SYNTHESIS OF BIRECIPROCAL WAVE DIGITAL FILTERS WITH EQUIRIPPLE AMPLITUDE AND PHASE

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ABSTRACT

In this contribution, robust simultaneous amplitude and phase approximations are delivered for bireciprocal wave digital filters exhibiting arbitrary amplitude and linear phase responses. Moreover, the resulting amplitude and phase characteristics are equiripple. The approximation is relying on translating the given amplitude and phase specifications into corresponding specifications for the difference and sum phase functions of the two branch bireciprocal polynomials. Consequently, these difference and sum functions are approximated in an alternative and iterative procedure such that they converge into their optimal behavior. Accordingly, the two branch polynomials are finally obtained and the synthesis of the filter structure in the wave digital domain is determined.

1. INTRODUCTION

The problem of designing wave digital filter structures satisfying arbitrary amplitude and linear phase specifications is important and interesting. Recently, some methods have been delivered [1-3] for simultaneously approximating the amplitude and phase of wave digital filters. It can be concluded that none of these methods is the optimal solution for the problem. The reason of this statement is that these methods are relying on restricting one of the two branch allpass functions to exhibit exact linear phase. Consequently, the phase of the other allpass functions is designed to control both the amplitude and phase. Of coarse, this results in degree consuming and non economical structures.

Recently, robust simultaneous amplitude and phase approximations have been established [4] for wave digital lattice structures. The robustness of this method is ensured by two sources. On one hand, none of the two allpass functions is restricted to have exact linear phase. On the other hand, the resulting amplitude and passband phase characteristics are controlled to be equiripple.

Now, the approximation concept of this method [4] is modified to cover the bireciprocal structures. The given amplitude and phase specifications are translated into corresponding specifications for the difference and sum phase functions of the two branch bireciprocal polynomials. These difference and sum functions are approximated in an alternative and iterative procedure such that they converge into their optimal response. Interpolation methods combined with the Remez-exchange algorithm [5] are employed for this purpose. Due to the bireciprocal property, the stopband is depending on the passband [6-8] and the interpolation is carried out only in the passband. Finally, the two branch bireciprocal polynomials are obtained and the wave digital realization for the filter structure is determined.

2. THE APPROXIMATION PROBLEM

Let that amplitude and linear phase specifications are given in the wave digital domain. It is required to approximate these specifications by a bireciprocal wave digital structure. If we express the reflection and transmission functions as :

$$S_{11}(\psi) = \frac{h(\psi)}{g(\psi)} \quad \text{and} \quad S_{21}(\psi) = \frac{f(\psi)}{g(\psi)} \tag{1}$$

where $\psi = \Sigma + j\phi$ is the complex frequency variable in the reference domain. For a lossless and reciprocal lattice structure, $h(\psi)$ and $f(\psi)$ are odd and even polynomials respectively [8]. Moreover, the following complementary relationship holds:

$$S_{11}(\psi)S_{11}(-\psi) + S_{21}(\psi)S_{21}(-\psi) = 1 \tag{2}$$

which for real frequencies becomes:

$$|S_{11}(j\phi)|^2 + |S_{21}(j\phi)|^2 = 1 \tag{3}$$

The corresponding characteristic function is

$$\Psi(\psi) = \frac{h(\psi)}{f(\psi)} \tag{4}$$

For bireciprocal structures, the characteristic function satisfies the following condition:

$$\Psi(\frac{1}{\psi}) = \frac{1}{\Psi(\psi)} \tag{5}$$

Accordingly, the complementary property of Eq.(2) reduces to:

$$S_{11}(\psi)S_{11}(-\psi) + S_{11}(\frac{1}{\psi})S_{11}(-\frac{1}{\psi}) = 1 \tag{6}$$

or alternatively:

$$S_{21}(\psi)S_{21}(-\psi) + S_{21}(\frac{1}{\psi})S_{21}(-\frac{1}{\psi}) = 1 \tag{7}$$

