ON-LINE MULTICHANNEL BLIND EQUALIZATION ALGORITHM WITH NONSTATIONARY SIGNALS

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ABSTRACT

Multichannel blind equalization is an important task for numerous applications such as speech separation, dereverberation, communication, signal processing and control, etc. In this paper, a cost function is constructed by the knowledge of correlation. Then a novel on-line algorithm is derived with natural gradient search method for multichannel blind source separation of convolved signal mixtures in time domain. Furthermore, equivariance property is possessed by the algorithm such that its asymptotic performance depends only on the correlation of the source signals, and not on the characteristics of the unknown channel. Simulations indicate the ability of the algorithm to perform equalization.

1. INTRODUCTION

Multichannel blind equalization is an important task for numerous applications such as speech separation, dereverberation, communication, signal processing and control, etc. Therefore, it has received extensive interests all over the world. The task of the blind equalization is to recover original sources, given only their convolved mixtures [1].

The key assumption in source separation lies in the statistical independent of sources. When sources are mutually independent and are also temporally independent and identical distributed (i.i.d) non-Gaussian signals, it is necessary to use higher order statistics (HOS) to achieve source blind deconvolution. Along this assumption, many blind deconvolution algorithms have been developed [1] [6] (and reference therein). In these algorithms, stationary sources were considered and HOS was necessary implicitly or explicitly.

In the other hand, blind equalization is studied only with second order statistics (SOS). For example, Hua and Tugnait [3] assumed that the FIR system is irreducible and the input signals are spatially uncorrelated with distinct power spectra, and then gave blind deconvolution algorithm using SOS. Mitusuru Kawamoto and Yujiro Inouye [4] also derived an algorithm in the frequency domain under a weaker condition that the FIR system is equalizable by means of the SOS of the outputs, and gave a necessary and sufficiently condition for solving the blind deconvolution.

In this paper, we extend the existing techniques for blind signal separation of additive instantaneous mixtures [5] to the task of joint signal separation and deconvolution with correlation of source signals, which is blind equalization. We reconstruct a cost function adapting to blind equalization and then derive the novel blind equalization algorithm with natural gradient search method based only on correlation of the output signals and a nontrivial item. However, differently from the above algorithms, our algorithm is developed in the time domain, and can be implemented on-line easily. We also show that our algorithm possesses the equivariant property. Simulations indicate the ability of the algorithm to perform equalization.

2. PROBLEM FORMULATION

In multichannel blind deconvolution and equalization, an n-dimensional vector of received discrete-time signals $\mathbf{x}(t)=\left[x_1(t),x_2(t),\cdots,x_n(t)\right]^{\mathrm{T}}$ is assumed to be produced from an m-dimensional vector of source signals $\mathbf{s}(t)=\left[s_1(t),s_2(t),\cdots,s_m(t)\right]^{\mathrm{T}}, m\le n$ using the mixture model

$$\mathbf{x}(t)=\sum_{p=-\infty}^{+\infty}\mathbf{H}_p\mathbf{s}(t-p) \tag{1}$$

where $\mathbf{H}_p$ is an $n\times m$-dimensional matrix of mixing coefficients at lag p. The goal is to calculate possibly scaled and delayed estimates of the source signals $\mathbf{s}(t)$ from the received signals $\mathbf{x}(t)$ only using the approximate knowledge of SOS.

In this paper, we estimate the source signals directly using a truncated version of a doubly-infinite multichannel equalizer of the form [6]

$$\mathbf{y}(t)=\sum_{p=-\infty}^{+\infty}\mathbf{W}_p(t)\mathbf{x}(t-p) \tag{2}$$

where $\mathbf{y}(t)=\left[y_1(t),y_2(t),\cdots,y_m(t)\right]^{\mathrm{T}}$ is an m-dimensional vector of outputs and $\mathbf{W}_p(t),-\infty\le p\le+\infty$ is a sequence of $m\times n$-dimensional coefficient matrices. In operator form, the input and output of the equalizer satisfy

$$\mathbf{x}(t)=\mathbf{H}(z)\left[\mathbf{s}(t)\right] \quad (3)$$

$$\mathbf{y}(t)=\mathbf{W}(z,t)\left[\mathbf{x}(t)\right]=\mathbf{C}(z,t)\left[\mathbf{s}(t)\right] \quad (4)$$

