

# A SOLUTION TO THE BEST WAVELET FILTER BANK PROBLEM IN THE WAVELET-BASED IMAGE WATERMARKING

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## ABSTRACT

The digital watermark technology is now drawing attention as a useful method of protecting copyrights of digital contents such as audio, image, and video. Especially, efficient image watermarking methods have been developed in the DWT (discrete wavelet transform) domain. In the wavelet-based watermarking, similar to the wavelet-based coding, the choice of wavelet filter banks generally affects the performance of watermarking methods. Hence a natural question arises for the wavelet-based watermarking : what is the best wavelet filter bank for use ? In this paper, we evaluate wavelet filter banks used in watermarking theoretically from the viewpoint of the quality of watermarked image and the robustness of watermark, and then try to answer the 'best wavelet filter bank' problem in the wavelet-based watermarking.

## 1 Introduction

With the rapid spread of computer networks and the further development of multimedia technologies, the copyright protection of digital contents such as audio, image and video, has been one of the most serious problems because digital copies can be made identical to the original. The digital watermark technology is now drawing attention as a useful method of protecting copyrights of digital contents. A digital watermark is realized by embedding information data, e.g., owner, distributor, or recipient identifiers, transaction dates, serial number, etc., directly into digital contents with an imperceptible form for human audio/visual systems, and should satisfy the following requirements: The embedded watermark should not spoil the quality of the original contents and should not be perceptible. It should be difficult for an attacker to remove the watermark and should be robust to signal processing and geometric distortions.

A significant number of watermarking methods have been recently reported[1],[2]. Most of these methods embed the watermark into the spectral coefficients of images by using signal transformation such as discrete cosine transformation (DCT) or discrete wavelet transformation (DWT) because embedding in the frequency

domain is more tolerant to attacks and image processing than embedding in the spatial domain. Especially, efficient image watermarking methods have been developed in the DWT domain.

In the wavelet-based watermarking, similar to the wavelet-based coding, the choice of wavelet filter banks generally affects the performance of watermarking methods. Hence a natural question arises for the wavelet-based watermarking : what is the best wavelet filter bank for use ? But it seems that this question has not been answered for image watermarking yet.

In this paper, we concentrate on the correlation-based watermarking methods in the DWT domain, in which a watermark code (pseudorandom sequence) is perceptually weighted and added to the wavelet coefficient, and is detected by computing the correlation between the watermarked coefficient and the watermark code to be checked for the presence, and discuss the performance of the watermarking methods theoretically from the viewpoint of the quality of watermarked image and the robustness of watermark. Based on the result, we evaluate wavelet filter banks used in watermarking and then try to answer the 'best wavelet filter bank' problem in the wavelet-based watermarking.

## 2 Wavelet Transform of Images using Wavelet Filter Banks

An obvious way to do the wavelet transform (wavelet decomposition) of two-dimensional (2D) signals (e.g., images)[3] is to use separable wavelets obtained from products of one-dimensional (1D) wavelets and scaling functions. Taking a lowpass filter  $H_0(z)$  and a high-pass filter  $H_1(z)$  corresponding, respectively, to a 1D scaling function and a 1D wavelet, we can construct 2D filters:  $H_{LL}(z_1, z_2) = H_0(z_1)H_0(z_2)$ ,  $H_{LH}(z_1, z_2) = H_0(z_1)H_1(z_2)$ ,  $H_{HL}(z_1, z_2) = H_1(z_1)H_0(z_2)$ , and  $H_{HH}(z_1, z_2) = H_1(z_1)H_1(z_2)$ . The function  $H_{LL}(z_1, z_2)$  is a separable 2D scaling function (that is, a 2D lowpass filter), while the function  $H_{LH}(z_1, z_2)$ ,  $H_{HL}(z_1, z_2)$ , and  $H_{HH}(z_1, z_2)$  are 2D separable wavelets.

