UNITARY CYCLIC MUSIC ALGORITHM IN A MULTIPATH ENVIRONMENT

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ABSTRACT

A signal selective Unitary Cyclic MUSIC algorithm is presented by introducing a new forward backward smoothed covariance matrix. Compared with Cyclic MUSIC algorithm, the suggested approach has a better performance in the presence of multipath propagation. In addition,this approach not only reduces the computational complexity by real-valued eigendecomposition, but also allows to select desired signals and to ignore interferences by exploiting the cyclostationarity property of signals of interest (SOIs). Simulation results that illustrate the performance of this approach in conjunction with Cyclic MUSIC algorithm are described.

1. INTRODUCTION

The problem of estimating the direction of arrival (DOA) of signals of interest (SOIs) from the noisy measurements is a fundamental one in array signal processing, which is used in many applications such as radar, sonar, etc. Popular DOA methods, such as MUSIC [1] and ESPRIT [2] are known to yield high resolution but suffer from three drawbacks. Firstly, the total number of signals impinging on the array, including both SOIs and interference, is less than the number of sensors or the characteristics of interfering signals are known so that their effects can be subtracted; secondly,it is impossible to resolve two signals spaced more closely than the resolution threshold of the array when only one signal is a SOI; thirdly the noise characteristics of the sensors and the environment are known or they are accurately modelled as independent and identically distributed Gaussian random processes. Therefore the signal selective Cyclic MUSIC algorithms[3]-[4] presented effectively overcome these drawbacks by exploiting the differing spectral correlation characteristics of the different signals.

These Cyclic MUSIC algorithms perform poorly when coherent or highly correlated signals are present. Thus a signal selective Unitary Cyclic MUSIC algorithm is proposed to circumvent the drawback,which has a better performance in the presence of multipath propagation in this paper. This algorithm not only reduces the computational complexity by real-valued eigendecomposition, but also allows to select desired signals and to ignore interferences by exploiting the cyclostationarity property of the SOIs.

2. SIGNAL MODEL

Consider a Uniform Linear Array (ULA) composed of m omnidirectional sensors.Suppose d narrowband farfield sources with center frequency w_0 impinging from the directions $\theta_1, \cdots, \theta_d$.Assume that there are N snapshots $\mathbf{x}(1), \mathbf{x}(2), \cdots, \mathbf{x}(N)$ available, the observation vector can be modelled as

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) \tag{1}$$

where

$$\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_d)] \tag{2}$$

is the $m \times d$ matrix of the signal direction vectors, and $\mathbf{a}(\theta_i)$ is the $m \times 1$ steering vector.$\mathbf{s}(k) = [s_1(k), \cdots, s_d(k)]^T$ is the vector of cyclostationary signals with cycle frequency α. $\mathbf{i}(k)$ is the vector of interfering sources with cyclostationary property, and $\mathbf{n}(k)$ is the vector of sensor noise. Hence for some cyclic frequency α and some lag parameter τ, the cyclic autocorrelation matrix of the observation vector is defined by

$$\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau) = \langle \mathbf{x}(k)\mathbf{x}^H(k+\tau)e^{-j2\pi\alpha k} \rangle = \mathbf{A}\mathbf{R}^{\alpha}_{\mathbf{ss}}(\tau)\mathbf{A}^H \tag{3}$$

where

$$\mathbf{R}^{\alpha}_{\mathbf{ss}}(\tau) = \langle \mathbf{s}(k)\mathbf{s}^H(k+\tau)e^{-j2\pi\alpha k} \rangle \tag{4}$$

is the $d \times d$ cyclic autocorrelation matrix of the cyclostationary signal vector. $\langle \cdot \rangle$ denotes the finite time average operator, and superscript H denotes complex conjugate transpose of a vector or matrix.