where

$$\mathbf{W}(z,t)=\sum_{p=-\infty}^{+\infty}\mathbf{W}_p(t)z^{-p} \quad (5)$$

$$\mathbf{H}(z,t)=\sum_{p=-\infty}^{+\infty}\mathbf{H}_p z^{-p} \quad (6)$$

$$\mathbf{C}(z,t)=\mathbf{W}(z,t)\mathbf{H}(z) \quad (7)$$

are the z -transforms of the equalizer, channel, and combined channel-plus-equalizer impulse responses respectively. In the formulations, z^{-1} is the delay operator, and $z^{-p}\left[s_i(t)\right]=s_i(t-p)$. Then, the goal of the deconvolution or equalization task is to adjust $\mathbf{W}(z,t)$ such that the zero-forcing condition

$$\lim_{t\to\infty}\mathbf{C}(z,t)=\mathbf{PD\Lambda}(z) \quad (8)$$

is satisfied, where $\mathbf{P}$ is an $m\times m$ -dimensional permutation matrix with a single unity entry in any of its rows or columns, $\mathbf{D}$ is a regular diagonal matrix with $m\times m$ dimension, and $\mathbf{\Lambda}(z)$ is an $m\times m$ regular diagonal matrix with diagonal entries being monic monomials.

The following assumptions we need to consider are very modest.

Definition 1. Correlative matrix $\mathbf{R}(t,\tau)$ of $\mathbf{y}(t)$

$$\mathbf{R}(t,\tau)=\left\langle \mathbf{y}(t)\mathbf{y}^{\mathrm{T}}(t-\tau)\right\rangle \quad (9)$$

and $\langle * \rangle$ denotes the ensemble average of *.

Assumption 1. Source signals vector $\mathbf{s}(t)$ are non-stationary with zero mean that means its covariance matrix

$$\mathbf{R}_s(t,\tau)=\left\langle \mathbf{s}(t)\mathbf{s}^{\mathrm{T}}(t-\tau)\right\rangle \quad (10)$$

varies with time (t,τ), and is not a constant value, where $\langle\cdot\rangle$ means ensemble average with the entry.

Assumption 2. Source signals $\mathbf{s}(t)$ are statistically mutually uncorrelated with each other. This implies that its covariance matrix $\mathbf{R}_s(t,\tau)$ is a diagonal one.

Many researchers have proved that with the above two assumptions, only SOS is sufficient for blind deconvolution and equalization [2].

3. MULTICHANNEL BLIND EQUALIZATION ALGORITHM

3.1 Cost function

Our cost function as follows is the direct consequence of the knowledge of correlation, which is defined in equation (10).

$$J\left(W_p(t)\right)=\frac{1}{2}\sum_{i=1}^{m}\log\left[\sum_{\tau}\left|R_{ii}(t,\tau)\right|\right] -\frac{1}{2\pi j}\oint\log\left|\det W(z,t)\right|z^{-1}dz \quad (11)$$

where $j=\sqrt{-1}$. The first term is a correlation measure of the equalizer output signal. The second term on the right hand side of (11) is a constraint term that guarantees that $\mathbf{W}(z,t)=0$ is not a minimizing point of(11). And one can get easily that (11) has the minima value when $R_{ij}(t,\tau)=\left\langle y_i(t)y_j(t-\tau)\right\rangle=0$ for $i\neq j$.

3.2 Natural gradient-based algorithm

We now derive the algorithm with nature gradient, and due to the simplification, we omit the details which can be easily obtained followed by the Amari's paper [6]. First, we give the following definition.

Definition 2. A modified differential matrix

$$d\mathbf{X}(z,t)=\sum_{p=-\infty}^{+\infty}d\mathbf{X}_p(t)=d\mathbf{W}(z,t)\mathbf{W}(z,t)^{-1} \quad (12)$$

Thus, we can get from (11) with all the assumptions and (12)

$$\frac{dJ}{d\mathbf{X}_p(t)}=\sum_{\tau}\Big\langle diag\left[\gamma_i(t,\tau)\right] \left[\mathbf{y}(t-\tau)\mathbf{y}(t-p)^{\mathrm{T}}+\mathbf{y}(t)\mathbf{y}(t-\tau-p)^{\mathrm{T}}\right]\Big\rangle-\mathbf{I}\delta_p \quad (13)$$

Here,

$$\gamma_i(t,\tau)=\frac{\operatorname{sgn}\left(R_{ii}(t,\tau)\right)}{\sum_{\tau}R_{ii}(t,\tau)^2} \quad (14)$$

The differential in (13) is in terms of the modified coefficient differential matrix $d\mathbf{X}(z,t)$. Note that $d\mathbf{X}(z,t)$ is a linear combination of the coefficient differentials $dW_{ij}(z,t)$ in the matrix polynomial $d\mathbf{W}(z,t)$. As long as $\mathbf{W}(z,t)$ is non-singular, $d\mathbf{X}(z,t)$ represents a valid search direction to minimize(11), because $d\mathbf{X}(z,t)$ spans the same tangent space of matrices as spanned by $d\mathbf{W}(z,t)$ [6][7]. For these reasons, an alternative stochastic gradient search method is used as the following form

$$\mathbf{W}_p(t+1) = \mathbf{W}_p(t) - \mu(t)\left[\frac{dJ(\mathbf{W}(z,t))}{d\mathbf{X}_p(t)}\right]\mathbf{W}(z,t) \quad (15)$$

where the right-sided operator $\mathbf{W}(z,t)$ acts on the gradient term in brackets only in the time dimension p. The search direction employed by (15) is nothing more than the natural gradient search direction using the Riemannian metric tensor of the space of all matrix filters of the form of $\mathbf{W}(z,t)$.