The 2D input signal  $X(z_1, z_2)$  (e.g., image) is processed by the 2D filters  $H_{LL}$ ,  $H_{LH}$ ,  $H_{HL}$ , and  $H_{HH}$

and then subsampled by 2 in each dimension, i.e., overall subsampled by 4. As the result,  $X(z_1, z_2)$  is divided into four subband signals  $X_{LL}^1(z_1, z_2)$ ,  $X_{LH}^1(z_1, z_2)$ ,  $X_{HL}^1(z_1, z_2)$ , and  $X_{HH}^1(z_1, z_2)$ . The subbands  $X_{LH}^1$ ,  $X_{HL}^1$ , and  $X_{HH}^1$  represent the finest scale wavelet coefficients. To obtain the next coarser scale of wavelet coefficients, the subband  $X_{LL}^1(z_1, z_2)$  is further processed and subsampled. The process continues until some final scale  $N$  is reached, and we have  $3N + 1$  subband signals consisting of the multiresolution approximation (MRA) component  $X_{LL}^N(z_1, z_2)$  and the multiresolution representation (MRR) components  $X_{LH}^n(z_1, z_2)$ ,  $X_{HL}^n(z_1, z_2)$ ,  $X_{HH}^n(z_1, z_2)$  ( $n = 1, 2, \dots, N$ ).

The inverse wavelet transform (wavelet reconstruction) is as follows. Let  $G_0(z)$  and  $G_1(z)$  be lowpass and highpass filters, respectively, and be the dual of  $H_0(z)$  and  $H_1(z)$ , i.e.,  $H_0(z)$ ,  $H_1(z)$  and  $G_0(z)$ ,  $G_1(z)$  satisfy the perfect reconstruction condition for analysis and synthesis filter banks[3]. Then we can construct 2D filters:  $G_{LL}(z_1, z_2) = G_0(z_1)G_0(z_2)$ ,  $G_{LH}(z_1, z_2) = G_0(z_1)G_1(z_2)$ ,  $G_{HL}(z_1, z_2) = G_1(z_1)G_0(z_2)$ , and  $G_{HH}(z_1, z_2) = G_1(z_1)G_1(z_2)$ . After upsampling the four subband signals  $X_{LL}^N$ ,  $X_{LH}^N$ ,  $X_{HL}^N$ , and  $X_{HH}^N$  by 2 in each direction, we process them by the 2D filters  $G_{LL}$ ,  $G_{LH}$ ,  $G_{HL}$ , and  $G_{HH}$ , respectively, and reconstruct the  $(N - 1)$ -th MRA component  $X_{LL}^{N-1}(z_1, z_2)$ . Next, the same procedure is applied to  $X_{LL}^{N-1}$ ,  $X_{LH}^{N-1}$ ,  $X_{HL}^{N-1}$ , and  $X_{HH}^{N-1}$ , and the  $(N - 2)$ -th MRA component  $X_{LL}^{N-2}(z_1, z_2)$  is obtained. This wavelet reconstruction is repeated  $N$  times and the 2D signal  $X(z_1, z_2)$  is reconstructed.

### 3 Watermarking Method using Wavelet Filter Banks

We decompose an image  $X(z_1, z_2)$  using a wavelet filter bank until the scale  $N$  and obtain the MRA component  $X_{LL}^N$  and the  $3N$  MRR components  $X_{LH}^n$ ,  $X_{HL}^n$ ,  $X_{HH}^n$  ( $n = 1, 2, \dots, N$ ). In the watermarking method with the MRA component (MRA method),  $B_A$  wavelet coefficients  $x_A^N(i_k, j_k)$  ( $1 \leq k \leq B_A$ ) are appropriately selected from  $X_{LL}^N$  and then a watermark  $w(k)$  ( $1 \leq k \leq B_A$ ) is embedded into  $x_A^N(i_k, j_k)$ , while in the watermarking method with the MRR component (MRR method),  $B_R$  wavelet coefficients  $x_R^N(i_k, j_k)$  ( $1 \leq k \leq B_R$ ) are properly choosed from  $X_{LH}^N$  and/or  $X_{HL}^N$  and then a watermark  $w(k)$  ( $1 \leq k \leq B_R$ ) is embedded into  $x_R^N(i_k, j_k)$ .

