COMPLEXITY REDUCED BLIND MULTICHANNEL EQUALIZATION

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ABSTRACT

Recently, an algorithm for blind (without training sequence) and direct (without prior estimation of the channel response) estimation of (all) Zero-Forcing (ZF) equalizers of a Single Input Multiple Output (SIMO) channel has been proposed. These span a linear subspace, making it necessary to introduce a criterion to select the *best* equalizer within the so-estimated subspace. This selection step increases considerably the algorithm complexity. We propose two modifications to the algorithm that achieve the same (asymptotic) equalization performances at a reduced complexity.

1. INTRODUCTION

Intersymbol interference (ISI) is the main limitation for high-rate wireless communication. To identify (then remove) the ISI pattern, training sequences are used. These are inadequate with the growing demand on bandwidth, while, at the same time, increase in length when using the, now popular, multiple antennas [1]. This has been motivating an intensive interest in the so-called *blind* techniques, where the knowledge of the statistics of the transmitted symbols is exploited, rather than that of the transmitted symbols themselves. Naturally, techniques based on Second Order Statistics (SOS) are preferred. Among these, a number enable one to directly estimate one [2, 3] or many [4, 5, 6, 7] ZF equalizer(s). Since the estimation of the channel response is not performed, exact knowledge of the channel order is not required. Hence, such techniques are inherently robust to order modeling errors.

Among these direct equalization techniques, the GRDA algorithm [6] has the advantage of determining all no-delay ZF equalizers. These form a linear subspace, making it necessary to select some *optimum* equalizer. This is achieved using the so-called Equalization Peak Criterion (EPC), hence estimating the *best* no-delay equalizer possible. This contrasts with other algorithms which estimate only one or some equalizers, with no guarantee of optimality.

The application of the EPC criterion requires an additional Eigen Vector Decomposition (EVD), hence significantly increasing the complexity. We rewrite the GRDA algorithm to estimate the same optimum equalizer with a reduced complexity. The first solution (Sec. 4) directly estimates the EPC equalizer without explicitly applying the EPC criterion (and the associated EVD). This solution, however, is not robust to order over estimation, contrarily to the original algorithm. A second solution (Sec. 5) preserves the robustness property and, at the same time, decreases the complexity, less significantly than the first version however.

2. THE CHANNEL MODEL

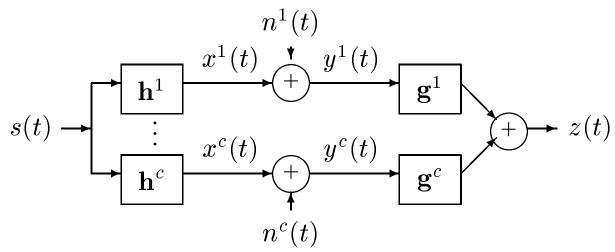


Fig. 1. Single input multiple output channel

Receivers with multiple antennas and/or using fractional sampling are often modeled as Single Input Multiple Output (SIMO) channels as depicted in Fig. 1. A set of c filters are driven by a common scalar input $s(t)$. The impulse response of the c'-th filter is given by $\mathbf{h}^{c'} \stackrel{\text{def}}{=} [h^{c'}(0), h^{c'}(1), \cdots]^T$. The order m of the SIMO channels is the maximum among those of $\mathbf{h}^1 \cdots \mathbf{h}^c$. We define the channel c-dim k-th tap $\mathbf{h}(k)$ as $\left[h^1(k) \cdots h^c(k)\right]^T$ and its (impulse) response as $\mathbf{h} \stackrel{\text{def}}{=} \left[\mathbf{h}^T(0) \cdots \mathbf{h}^T(m)\right]^T$. The noise corrupted output $\mathbf{y}(t) \stackrel{\text{def}}{=} \left[y^1(t) \cdots y^c(t)\right]^T$ equals $\mathbf{x}(t)+\mathbf{n}(t) = \mathbf{H}\mathbf{s}_{m+1}(t)+\mathbf{n}(t)$ where $\mathbf{H} \stackrel{\text{def}}{=} [\mathbf{h}(0) \cdots \mathbf{h}(m)]$ and $\mathbf{s}_k(t) \stackrel{\text{def}}{=} [s(t) \cdots s(t-k+1)]^T$ for any k. The channel output is observed over many (say l) symbol periods and stacked into $\mathbf{y}_l^T(t) \stackrel{\text{def}}{=} [\mathbf{y}^T(t) \cdots \mathbf{y}^T(t-l+1)]^T$. We have