Consequently, for bireciprocal structures, the passband and stopband amplitudes are depending on each other. So, the amplitude specifications for a bireciprocal filter are given in the stopband only. If we define $p = \sigma + j\omega$, as the complex

frequency variable in the digital domain , then , it is related to ψ by the bilinear transformation [9]:

$$\psi = \tanh(pT/2) = \frac{z-1}{z+1}, z = e^{pT}$$
$$\phi = \tan(\omega T/2) \quad (8)$$

where T is the sampling period. If we define the following loss functions:

$$\alpha_{11}(\omega) = -20\mathrm{Log}(|S_{11}|)$$
$$\alpha_{21}(\omega) = -20\mathrm{Log}(|S_{21}|) \quad (9)$$

then , according to Eqs (6-8) , for a bireciprocal structure the above two loss functions are the mirror image of each other around the quarter sampling frequency, i.e.,

$$\alpha_{11}(\omega) = \alpha_{21}(\frac{\pi}{T} - \omega), \ \alpha_{21}(\omega) = \alpha_{11}(\frac{\pi}{T} - \omega) \quad (10)$$

The transmission function of a bireciprocal structure can be expressed as:

$$S_{21}(\psi) = \frac{f(\psi)}{g(\psi)} \quad (11)$$

where $f(\psi)$ is an even polynomial of degree = N-1. On the other hand, $g(\psi)$ is a strictly Hurwitz and bireciprocal polynomial of odd degree N. The corresponding two branch all-pass functions are:

$$S_1(\psi) = -\frac{g_1(-\psi)}{g_1(\psi)} \text{ and } S_2(\psi) = \frac{g_2(-\psi)}{g_2(\psi)} \quad (12)$$

where both $g_1(\psi)$ and $g_2(\psi)$ are strictly Hurwitz and bireciprocal polynomials. The transmission function is given by:

$$S_{21}(\psi) = \frac{S_2(\psi) - S_1(\psi)}{2} \quad (13)$$

For real reference frequencies, the two branch all-pass functions can be formulated as:

$$S_1(j\phi) = e^{-j2\theta_1(\phi)} \text{ and } S_2(j\phi) = e^{-j2\theta_2(\phi)} \quad (14)$$

Consequently, $\theta_1(\phi)$ and $\theta_2(\phi)$ are the corresponding phase functions exhibited by $g_1(j\phi)$ and $g_2(j\phi)$ respectively. Now, by defining the following functions:

$$\alpha(\phi) = \theta_2(\phi) + \theta_1(\phi) \text{ and } \beta(\phi) = \theta_2(\phi) - \theta_1(\phi) \quad (15)$$

So, $\alpha(\phi)$ and $\beta(\phi)$ are respectively the sum and difference phase functions of the two branch polynomials $g_1(\phi)$ and $g_2(\phi)$. Simple mathematical processing yields:

$$\beta(\phi) = \mathrm{Cos}^{-1}|S_{21}(j\phi)| \quad (16)$$
$$\alpha(\phi) = -\mathrm{Arg}[S_{21}(j\phi)] \quad (17)$$

According to these two equations, the amplitude and phase specifications are translated into corresponding specifications for $\alpha(\phi)$ and $\beta(\phi)$. It is evident that to approximate unity transmission (zero loss) in the passband, $\beta(\phi)$ must approximate zero within allowed error margin $\pm\varepsilon_1$. On the other hand, to approximate zero transmission (infinite loss) in the stopband, $\beta(\phi)$ must approximate $\pm\pi/2$ within another error margin $\pm\varepsilon_2$. Moreover, to approximate linear phase in the passband, $\alpha(\phi)$ must approximate linearity within a third error margin $\pm\varepsilon_3$. Of course, the three error margins ε_1, ε_2 and ε_3 are depending on the original specifications .

Now, the approximation is relying on the determination of the specifications of $\theta_1(\phi)$ and $\theta_2(\phi)$ .. This can be achieved since from Eq.(15), we have:

$$\theta_2(\phi) = \frac{\alpha(\phi)+\beta(\phi)}{2} \text{ and } \theta_1(\phi) = \frac{\alpha(\phi)-\beta(\phi)}{2} \quad (18)$$

As a consequence, the approximation problem has been reduced to the generation of two strictly-Hurwitz and bireciprocal polynomials $g_1(\psi)$ and $g_2(\psi)$ specified by the two phase functions $\theta_1(\phi)$ and $\theta_2(\phi)$ respectively. The objective is to determine these two polynomials such that the two functions $\alpha(\phi)$ and $\beta(\phi)$ are satisfied. So, the approximation procedure will depend on the generation of $g_1(\psi)$ and $g_2(\psi)$ such that the two functions $\beta(\phi)$ and $\alpha(\phi)$ are approximated alternatively and iteratively. This means that while one of $\beta(\phi)$ or $\alpha(\phi)$ is approximated, the other one is fixed. By iterating this process, the two functions will be converge into their optimal responses.