Watermark embedding of both the methods is carried out as follows : Let  $W = \{\mathbf{w}_l, 1 \leq l \leq L\}$  be a set of watermarks, where  $\mathbf{w}_l = [w_l(k)]$  ( $1 \leq k \leq B$ ) and  $x_l(k)$ 's are pseudorandom sequences drawn from the Gaussian distribution with zero-mean and variance  $\sigma^2 = 1$ . Then we choose one watermark  $\mathbf{w} = [w(k)]$  from  $W$  and embed  $w(k)$ 's into the wavelet coefficients  $x(i_k, j_k)$ 's in the form of

$$x'(i_k, j_k) = x(i_k, j_k) + Qw(k), \quad 1 \leq m \leq B, \quad (1)$$

where  $Q(> 0)$  is a weight called the embedded intensity.

Watermark detection of both the methods is as follows : We set a threshold  $T(> 0)$  to decide whether the watermark is detected or not, and calculate the correlation  $z$  between the watermarked wavelet coefficients  $x'(i_k, j_k)$ 's and all candidates  $\mathbf{w}_l \in W$  of the embedded watermark as

$$z = z(l) = \frac{1}{B} \sum_{k=1}^B x'(i_k, j_k)w_l(k), \quad 1 \leq l \leq L. \quad (2)$$

If  $z(l)$  exceeds  $T$ , then the watermark detector determines that the watermarked image contains  $\mathbf{w}_l$ . That is, if  $z(l) > T$ , then  $\mathbf{w}_l$  is detected, else no  $\mathbf{w}_l$  detected.

### 4 On the Quality of Watermarked Images

We discuss the relationship between the characteristics of wavelet filter banks and the quality of watermarked images. Let  $\Delta x(i_k, j_k) = x'(i_k, j_k) - x(i_k, j_k)$  be the noise that arises from the watermark embedding. Then  $\Delta x(i_k, j_k)$ 's are Gaussian noise with zero-mean and variance  $\sigma^2 = Q^2$ . The random noise  $\Delta x(i_k, j_k)$ 's pass through the synthesis filter bank consisting of upsampling followed by 2D filtering, spread to a whole of image, and degrade the quality of watermarked image. The noise power at the output of the filter bank, denoted by  $N_A$  and  $N_R$ , respectively, for the MRA and MRR methods, is given as follows:

For the MRA method,

$$N_A = B_A Q_A^2 \|G_A\|^2, \quad (3)$$

where  $Q_A$  is the embedded intensity,

$$G_A(z_1, z_2) = \prod_{k=0}^{N-1} G_0(z_1^{2^k})G_0(z_2^{2^k}), \quad (4)$$

and  $\|G\|$  denotes the  $L^2$ -norm of  $G(e^{j\omega_1}, e^{j\omega_2})$ :

$$\|G\|^2 = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |G(e^{j\omega_1}, e^{j\omega_2})|^2 d\omega_1 d\omega_2. \quad (5)$$

For the MRR method,

$$N_R = B_R Q_R^2 \|G_R\|^2, \quad (6)$$

in which  $Q_R$  is the embedded intensity,

$$\begin{aligned} G_R(z_1, z_2) &= \{\alpha G_0(z_1^{N-1})G_1(z_2^{N-1}) \\ &+ (1 - \alpha)G_1(z_1^{N-1})G_0(z_2^{N-1})\} \\ &\times \prod_{k=0}^{N-2} G_0(z_1^{2^k})G_0(z_2^{2^k}), \end{aligned} \quad (7)$$

and we assume that the ratio of amount of watermarks embedded into  $X_{LH}^N$  and  $X_{HL}^N$  is  $\alpha : (1 - \alpha)$  ( $0 < \alpha < 1$ ). It can be seen that the watermarked image quality depends on not only amount of watermarks and the embedded intensity but also the characteristics of wavelet filter bank.

## 5 On the Robustness of Watermark

We consider the case where a watermarked image is processed by an operation like lossy compression and filtering, etc. Then the noise that arises from image processing passes through the analysis filter bank consisting of 2D filtering followed by downsampling and is transmitted to every subband. Especially, the noise at the subbands  $X_{LL}^N$ ,  $X_{LH}^N$ , and  $X_{HL}^N$  causes watermark detection errors and hence deterioration of accuracy of watermark detection.