From(13), we can further

$$\mathbf{W}_p(t+1) = \mathbf{W}_p(t) + \mu(t)\left[\mathbf{W}_p(t) - \sum_{\tau}\left\langle diag\left[\gamma_i(t,\tau)\right]\mathbf{G}_p(t,\tau)\right\rangle\right]. \quad (16)$$

Here,

$$\mathbf{G}_p(t,\tau) = \mathbf{y}(t-\tau)\mathbf{u}_p(t)^{\mathrm{T}} + \mathbf{y}(t)\mathbf{u}_p(t-\tau)^{\mathrm{T}} \quad (17)$$

and

$$\mathbf{u}_p(t) = \sum_{q=0}^{L}\mathbf{W}_q^{\mathrm{T}}(t)\mathbf{y}(t-p+q) \quad (18)$$

3.3 Practical implementation

In practice, the double-infinite non-causal equalizer cannot be implemented, and the finite-impulse-response (FIR) causal equalizer is used to approximate the double-infinite one, which is given from (2) by

$$\mathbf{y}(t) = \sum_{p=0}^{L}\mathbf{W}_p(t)\mathbf{x}(t-p) \quad (19)$$

However, even with this restriction, the pth coefficient matrix updating in (16) depends on future equalizer outputs $\mathbf{y}(t-p+q), 0 \le q \le L$, when $p=0$ through the definition of $\mathbf{u}_p(t)$ in (18) for a truncated equalizer. Instead of using approximation and data storage to estimate $\mathbf{u}_p(t)$, the last term in (16) is delayed by L samples. This delayed update maintains the same statistical relationships between the signals in the updates and provides similar performance to (16) for the small step sizes.

In addition, when t is big enough, we can assume that $\mathbf{W}_p(k) \approx \mathbf{W}_p(k-1) \approx \mathbf{W}_p(k-2L)$, such that from (18)

$$\mathbf{u}_p(t) \approx \mathbf{u}_0(t-p) \quad (20)$$

With these changes, the proposed algorithm in the general case of signal is

$$\mathbf{W}_p(t+1) = \mathbf{W}_p(t) + \mu(t)\left[\mathbf{W}_p(t) - \sum_{\tau}\left\langle diag\left[\gamma_i(t-L,\tau)\right]\mathbf{G}_p(t,\tau)\right\rangle\right]. \quad (21)$$

where $\mathbf{G}_p(t,\tau)$ and the corresponding $\mathbf{u}_p(t)$ is modified as

$$\mathbf{G}_p(t,\tau) = \mathbf{y}(t-L-\tau)\mathbf{u}(t-p)^{\mathrm{T}} + \mathbf{y}(t-L)\mathbf{u}(t-\tau-p)^{\mathrm{T}} \quad (22)$$

$$\mathbf{u}(t) = \sum_{q=0}^{L}\mathbf{W}_{L-q}^{\mathrm{T}}(t)\mathbf{y}(t-p-q) \quad (23)$$

4. EQUIVARIANT PERFORMANCE

We now study the algorithm in (16) in the combined channel-plus-equalizer parameter space of $\mathbf{C}(z,t)$ defined in(7). A deconvolution algorithm is equivariant if its behaviour only depends on the combined filter $\mathbf{C}(z,t)$. Such a result can only hold for the doubly-infinite equalizer in (2) as the ability of an FIR equalizer to properly compensate for an IIR channel depends on the length of the equalizer coefficients.

Let us consider the algorithm in the form

$$\Delta\mathbf{W}(z,t) = -\mu(t)\left[\frac{dJ(\mathbf{W}(z,t))}{d\mathbf{X}(z,t)}\right]\mathbf{W}(z,t) \quad (24)$$

Post-multiplying both sides of this equation by $\mathbf{H}(z)$ and noting the definition in (7), we have

$$\Delta\mathbf{C}(z,t) = -\mu(t)\left[\frac{dJ(\mathbf{W}(z,t))}{d\mathbf{X}(z,t)}\right]\mathbf{C}(z,t) \quad (25)$$

As in(4), it is easily seen from (13) that the update in (25) is independent of the channel $\mathbf{H}(z)$. This result proves the equivariance of the proposed multichannel deconvolution algorithm.