3. CYCLIC MUSIC ALGORITHM

The cyclic autocorrelation matrix at cycle frequency α for some lag parameter τ is nonzero and can be estimated by

$$\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau) = \frac{1}{N}\sum_{k=1}^{N} \mathbf{x}(k)\mathbf{x}^H(k+\tau)e^{-j2\pi\alpha k} \tag{5}$$

where depending on type of modulation used, the cycle frequency α is usually equal to the twice of the carrier frequency, multiple of the baud rate, spreading codes repetition rate, chip rate or combinations of these. Compared with the covariance matrix exploited by the MUSIC algorithm, the cyclic autocorrelation matrix exploited by the Cyclic MUSIC method is generally not Hermitian. Then, instead of using the eigenvalue decomposition (EVD), Cyclic MUSIC algorithm uses the singular value decomposition (SVD)

$$\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau) = [\mathbf{U}_s\ \mathbf{U}_n] \begin{bmatrix} \Sigma_s & \mathbf{0} \\ \mathbf{0} & \Sigma_n \end{bmatrix} [\mathbf{V}_s\ \mathbf{V}_n]^H \tag{6}$$

Thus the Cyclic MUSIC spatial spectrum can be written as

$$\mathbf{P}_{CM}(\theta) = \frac{\|\mathbf{a}(\theta)\|^2}{\|\mathbf{U}_n^H\mathbf{a}(\theta)\|^2} \tag{7}$$

For finite number of time samples, the algorithm can be summarized as the follows:

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- Step 1: Choose α a cycle frequency of the desired signals;
- Step 2: Find the null space $\mathbf{U}_n$ of $\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau)$ by eigendecomposition and its rank Z;
- Step 3: Determine the number of SOIs, $d = m - Z$;
- Step 4: Search over θ for the d highest peaks in $P_{CM}(\theta)$.

4. UNITARY CYCLIC MUSIC ALGORITHM

Let a ULA with L identical sensors $\{1, \cdots, L\}$ be divided into overlapping subarrays of size m, with sensors $\{1, \cdots, m\}$ forming the first subarray, sensors $\{2, \cdots, m+1\}$ forming the second subarray, etc. Thus the vector of received signals at the pth subarray can be written as follows

$$\mathbf{x}_p(k) = \mathbf{A}\mathbf{\Phi}^{(p-1)}\mathbf{s}(k) + \mathbf{i}_p(k) + \mathbf{n}_p(k) \quad (8)$$

where $\mathbf{\Phi}$ denotes the pth power of $d \times d$ diagonal matrix

$$\mathbf{\Phi} = \text{diag}\{e^{-jw_0\tau_1}, \cdots, e^{-jw_0\tau_d}\} \quad (9)$$

The forward covariance matrix of the pth subarray is defined by

$$\mathbf{R}^{\alpha}_p(\tau) = \mathbf{A}\mathbf{\Phi}^{(p-1)}\mathbf{S}^{\alpha}_p(\tau)[\mathbf{\Phi}^{(p-1)}]^H\mathbf{A}^H \quad (10)$$

where $\mathbf{S}^{\alpha}_p(\tau)$ is therefore given by

$$\mathbf{S}^{\alpha}_p(\tau) = \mathbf{R}^{\alpha}_{ss}(\tau)[\mathbf{\Phi}^{(p-1)}]^H\mathbf{A}^H\mathbf{A}\mathbf{\Phi}^{(p-1)}[\mathbf{R}^{\alpha}_{ss}(\tau)]^H \quad (11)$$

and the backward covariance matrix of the pth subarray is defined as follows

$$\begin{aligned}\bar{\mathbf{R}}^{\alpha}_p(\tau) &= \mathbf{J}\mathbf{A}^*[\mathbf{\Phi}^{(p-1)}]^*[\mathbf{S}^{\alpha}_p(\tau)]^*[\mathbf{\Phi}^{(p-1)}]^{*H}\mathbf{A}^{*H}\mathbf{J} \\ &= \mathbf{A}\mathbf{\Phi}^{(2-m-p)}[\mathbf{S}^{\alpha}_p(\tau)]^*[\mathbf{\Phi}^{(2-m-p)}]^H\mathbf{A}^H \quad (12)\end{aligned}$$