Assuming that the noise caused by image processing is random and additive noise with zero-mean and variance  $\sigma^2$ , the correlation  $z$  can be written as

$$z = z(l) = \frac{1}{B} \sum_{k=1}^B x''(i_k, j_k) w_l(k), \quad 1 \leq l \leq L, \quad (8)$$

in which

$$\begin{aligned} x''(i_k, j_k) &= x'(i_k, j_k) + \delta(i_k, j_k) \\ &= x(i_k, j_k) + Qw(k) + \delta(i_k, j_k), \end{aligned} \quad (9)$$

and  $\delta(i_k, j_k)$  represents the noise at  $X_{LL}^N$ ,  $X_{LH}^N$ , and  $X_{HL}^N$ , and the noise power at  $X_{LL}^N$ ,  $X_{LH}^N$ , and  $X_{HL}^N$ , denoted by  $\sigma_{LL}^2$ ,  $\sigma_{LH}^2$ , and  $\sigma_{HL}^2$ , respectively, is given by

$$\sigma_\theta^2 = \sigma^2 \|E_\theta\|^2, \quad (10)$$

where  $\theta$  stands for  $LL$ ,  $LH$ , or  $HL$ ,

$$E_{LL}(z_1, z_2) = \prod_{k=0}^{N-1} H_0(z_1^{2^k}) H_0(z_2^{2^k}), \quad (11)$$

$$\begin{aligned} E_{LH}(z_1, z_2) &= H_0(z_1^{N-1}) H_1(z_2^{N-1}) \\ &\times \prod_{k=0}^{N-2} H_0(z_1^{2^k}) H_0(z_2^{2^k}), \end{aligned} \quad (12)$$

$$\begin{aligned} E_{HL}(z_1, z_2) &= H_1(z_1^{N-1}) H_0(z_2^{N-1}) \\ &\times \prod_{k=0}^{N-2} H_0(z_1^{2^k}) H_0(z_2^{2^k}), \end{aligned} \quad (13)$$

and it is noted that  $\|E_{LH}\| = \|E_{HL}\|$  holds.

The robustness of watermark is measured by the error rate

$$\varepsilon = \int_T^\infty p_0(z) dz + \int_{-\infty}^T p_1(z) dz \quad (14)$$

where  $p_1(z)$  is the probability density function (pdf) of  $z(l)$  when  $w_l$  corresponds to the watermark embedded into the image, while  $p_0(z)$  the pdf of  $z(l)$  when  $w_l$  does not. Here we can calculate the expectation  $E[z(l)]$  and the variance  $\text{Var}[z(l)]$ . Let's suppose that  $x(i_k, j_k)$ ,  $w(k)$ , and  $\delta(i_k, j_k)$  are uncorrelated mutually. Then  $E[z(l)]$  and  $\text{Var}[z(l)]$  are given by, respectively,

$$E[z(l)] = \begin{cases} Q_\lambda & , \quad w_l = w \\ 0 & , \quad w_l \neq w \end{cases} \quad (15)$$

$$\text{Var}[z(l)] = \begin{cases} (2Q_\lambda^2 + \sigma^2 \|E_\lambda\|^2) / B_\lambda & , \quad w_l = w \\ (Q_\lambda^2 + \sigma^2 \|E_\lambda\|^2) / B_\lambda & , \quad w_l \neq w \end{cases} \quad (16)$$

where  $\lambda$  represents  $A$  for the MRA method or  $R$  for the MRR method,  $\|E_A\| = \|E_{LL}\|$ ,  $\|E_R\| = \sqrt{\alpha \|E_{LH}\|^2 + (1-\alpha) \|E_{HL}\|^2}$  ( $= \|E_{HL}\| = \|E_{LH}\|$ ), and  $Q_\lambda$ ,  $B_\lambda$  and  $\alpha$  are the same as those in Sect. 4. Putting  $T = Q_\lambda/2$  and approximating  $p_0(z)$  and  $p_1(z)$  by the Gaussian pdf with the above statistics, we can evaluate the robustness of watermark by

$$\begin{aligned} \varepsilon_\lambda &= \text{erfc} \left( \frac{1}{2} \sqrt{\frac{B_\lambda}{1 + (\sigma \|E_\lambda\| / Q_\lambda)^2}} \right) \\ &+ \text{erfc} \left( \frac{1}{2} \sqrt{\frac{B_\lambda}{2 + (\sigma \|E_\lambda\| / Q_\lambda)^2}} \right), \end{aligned} \quad (17)$$

in which  $\lambda = A$  or  $R$ , and  $\text{erfc}(x)$  is the error function defined by

$$\text{erfc}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt. \quad (18)$$

It is noted that the error rate is also dependent on not only the embedded intensity  $Q_\lambda$  and the length of watermark  $B_\lambda$  but also the characteristics of wavelet filter bank.

