tilde{s}_{1,p}[\kappa]$ (spreading factor N):

$$\mathbf{c}_{1,p}[\kappa] = \begin{bmatrix} c_{1,p}[(\kappa \bmod \rho)N] & \cdots & c_{1,p}[(\kappa \bmod \rho)N + N - 1] \end{bmatrix},$$

and $\mathbf{C}_{1,p}$ is the $K \times KN$ code matrix, defined as

$$\mathbf{C}_{1,p} = \begin{bmatrix} \mathbf{c}_{1,p}[0] & & \\ & \ddots & \\ & & \mathbf{c}_{1,p}[K-1] \end{bmatrix}. \quad (14)$$

The vector $\tilde{\mathbf{x}}_{1,p}$ is a row of every input matrix from the set $\{\tilde{\mathbf{X}}^{red}_{p;a}\}^{L_1}_{a=-Q-\mu}$ and is therefore 'contained' in every output matrix from the set $\{\tilde{\mathbf{Y}}_{p;a}\}^{L_1}_{a=-Q-\mu}$. The problem addressed here is to compute $\tilde{\mathbf{r}}_{1,p}$ from $\{\tilde{\mathbf{Y}}_{p;a}\}^{A_2}_{a=A_1}$, with $-Q-\mu \leq A_1 \leq A_2 \leq L_1$ ($A = A_2 - A_1 + 1$), based only on the knowledge of the code sequence $c_{1,p}[\nu]$. To solve this problem we make the following assumptions:

Assumption 1. *The channel matrix $\mathcal{G}^{red}_{p;a}$ has full column rank r^{red}_a (r^{red}_a is then called the delay-a system order), for $a = A_1, A_1+1, \ldots, A_2$.*

Assumption 2. *The input matrix $\tilde{\mathbf{X}}^{red}_{p,0;a}$ has full row rank r^{red}_a, for $a = A_1, A_1+1, \ldots, A_2$.*

The first assumption requires that

$$(Q+1)M \geq r^{red}_a, \quad \text{for } a = A_1, A_1+1, \ldots, A_2. \quad (15)$$

The second assumption, on the other hand, requires that

$$KN \geq r^{red}_a, \quad \text{for } a = A_1, A_1+1, \ldots, A_2.$$

We can then use a method that is similar to the method presented in [3]. However, instead of working with the noise subspace, we will work with the signal subspace. For a large per tone burstlength, this leads to a smaller computational complexity. Computing the SVD of the $(Q+1)M \times KN$ matrix $\tilde{\mathbf{Y}}_{p;a}$ ($a = A_1, A_1+1, \ldots, A_2$) and defining the collection of the r^{red}_a right singular vectors corresponding to the r^{red}_a largest singular values as the $KN \times r^{red}_a$ matrix $\tilde{\mathbf{V}}^s_{p;a}$, we solve

$$\max_{\tilde{\mathbf{s}}_{1,p}} \{ \| \tilde{\mathbf{s}}_{1,p} \begin{bmatrix} \mathbf{C}_{1,p}\tilde{\mathbf{V}}^s_{p;A_1} & \cdots & \mathbf{C}_{1,p}\tilde{\mathbf{V}}^s_{p;A_2} \end{bmatrix} \|^2 \},$$
$$\text{subject to } \|\tilde{\mathbf{s}}_{1,p}\|^2 = 1.$$

We know that when we increase μ, the value of r^{red}_a, for $a = A_1, A_1+1, \ldots, A_2$, decreases (see section 3) and, hence, the minimal required value of Q decreases (see (15)). Therefore, when we increase μ, using the minimal required value of Q, the computational complexity of the proposed method decreases.

5 SIMULATION RESULTS

In this section, we will perform some simulations on a DMT-DS-CDMA system. We consider QPSK modulation ($\tilde{s}_{j,p}[\kappa] = \pm 1/\sqrt{2} \pm 1/\sqrt{2}i$, for $\kappa = 0, 1, \ldots, K-1$). We assume that the additive noises $\{e^{(m)}[n]\}^M_{m=1}$ are mutually independent and zero-mean white with variance σ^2_e. We further assume that all the users have equal received powers and define the SNR as

$$SNR = (\sum_{j=1}^{J} \sum_{n=-\infty}^{+\infty} \|\mathbf{g}^{st}_j[n]\|^2)/(JM\sigma^2_e).$$

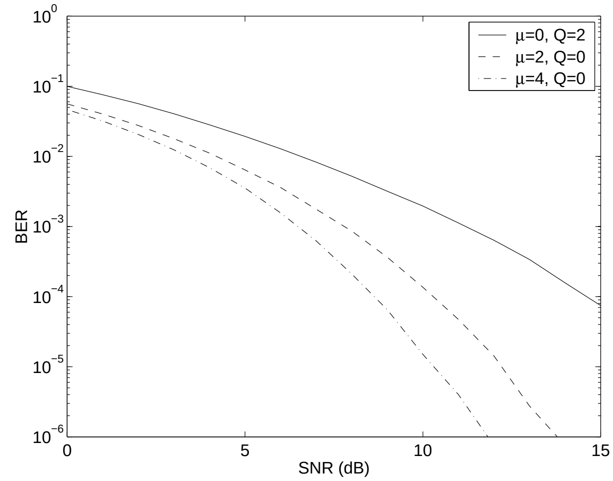

Figure 2: Average BER per user versus SNR.

The parameters we use are $M = 4$, $J = 2$, $N = 2$, $P = 20$, $K = 100$, $L_1 = 2$, $L_2 = 4$ and $\delta_1 = \delta_2 = 0$ (synchronous users).

We take $A_1 = A_2 = 0$ ($A = 1$) for every tone of every user. We will consider three values of μ: $\mu = 0$ (no cyclic prefix), $\mu = 2$ and $\mu = 4$. For each of these values of μ, we use the minimal required value of Q, which can be calculated from (15) and example 1. When $\mu = 0$ (we then have $J' = 0$), the minimal required value of Q is 2 (we then get $r^{red}_0 = (Q+1)M = 12$). When $\mu = 2$ (we then have $J' = 1$) or $\mu = 4$ (we then have $J' = 2$), the minimal required value of Q is 0 (for $\mu = 2$, we then get $r^{red}_0 = (Q+1)M = 4$, while for $\mu = 4$, we then get $r^{red}_0 = 2 < (Q+1)M = 4$). The performance results for all these scenarios are shown in figure 2. We observe that when we increase μ, using the minimal required value of Q, the performance improves.

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