where $\mathbf{J} \in C^{d\times d}$ is the exchange matrix with ones on its antidiagonal and zeros elsewhere and superscript $*$ denotes complex conjugation. So the forward backward smoothed covariance matrix is introduced by

$$\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau) = \frac{1}{2P}\sum_{p=1}^{P}(\mathbf{R}^{\alpha}_p(\tau) + \bar{\mathbf{R}}^{\alpha}_p(\tau)) \quad (13)$$

or more compactly as

$$\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau) = \mathbf{A}\mathbf{S}^{\alpha}_{\mathbf{FB}}(\tau)\mathbf{A}^H \quad (14)$$

where $\mathbf{S}^{\alpha}_{FB}(\tau)$, the modified cyclic autocorrelation matrix [5]-[6] of the cyclostationary signals, is given by

$$\begin{aligned}\mathbf{S}^{\alpha}_{FB}(\tau) &= \frac{1}{2P}\sum_{p=1}^{P}(\mathbf{\Phi}^{(p-1)}\mathbf{S}^{\alpha}_p(\tau)[\mathbf{\Phi}^{(p-1)}]^H \\ &\quad +\mathbf{\Phi}^{(2-m-p)}[\mathbf{S}^{\alpha}_p(\tau)]^*[\mathbf{\Phi}^{(2-m-p)}]^H) \quad (15)\end{aligned}$$

Theorem: The modified cyclic autocorrelation matrix of the array $\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau)$ is called centro-Hermitian.

Proof: Firstly, by multiplying the complex conjugation form of the modified cyclic autocorrelation matrix $\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau)$ on both sides with the exchange matrix $\mathbf{J}$, we obtain a matrix in the following fashion:

$$\begin{aligned}\mathbf{J}[\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau)]^*\mathbf{J} &= \mathbf{J}\mathbf{A}^*[\mathbf{S}^{\alpha}_{\mathbf{FB}}(\tau)]^*[\mathbf{J}\mathbf{A}^*]^H \\ &= \mathbf{A}\mathbf{\Phi}^{-(m-1)}[\mathbf{S}^{\alpha}_{FB}(\tau)]^*\mathbf{\Phi}^{(m-1)}\mathbf{A}^H (16)\end{aligned}$$

where the complex conjugation form of $\mathbf{S}^{\alpha}_{FB}(\tau)$ is rewritten as

$$\begin{aligned}[\mathbf{S}^{\alpha}_{FB}(\tau)]^* &= \frac{1}{2P}[\sum_{p=1}^{P}(\mathbf{\Phi}^{(p-1)}\mathbf{S}^{\alpha}_p(\tau)[\mathbf{\Phi}^{(p-1)}]^H \\ &\quad +\mathbf{\Phi}^{(2-m-p)}[\mathbf{S}^{\alpha}_p(\tau)]^*[\mathbf{\Phi}^{(2-m-p)}]^H)]^* \\ &= \mathbf{\Phi}^{(m-1)}\mathbf{S}^{\alpha}_{FB}(\tau)\mathbf{\Phi}^{-(m-1)} \quad (17)\end{aligned}$$

Obviously, using (16) and (17) the important equation can be drawn as

$$\mathbf{J}[\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau)]^*\mathbf{J} = \tilde{\mathbf{R}}^{\alpha}_{FB}(\tau) \quad (18)$$

Secondly, a complex matrix $\mathbf{G}$ is called centro-Hermitian [7]-[8] if

$$\mathbf{G} = \mathbf{J}\mathbf{G}^*\mathbf{J} \quad (19)$$

Thus, the modified cyclic autocorrelation matrix $\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau)$ is centro-Hermitian.

