THE USE OF STOCHASTIC MATCHED FILTER IN ACTIVE SONAR

J.-L. MORI, P. GOUNON
LIS, ENSIEG, Domaine universitaire, BP 46
38402 Saint Martin d’Hères, France
e-mail: jean-louis.mori@lis.inpg.fr, patrick.gounon@lis.inpg.fr

ABSTRACT

In this present study we propose to evaluate a kind of matched filter, called "stochastic matched filter", first described by J-F Cavassilas and originated to detect short stationary stochastic “pulses” in stationary noise.

We can show in this paper that this filter, with minor changes in its definition, can be applied on frequency time-varying signals, such as wide band modulated sonar signal propagated in shallow water.

The stochastic matched filter is able to take into account uncertainties and variations of the propagation channel, thus enhancing the detection efficiency. The main characteristics of this treatment is shown up with simulated data.

1. INTRODUCTION

Assuming only one single path of propagation in active sonar systems leads naturally to the classic matched filter. In that case this is the optimal way of detection (considering no Doppler effect present). However, when sound propagates along multiple paths, the actual received signal depends on the characteristics of the medium, that we have to know exactly to design a matched filter.

If the impulse response of the system is perfectly known, there is no problem to design a filter matching the multipath signal. But if there are in the system some random parameters (or some deterministic but unknown parameters such as bottom geometry, depths of source and receivers, …), the matched filter cannot be used and that leads to degradations of the detector’s performance [1].

The only thing we can do is to modelise these unexpected variations of the received signal. For example, we can consider a probability density function for the propagation time for each multipath. More simply, we can use the covariance matrix of the impulse response of the system.

In this present study we propose to use a kind of matched filter, called “stochastic matched filter” first described by J-F Cavassilas and originated to detect stationary stochastic signals in stationary noise [2]. We show that stochastic matched filter can apply on frequency time-varying signals, such as wide band modulated sonar signal propagated in shallow water.

In the first part of this paper, we present the principle of the stochastic matched filter in the case of stationary signals. Then, we show an extension of this filtering to non-stationary signals. The third part presents some illustrations of this processing with simulated sonar data.

2. STOCHASTIC MATCHED FILTER

2.1. Insight of theory

Assuming $s(t)$, the signal to detect, deterministic and of finite duration $T$. The classical matched filtering correspond to the maximization of the following Signal to Noise Ratio $\rho$ (S.N.R.) [3]:

$$\rho = \frac{\|y_s(T)\|^2}{E\{y_s(T)\|^2\}}$$

(1)
Where: $y_s(T) = \int_{0}^{T} s(t) h'(t) dt$

and $y_b(T) = \int_{0}^{T} b(t) h'(t) dt$

$h(T-t)$ being the impulse response of the filter and $^*$ denotes complex conjugation.

If the signal to detect is not deterministic, the previous expression must be modified. Consider $S(t,l)$, a known signal of finite duration $T$, function of $l$, an occurrence of a random process $L$. We assume that the observed signal is perturbed by an additive, zero-mean, stationary Gaussian noise process $b(t)$.

The criterion used to define the stochastic matched filter is the maximization of the following signal to noise ratio:

$$\rho = \frac{\int_{0}^{T} s(t) h^*(t) dt}{\int_{0}^{T} b(t) h^*(t) dt}$$

where: $y_s(T) = \int_{0}^{T} s(t,l) h'(t) dt$

and $y_b(T) = \int_{0}^{T} b(t) h^*(t) dt$

Assuming that the second order moments of the signal are not time varying, it can be shown that $\rho$ is equal to

$$\rho = \frac{\int_{0}^{T} \int_{0}^{T} h(t) \left( \int G_s(t-t',l) p(t') dt' \right) h^*(t') dt dt'}{\int_{0}^{T} \int_{0}^{T} h(t) G_b(t-t',l) h^*(t') dt dt'}$$

where $G_s(t-t',l)$ is the autocorrelation function of $S(t,l)$ and $G_b(t-t',l)$ is the autocorrelation function of $B(t)$.

3. DISCRETE SIGNALS

For numerical signals, the ratio $\rho$ can be expressed as a quotient of two quadratic expressions:

$$\rho = \frac{h^* R_b h}{h^* R_b h}$$

Where $R_b$ is the correlation matrix of $s(t,l)$, $R_b$ is the correlation matrix of $b(t)$ and $^*$ denotes transposition and conjugation for matrices. This quotient is known as "Rayleigh's quotient", and is maximal for the vector $h$ which is the eigenvector associated to the highest eigenvalue of $R_b = R_b^{-1} R_s$ [4].

In that case, $\rho$ is equal to the highest eigenvalue.