## 6 Comparison between the MRA and MRR Methods using Filter Bank of 5/3 Taps SSKF

In the case of filter bank of 5/3 taps SSKF (symmetric short kernel filter)[4]:  $H_0(z) = -0.125 + 0.25z^{-1} + 0.75z^{-2} + 0.25z^{-3} - 0.125z^{-4}$  (LPF),  $H_1(z) = 0.5 - z^{-1} + 0.5z^{-2}$  (HPF),  $G_0(z) = H_1(-z)$ ,  $G_1(z) = -H_0(-z)$  and the scale  $N = 4$ , we have  $\|G_A\|^2 = 114.2227$ ,  $\|G_R\|^2 = 32.5217$  and  $\|E_A\|^2 = 0.6171$ ,  $\|E_R\|^2 = 1.5009$ . From Eqs. (3) and (6), the following relation holds.

$$N_A/N_R = 3.512 (B_A/B_R)(Q_A/Q_R)^2. \quad (19)$$

Eq. (19) implies that when  $Q_A = Q_R$  and  $B_A = B_R$ , by using the MRR method, we can obtain better quality of watermarked image, i.e., we can expect the improvement of watermarked image quality of about 5.5 [dB], in comparison with the MRA method.

On the other hand, in order to obtain the same rate of watermark detection in both the methods, i.e.,  $\varepsilon_A = \varepsilon_R$ , we need to set  $Q_A$  and  $Q_R$  as

$$Q_A/Q_R = \|E_A\|/\|E_R\| = 0.6412 \quad (20)$$

Thus, when  $B_A = B_R$ , we have, from Eq. (19),  $N_A/N_R = 1.444$ . This implies that the MRR method also presents better quality of watermarked image than the MRA method under the condition that  $\varepsilon_A = \varepsilon_R$ , that is, by using the MRR method, the improvement of watermarked image quality of about 1.6 [dB] is expected in comparison with the MRA method.

Numerical experiments are performed to show the validity of the above theoretical results, where the test image is “Lena” (256 × 256 pixels, 8 bits/pixel) and the number of watermarks is 256 bits ( $B_A = B_R = 256$ ). We first investigate the watermarked image quality. Figure 1 illustrates the relationship between the watermarked image quality (PSNR) and the embedded intensity. It is noticed that the MRR method presents the improvement of 5.1 ~ 5.6 [dB] in comparison with the MRA method. Next we evaluate the robustness against JPEG compression with standard quality. The result is shown in Fig. 2, from which we can see that both the methods are robust if  $Q_A \geq 6$  and  $Q_R \geq 10$ . As the watermarked image quality (PSNR) is 47.8 [dB] for  $Q_A = 6$  and 49.2 [dB] for  $Q_R = 10$ , the improvement of 1.4 [dB] is obtained by the MRR method in comparison with the MRA method. From the above experimental results, the theoretical results are verified.

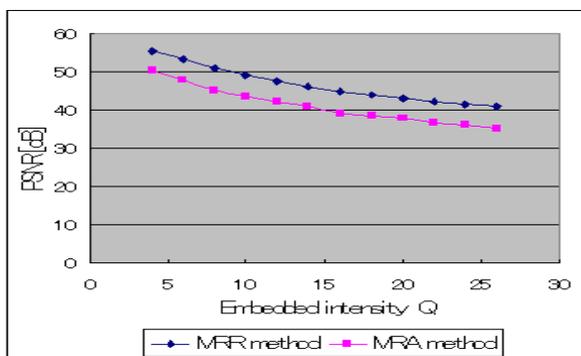


Figure 1 : Quality of watermarked image ( $Q$  versus PSNR).