By exploiting the centro-Hermitian property of $\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau)$, we introduce the real-valued covariance matrix as

$$\mathbf{C} = \mathbf{Q}^H\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau)\mathbf{Q} \quad (20)$$

where $\mathbf{Q}$ is any unitary column conjugate symmetric matrix, for example

$$\mathbf{Q} = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I} & j\mathbf{I} \\ \mathbf{J} & -j\mathbf{J}\end{bmatrix}$$

and

$$\mathbf{Q} = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I} & \mathbf{0} & j\mathbf{I} \\ \mathbf{0}^T & \sqrt{2} & \mathbf{0}^T \\ \mathbf{J} & \mathbf{0} & -j\mathbf{J}\end{bmatrix}$$

can be chosen for arrays with an even and odd number of sensors, respectively, where $\mathbf{I}$ is the identity matrix and $\mathbf{0}$ is the vector $[0, 0, \cdots, 0]^T$. Compared with the modified cyclic autocorrelation matrix (14), the real-valued covariance matrix is obtained as

$$\mathbf{C} = \tilde{\mathbf{A}}\mathbf{S}^{\alpha}_{FB}(\tau)\tilde{\mathbf{A}}^H \quad (21)$$

where

$$\tilde{\mathbf{A}} = \mathbf{Q}^H\mathbf{A} \quad (22)$$

denotes the relationship between the former and new manifolds.

Contrary to the cyclic covariance matrix $\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau)$ exploited by Cyclic MUSIC algorithm, the forward backward smoothed covariance matrix $\tilde{\mathbf{R}}^{\alpha}_{FB}(\tau)$ presented is Hermitian. Let the eigendecompositions of the matrices (13) and (20) be defined as

$$\mathbf{R}^{\alpha}_{FB}(\tau) = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H = \mathbf{U}_s\Lambda_s\mathbf{U}_s^H + \mathbf{U}_n\Lambda_n\mathbf{U}_n^H \quad (23)$$

$$\mathbf{C} = \mathbf{E}\mathbf{\Gamma}\mathbf{E}^H = \mathbf{E}_s\Gamma_s\mathbf{E}_s^H + \mathbf{E}_n\Gamma_n\mathbf{E}_n^H \quad (24)$$

where

$$\begin{aligned}\mathbf{U}_s &= [u_1, \cdots, u_d] \\ \mathbf{U}_n &= [u_{d+1}, \cdots, u_m] \\ \mathbf{\Lambda}_s &= \text{diag}\{\lambda_1, \cdots, \lambda_d\} \\ \mathbf{\Lambda}_n &= \text{diag}\{\lambda_{d+1}, \cdots, \lambda_m\} \\ \mathbf{E}_s &= [\varepsilon_1, \cdots, \varepsilon_d] \\ \mathbf{E}_n &= [\varepsilon_{d+1}, \cdots, \varepsilon_m] \\ \mathbf{\Gamma}_s &= \text{diag}\{\gamma_1, \cdots, \gamma_d\} \\ \mathbf{\Gamma}_n &= \text{diag}\{\gamma_{d+1}, \cdots, \gamma_m\}\end{aligned}$$

and the subscripts s and n stand for signal- and null-space, respectively. The characteristic equation for the matrix (13) is written as

$$\tilde{\mathbf{R}}_{FB}^{\alpha}(\tau)u = \lambda u \tag{25}$$

which can be further rewritten as

$$\begin{aligned} \mathbf{Q}^H\tilde{\mathbf{R}}_{FB}^{\alpha}(\tau)u &= \mathbf{Q}^H\tilde{\mathbf{R}}_{FB}^{\alpha}(\tau)\mathbf{Q}\mathbf{Q}^H u \\ &= \mathbf{C}\mathbf{Q}^H u \\ &= \lambda u \end{aligned} \tag{26}$$