The generation of $g_1(\psi)$ and $g_2(\psi)$ will be achieved by applying the interpolation combined with the Remez-exchange algorithm. We will assume $g_1(\psi)$ be of even degree m and $g_2(\psi)$ be of odd degree n:

$$N = m + n \quad (19)$$

So, one of the following cases holds:

$$m = (N-1)/2 \text{ and } n = (N+1)/2 \quad (20)$$

$$\text{or } m = (N+1)/2 \text{ and } n = (N-1)/2 \quad (21)$$

The polynomial $g_1(\psi)$ is formulated as:

$$g_1(\psi) = \psi^m + a_1\psi^{m-1} + \ldots a_{(m/2)}\psi^{m/2} \ldots + a_1\psi + 1$$

So, the coefficients are symmetrical around the central one and the total number of distinct coefficients is m/2 . Consequently, the phase of $g_1(\psi)$ must be specified at a total number r= m/2 of real frequencies ϕ_1 , …., ϕ_r . Then, the problem reduces to solution of a set of equations. On the other hand, the polynomial $g_2(\psi)$ is formulated as:

$$g_2(\psi) = \psi^n + b_1\psi^{n-1} + \ldots\ldots\ldots\ldots + b_1\psi + 1$$

So, the total number of distinct coefficients is (n-1)/2. If the interpolation will be applied, the phase of $g_2(\psi)$ must be specified at a total number q = (n-1)/2 of real frequencies ϕ_1 , ϕ_2,……, ϕ_q. Then, the problem reduces to solution of a set of equations.

3. THE APPROXIMATION PROCEDURE

Now, the approximation procedure can be summarized in steps as follows:

1- From, the given amplitude and phase specifications, determine the corresponding specifications for the difference and sum phase functions $\beta(\phi)$ and $\alpha(\phi)$. This is achieved by determining the three ripple factors ε_1, ε_2 and ε_3 .

2- At initial degree N for the filter. Consequently, the values of m and n can be determined. The starting value of N can be chosen to be equal to the degree of a corresponding bireciprocal structure satisfying the amplitude specifications only [7].

3- Now, we will assume that r=q . The case of r=q+1 will be discussed later. So, with r=q, get the specifications of the sum and difference phase functions $\alpha(\phi)$ and $\beta(\phi)$ at a number =r=q of interpolation points distributed over the passband only. The initial distribution of these points in the digital frequency domain can be achieved such that they become in an equidistant arrangement with the upper passband edge held as a fixed point. Note that the digital frequency values are translated into reference frequency values using the bilinear transformation.

4- From the specifications of $\alpha\phi)$ and $\beta(\phi)$, get the corresponding specifications of $\theta_1(\phi)$ and $\theta_2(\phi)$ by applying Eq. (18).

5- Now, both $g_1(\psi)$ and $g_2(\psi)$ are generated from their phase specifications by applying Eqs (23,28).

6- Get the resulting amplitude and phase responses or equivalently the β and α functions .

7- Now, Concentrating on the β function (or the amplitude) , apply the Remez-exchange algorithm to change the set of interpolation frequencies for optimal β (or amplitude). During this process the α function (or the phase) is fixed. This means that at any new interpolation frequency the α (or phase) value is set at its value obtained from the previous response. For every new set of frequencies , the polynomials $g_1(\psi)$ and $g_2(\psi)$ are newly generated.

8- .With the β (or the amplitude) specifications become satisfied, return into the α function (or the phase), apply the Remez-exchange algorithm for optimal α function. During this process, the β function (or the amplitude) is fixed. This means that at any new interpolation point, the β (or amplitude) value is set at its value obtained from the previous response. For every new set of frequencies, the two polynomials $g_1(\psi)$ and $g_2(\psi)$ are newly generated.