A center-tap initialization scheme is employed, that is,

$$\mathbf{W}(z,0) = \mathbf{I}z^{-j} \quad (26)$$

for some $0 \le j \le L$. It also may be necessary to use tap-centering scheme or better one to obtain good performance from this system to meet the additive assumption 3. Although equivariance indicates the useful convergence behavior of the algorithm, it does not guarantee that $\mathbf{W}(z,t)$ is adequate for equalizing the channel, nor does it guarantee good performance when the equalizer filter has a poor initialization [6]. In fact, initializations can cause convergence problems.

5. SIMULATIONS

We present simulation results for a two-input two-output system with impulse response length of 3. Two input source signals are two random signals, one is Gaussian random signals, and the other is $s_2(t) = \sin(\pi t)randn(t)$, where randn(t) is also a Gaussian random signals. All the experiments were carried out with artificial mixing.

The mixing filter matrix $\mathbf{H}_p$ is

h11=[0.5 0.2 0.2]; h12=[0.2 0.2 0.1];
h21=[0.1 0.2 -0.1]; h22=[0.5 0.2 0.1];

The algorithm in (21) is tested, and the performance is shown in terms of inter-symbol interference (ISI), which defined as in [6].

In the algorithm, we set the learning rate $\mu(t) = 0.001$. The simulation results are depicted in figure 1, 2, 3 and 4. Figure 1 shows sources signals, and figure 2 convolved and mixed signals, figure 3 the combined channel-plus-equalizer filter impulse response, and figure 4 the inter-symbol interference. It is easily seen that the ISI between channels is reduced. Therefore, the proposed algorithm has the ability to perform blind equalization.

We are making further development, and hope to publish it in future.

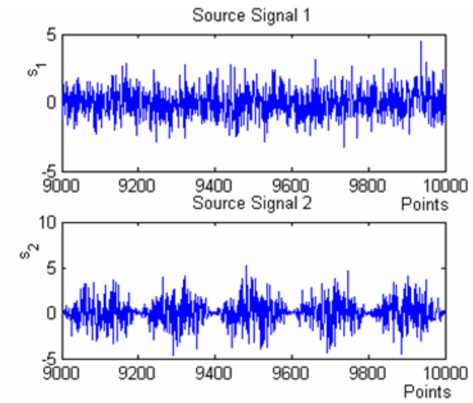

Fig. 1. Source Signals

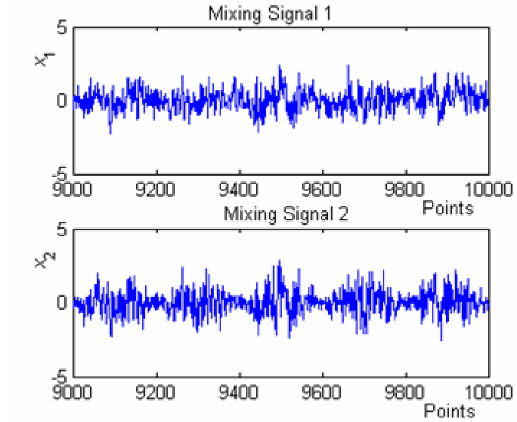

Fig. 2. Mixed Signals

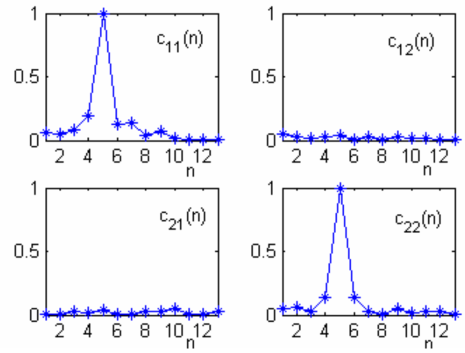

Fig.1. Equalizer impulse response

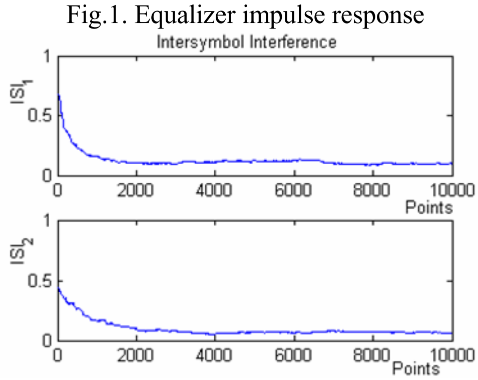

Fig.2. Inter-symbol interference

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