Equation (26) can be identified as the characteristic one for the real-valued covariance matrix (20) . Hence, the eigenvectors and eigenvalues of the matrices (13) and (20) are related as

$$\mathbf{E} = \mathbf{Q}^H\mathbf{U} \tag{27}$$

$$\mathbf{\Gamma} = \mathbf{\Lambda} \tag{28}$$

By simple manipulation with the conventional MUSIC algorithm, the spatial spectrum of Unitary Cyclic MUSIC algorithm is obtained as

$$\begin{aligned} P_{UCM}(\theta) &= \frac{\mathbf{a}^H(\theta)\mathbf{Q}\mathbf{Q}^H\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{Q}\mathbf{Q}^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{Q}\mathbf{Q}^H\mathbf{a}(\theta)} \\ &= \frac{\tilde{\mathbf{a}}^H(\theta)\tilde{\mathbf{a}}(\theta)}{\tilde{\mathbf{a}}^H(\theta)\mathbf{Q}^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{Q}\tilde{\mathbf{a}}(\theta)} \\ &= \frac{\tilde{\mathbf{a}}^H(\theta)\tilde{\mathbf{a}}(\theta)}{\tilde{\mathbf{a}}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\tilde{\mathbf{a}}(\theta)} \end{aligned} \tag{29}$$

which can be simplified to

$$P_{UCM}(\theta) = \frac{\|\tilde{a}(\theta)\|^2}{\|\mathbf{E}_n^H\tilde{a}(\theta)\|^2} \tag{30}$$

Summary of Unitary Cyclic MUSIC algorithm:

- Step 1: Choose the cycle frequency of desired signals α and the optimal τ ;
- Step 2: Estimate the forward backward smoothed covariance matrix $\tilde{\mathbf{R}}_{FB}^{\alpha}(\tau)$ from the received data of ULA;
- Step 3: Form the real-valued matrix $\mathbf{C}$ with the use of (20);
- Step 4: Find the null subspace $\mathbf{E}_n$ of the real-valued matrix $\mathbf{c}$, and detect the number d of SOIs based on AIC and MDL principle;
- Step 5: Search over θ for the d highest peaks in $P_{UCM}(\theta)$.

5. SIMULATION RESULTS

In this section, we present some simulation results to show the behavior of Unitary Cyclic MUSIC algorithm and to compare it with the Cyclic MUSIC algorithm. Assume a ULA with eight omnidirectional sensors spaced by a half wavelength of the coming signals. Incoming cyclostationary signals with central frequency 0.1 are generated with noise at cycle frequency 0.2 and the signal-to-noise ratio (SNR) is 10 dB for each signal.

In the first simulation, the signal selectivity and accuracy of Unitary Cyclic MUSIC algorithm are tested. Two uncorrelated SOIs arrive from $-15°$ and $-25°$, and one interferer arrives from $15°$. The resulting spatial spectra are shown in Fig. 1.

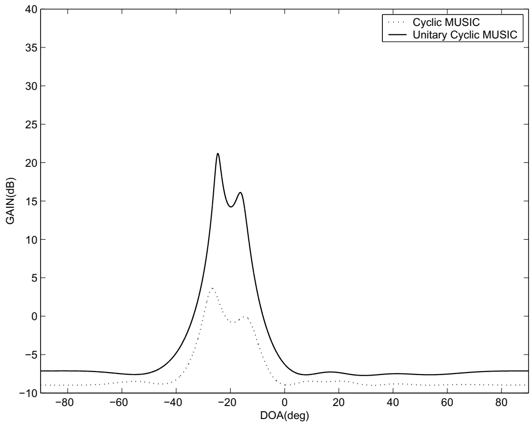

Figure 1: Spatial spectra for environment containing two uncorrelated SOIs with $-15°$ and $-25°$ DOA and one interferer with $15°$ DOA.

In the second simulation, Unitary Cyclic MUSIC algorithm accurately estimates two SOI DOAs in multipath propagation. Two coherent SOIs arrive from $-15°$ and $-25°$, one interferer arrives from $15°$. The resulting spatial spectra are shown in Fig. 2.