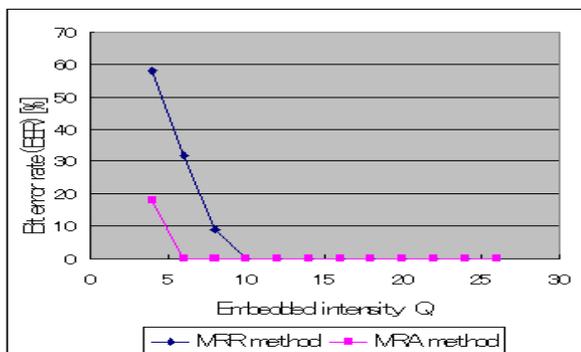


Figure 2 : Robustness against JPEG compression with standard quality ( $Q$  versus BER).

## 7 Answer to the ‘best wavelet filter bank’ problem – How to evaluate wavelet filter banks –

As a solution to the ‘best wavelet filter bank’ problem, we present how to evaluate wavelet filter banks for the wavelet-based watermarking based on the results described in Sects. 4 and 5. Here, because of the limitation of space, the evaluation method is explained using filter banks with the Daubechies wavelet having  $2p$  taps, abbreviated as  $db(p)$ , ( $1 \leq p \leq 10$ ) [5]. Let  $B_\lambda$ ,  $Q_\lambda$ ,  $N_\lambda$ ,

and  $\varepsilon_\lambda$  of  $N$ -octave wavelet filter bank with  $db(p)$  be denoted by, respectively,  $B_\lambda(p, N)$ ,  $Q_\lambda(p, N)$ ,  $N_\lambda(p, N)$ , and  $\varepsilon_\lambda(p, N)$ , where  $\lambda$  represents  $A$  or  $R$ , and let us take the filter of  $N = 4$  and  $db(1)$ , corresponding to the Haar wavelet, as a standard filter bank. The performance of the filter banks to the standard,  $r_\lambda(p, N)$ , is measured by the watermarked image quality when setting the amount of watermarks as  $B_\lambda(p, N) = B_\lambda(1, 4)$  and the embedded intensity  $Q_\lambda(p, N)$  and  $Q_\lambda(1, 4)$  as the same rate of watermark detection  $\varepsilon_\lambda(p, N) = \varepsilon_\lambda(1, 4)$ , i.e.,

$$\begin{aligned} r_\lambda(p, N) &= \frac{N_\lambda(p, N)}{N_\lambda(1, 4)} \Big|_{B_\lambda(p, N)=B_\lambda(1, 4), \varepsilon_\lambda(p, N)=\varepsilon_\lambda(1, 4)} \\ &= \frac{\|G_\lambda(p, N)\|^2}{\|G_\lambda(1, 4)\|^2} \cdot \frac{\|E_\lambda(p, N)\|^2}{\|E_\lambda(1, 4)\|^2}, \end{aligned} \quad (21)$$

$G_\lambda(p, N)$  and  $E_\lambda(p, N)$  ( $\lambda=A$  or  $R$ ) being  $G_\lambda$  and  $E_\lambda$  of  $N$ -octave filter bank with  $db(p)$ , respectively,  $\|G_A(1, 4)\| = \|G_R(1, 4)\|$ , and  $\|E_A(1, 4)\| = \|E_R(1, 4)\|$ . When  $r_\lambda(p, N)$  is computed for  $p = 1, 2, \dots, 10$  and  $N = 2, 3, 4$ , we find that  $r_R(6, 4) = 0.3024$ ,  $r_R(2, 4) = 0.3840$ ,  $r_R(8, 4) = 0.5824$ ,  $\dots$ , and hence the MRR method using the 4-octave wavelet filter bank with  $db(6)$  provides the best performance among the filter banks examined in this section.

## 8 Concluding Remarks

In this paper, a problem concerning the best wavelet filter bank for the correlation-based watermarking method in the DWT domain has been discussed and the method of evaluating wavelet filter banks has been presented on the basis of the theoretical analysis of the quality of watermarked image and the robustness of watermark. The evaluation method can also be applied to filter banks with another type of wavelet such as Symlets, Coiflets, and Spline wavelet [5]. The author is convinced that the approach used in this paper is effective when we study and realize the wavelet-based watermarking system. We would like to discuss several related problems in forthcoming papers.

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