9- The steps 7 and 8 are repeated iteratively until the α and β functions converge into their optimal response and the amplitude and phase specifications become satisfied.

10- If the chosen value for the degree not sufficient, increase it by two and go to step 3.

11- From the resulting two polynomials $g_1(\psi)$ and $g_2(\psi)$, obtain the realization of the resulting filter in the wave digital domain.

In case of r=q+1, the same procedure can be followed with some notes:

One- The number of interpolation points =r with the passband upper edge be the last point.

Two- From the specifications of α and β at the first q (q = r-1) interpolation points, a number = q phase specifications for either the polynomial $g_1(\psi)$ or $g_2(\psi)$ can be obtained. Consequently, $g_1(\psi)$ can be generated.

Three- Using the phase of $g_1(\psi)$, the phase specifications of $g_2(\psi)$ at the last point can be determined such that either the amplitude or phase is satisfied at this point., depending on which function is being under approximation. Consequently, $g_2(\psi)$ can be generated.

4. THE REALIZATION PROBLEM

As soon as the two bireciprocal polynomials $g_1(\psi)$ and $g_2(\psi)$ are obtained , the wave digital realization of the resulting structure can be realized in the wave digital domain. The realization is relying on the factorization of the two all-pass functions into cascaded sections of first, second and fourth sections [7,10]. A first order section is written as:

$$S = \frac{-\psi+1}{\psi+1} \tag{22}$$

and its realization needs no adaptors [7]. A second order section is formulated as:

$$S = \frac{\psi^2 - a\psi + 1}{\psi^2 + a\psi + 1} \tag{23}$$

and its need one adaptor [7] with coefficient value:

$$\gamma = \frac{a-2}{a=2} \tag{24}$$

A section of fourth order is formulated as :

$$S = \frac{\psi^4 - A\psi^3 + B\psi^2 - A\psi + 1}{\psi^4 + A\psi^3 + B\psi^2 + A\psi + 1} \tag{25}$$

In this case the use of the lowpass-bandpass transfomation:

$$\psi^{\sim} = 0.5(\psi + 1/\psi) \quad \text{or} \quad z^{\sim} = -z^2 \tag{26}$$

enables us to realize this section in the $z^{\sim}$ domain by two adaptors [10] with the following adaptor coefficient values:

$$\gamma_1 = \frac{2A-B-2}{2A+B+2} \qquad \gamma_2 = \frac{6-B}{2+B} \tag{27}$$

however, to return into the z domain, each delay element is doubled and cascaded by a multiplier of –1.

DESIGN EXAMPLE

The reliability of the introduced approximation method will now be shown by a typical example. Let us be given amplitude and phase specifications as follows:
The lower edge frequency of the stopband is 6 kHz.
The minimum allowed loss in the passband is 33 dB.
The sampling frequency is 20 kHz.
The maximum phase deviation from linearity is ± 0.02 rad.
By following the introduced method, the required degree for these specifications has been detected to be 13. The starting interpolation values are as follows:

Freq. kHz	β rad,	α dev. from linearity
1.3333333	0.02238964	0.02
2.6666666	-0.02238964	-0.02
4.0	0.02238964	0.02

The final interpolation values obtained after total number of 4 iterations are:

Freq. kHz	β rad.	α dev. from linearity
1.82	0.0204824	0.02
3.4	- 0.02238964	- 0.02
4.0	0.02238964	0.02

The following results have been obtained:

$$g_1(\psi) = \psi^6 + 5.952623\psi^5 + +14.97674\psi^4 + 19.86056\psi^3 + 14.97674\psi^2 + 5.952623\psi + 1$$

$$g_2(\psi) = \psi^7 + 6.089712\psi^6 + 15.912850\psi^5 + 23.253850\psi^4 + 23.253850\psi^3 + 15.912850\psi^2 + 6.089712\psi + 1$$

Accordingly, the resulting loss respons and the resulting phase deviation from linearity is shown in Fig. (1) . To realize the filter structure, the two polynomials are factorized into cascaded sections of first, second and fourth orders. Consequently, the wave digital realization is obtained by applying two-port parallel adaptors [7]

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