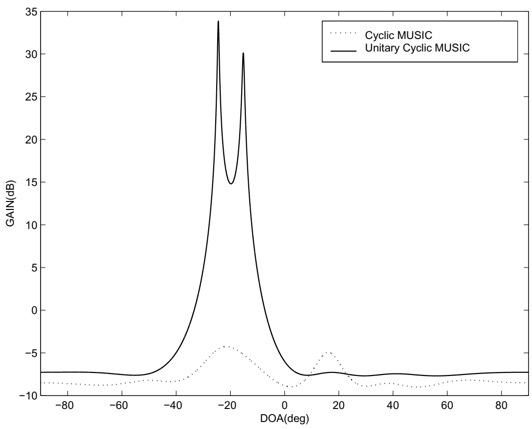

Figure 2: Spatial spectra for environment containing two coherent SOIs with $-15°$ and $-25°$ DOA and one interferer with $15°$ DOA.

In the third simulation, the performance of Unitary Cyclic MUSIC algorithm is affected by SNR. Two uncorrelated signals arrive from $-15°$ and $-25°$. The results for Cyclic MUSIC algorithm and Unitary Cyclic MUSIC algorithm versus the SNR are plotted in Fig. 3.

In the fourth simulation, the performance of Unitary Cyclic MUSIC algorithm is affected by SNR in multipath propagation. Two coherent signals arrive from $-15°$ and $-25°$. The results for Cyclic MUSIC algorithm and Unitary Cyclic MUSIC algorithm versus the SNR are plotted in Fig. 4.

From Figs.1-2, both Unitary Cyclic MUSIC algorithm and Cyclic MUSIC algorithm can separate uncorrelated signals, but only Unitary Cyclic MUSIC algorithm can separate coherent signals. As expected, the presence of the interferer has little effect on the SOIs. Figs.3-4 show how the SNR affects the DOA estimation. The performance of the method

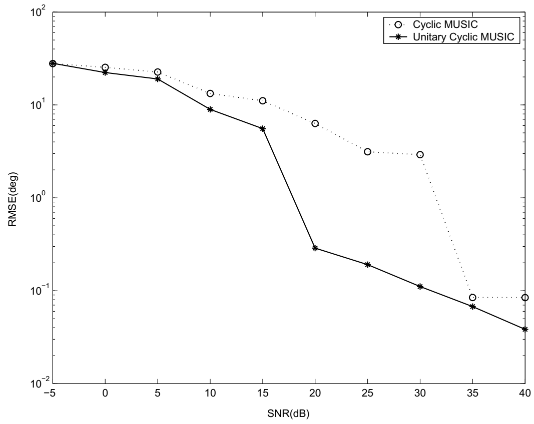

Figure 3: RMSE versus SNR for environment containing two uncorrelated SOIs with $-15°$ and $-25°$ DOA.

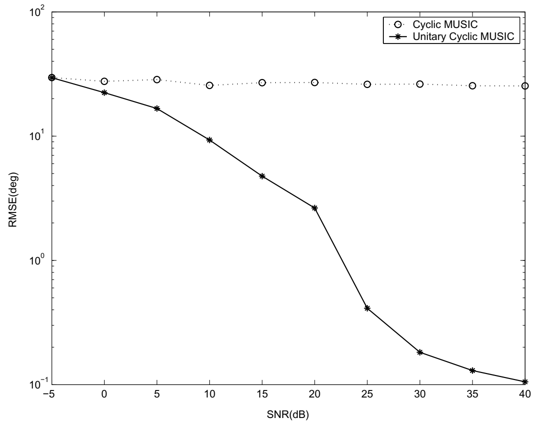

Figure 4: RMSE versus SNR for environment containing two coherent SOIs with $-15°$ and $-25°$ DOA.

is quantified by the root-mean-square-error (RMSE) of 100 independent DOA estimates. Both for correlated and uncorrelated source scenarios, Unitary Cyclic MUSIC algorithm performs better than the Cyclic MUSIC algorithm.

6. CONCLUSION

Unitary Cyclic MUSIC algorithm is proposed by constructing a new forward backward smoothed covariance matrix in this paper.Simulation results suggest that the proposed approach has a better signal selectivity and a better resolution power than Cyclic MUSIC algorithm, by exploiting the property of the cyclostationarity of incoming signals,both for correlated and uncorrelated source scenarios. At the same time this approach has low computational complexity because of real-valued computation.

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