3.1. SNR improvement

Assuming $\sigma_s^2$ be the power of the signal and $\sigma_b^2$ the power of the noise process.

Let $R_s = \frac{R_s}{\sigma_s}$ and $R_b = \frac{R_b}{\sigma_b}$.

With these notations, $\rho$ can be written:

$$\rho = \frac{\sigma_s^2}{\sigma_b^2} \alpha$$

with

$$\alpha = \frac{h^* R_s h}{h^* R_b h}$$

$\frac{\sigma_s^2}{\sigma_b^2}$ is the SNR before filtering, $\rho$ is the SNR after filtering and $\alpha$ is the gain of the filter.

We can easily see that there is not only one vector $h$ which improves the SNR: the eigenvector associated to the greatest eigenvalue of $R_b = R_b^{-1} R_s$ leads to the best improvement (in term of SNR) but all the eigenvectors associated to eigenvalues greater than one improve the SNR and can be considered.

It can be shown [5] that, considering all these filters, the sufficient statistic $\Lambda$ to detect the presence of $s(t)$ is:

$$\Lambda = \sum_{i=1}^{N} \left| z_i \right|^2 \frac{dn_i}{1 + dn_i}$$

where $dn_i$ are the eigenvalues corresponding to the vectors $h_i$, and $z_i$ are the output of the $i$th filter at time $T$.

The combination of the $N$ filters (by the use of 5 ) lead to the following S.N.R. improvement:
\[
\alpha = \frac{\sum_i \frac{dn_i^2}{1 + dn_i}}{\sum_i \frac{dn_i}{1 + dn_i}}
\] (6)

3.2. Non stationary signals

If the second order moments of the signal are time varying (non stationary signal), then the correlation function \( G \) is now:

\[
G_S(t,t') = \sum_i S_i(t)S_i(t')
\]

\( R_S \), a Toeplitz matrix in 3.1, is now only an Hermitian matrix. We now have to consider the new SNR before filtering:

\[\rho_1 = \frac{m_s^2}{\sigma_b^2}\]

where \( m_s^2 \) is the square of the maximum of \( s(t) \).

As we did before, we can now compute the eigenvectors that have the filtering gain greater than 1.

4. APPLICATION

We will now illustrate the performances of the filtering described above. The problem considered is the sonar detection problem in shallow water. The signal to be detected is the sum of multiple paths of propagation. The results are obtained using numerical simulations.

For all the simulations, the parameters are the following:
- Distance of the target : 2 km
- Transmitted signal : Linear Modulated Frequency
  - Duration : 1 s.
  - Bandwidth : 100 Hz
  - Central frequency : 1 kHz

The random part of the received signal is due to the uncertainties of the environment: the depth of the bottom (for all the distances between the sonar and the target) and the velocity are not perfectly known and are modeled by random processes.

The perturbing noise is composed by a white gaussian noise and a reverberation noise. In the considered bandwidth, the reverberation is white. So, \( R_b \) is equal to the identity matrix.

As we could not analytically calculate the signal correlation matrix, this matrix is estimated using the results of the simulations.

4.1. Results

3 kinds of filtering were used:
- the classic matched filter.
- the filter matching the "average signal", i.e. the signal received after multi-path propagation with the average values of the environment parameters.
- the stochastic matched filter.

40 snapshots of signal were simulated to characterize the stochastic matched filter. The reverberation was so chosen that the peak matching the target is near a peak of reverberation.

As expected, in the case of the classic matched filter, the response of signal and reverberation have quite the same level. This is because the classic matched filter reacts the same way against any echo (fig. 1 upper side).

In the second case, the peak of correlation should theoretically be higher than the previous one. But for a small difference of shape between the awaited signal and the real signal (i.e. for small variations of the environment parameters from their average values), the performance is highly decreased and gets close to the conventional matched filter (fig. 1 middle).

In the case of the F.A.S., the performance is better than in the previous filters: the level of the peak corresponding to the target is significantly enhanced (fig. 1 lower side).
4.2. Performances

In order to quantify the performances of stochastic matched filter, we established the ROC (Receiver Optimal Characteristic) curves of both detectors: classic matched filtering and stochastic matched filtering. The perturbing noise is reverberation added with white noise. Two hundred simulations were done in order to obtain an accurate result.

- Dashed curve: Classic matched filter
- Full curve: Stochastic matched filter.

As we can see (fig. 2), the stochastic matched filter is significantly more efficient than the classical matched filter.

5. CONCLUSION

We have presented in this paper a new approach to detect a partially known signal. This filtering, called stochastic matched filter, is based upon the second order statistics of the signal. It is globally more efficient than the classic matched filter. Its performances have been illustrated by simulations.

REFERENCE