$$\mathbf{y}_l(t) = \mathbf{H}_l \mathbf{s}_{l+m}(t) + \mathbf{n}_l(t)$$

where $\mathbf{H}_l \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{H} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H} & \cdots & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H} \end{bmatrix}$ is the $cl \times (l+m)$ filtering matrix, $\mathbf{n}_l(t)$ is defined similarly as $\mathbf{y}_l(t)$ and $\mathbf{0}$ is the c-dim null vector. The channel SOS are completely described by the correlation terms $\gamma(k) \stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{y}(t+k)\mathbf{y}^H(t)\right]$. $\gamma(k) = \mathbf{0}$ for $|k| > m$. (Blind) Channel identification and equalization is possible from the channel output SOS. The channel input is unknown but assumed to be i.i.d. and uncorrelated from the white noise components. σ_s^2 and σ_n^2 refer to symbol and noise variances, respectively. Under these assumptions, the correlation matrix is given by

$$
\begin{aligned}
\mathbf{R}_l &\stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{y}_l(t)\mathbf{y}_l^H(t)\right] \\
&= \begin{bmatrix} \gamma(0) & \gamma(1) & \cdots & \gamma(l-1) \\ \gamma^H(1) & \gamma(0) & \ddots & \gamma(l-2) \\ \vdots & \ddots & \ddots & \vdots \\ \gamma^H(l-1) & \gamma^H(l-2) & \cdots & \gamma(0) \end{bmatrix} \\
&= \sigma_s^2 \mathbf{H}_l \mathbf{H}_l^H + \sigma_n^2 \mathbf{I}
\end{aligned}
$$

where $\mathbf{I}$ stands for the identity matrix of appropriate dimensions. Finally, we recall that $\mathbf{H}_l$ is full column rank if $l \geq m$, a condition assumed throughout the paper.

3. THE GRDA ALGORITHM

An $(l-1)$-order d-delay ZF equalizer $\mathbf{g}$ verifies

$$\mathbf{H}_l^T \mathbf{g} = [\underbrace{0 \cdots 0}_{d}\, u\, 0 \cdots 0]^T \tag{1}$$

for some constant u. More interestingly for blind equalization, $\mathbf{g}$ is a minimum i.e. $d = 0$ (resp. maximum i.e. $d = l + m - 1$) delay equalizer iff it is right (resp. left) orthogonal to $R_l^* - \sigma_n^2 \mathbf{J}_l \otimes \mathbf{I}$ where R_l is the *shifted* correlation matrix defined as [6]

$$
\begin{aligned}
R_l &\stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{y}_l(t)\mathbf{y}_l^H(t+1)\right] \\
&= \begin{bmatrix} \gamma^H(1) & \gamma(0) & \cdots & \gamma(l-2) \\ & \gamma^H(1) & \ddots & \vdots \\ \vdots & & \ddots & \gamma(0) \\ \gamma^H(l) & \cdots & & \gamma^H(1) \end{bmatrix} \\
&= \sigma_s^2 \mathbf{H}_l \mathbf{J}_{l+m} \mathbf{H}_l^H + \sigma_n^2 \mathbf{J}_l \otimes \mathbf{I}
\end{aligned}
$$

where $\mathbf{J}_a$ denotes the a-dim up-shifting matrix and $\otimes$ stands for the Kronecker product. Hence, if N_l^R and N_l^L are $((c-1)l - m + 1)$-dim, respectively, right and left null spaces of $R_l - \sigma_n^2 \mathbf{J}_l \otimes \mathbf{I}$, then min and max delay ZF equalizers are given by the column span of $\left(N_l^R\right)^*$ and $\left(N_l^L\right)^*$, respectively. By computing a Singular Value Decomposition (SVD) of $R_l - \sigma_n^2 \mathbf{J}_l \otimes \mathbf{I}$, these have orthogonal unit-norm columns. All unit-norm linear combination of the columns of $\left(N_l^R\right)^*$ or of those of $\left(N_l^L\right)^*$ are equally unit-norm. Hence, the so-computed ZF equalizers are equivalent from the noise enhancement point of view [8, 9] and can be compared on the basis of the amplitude of the ISI-free restored symbol. For an equalizer $\mathbf{g}$, this is given [8] by (the square root of) $\mathbf{g}^H(\mathbf{R}_l^* - \sigma_n^2\mathbf{I})\mathbf{g}$. This lead to the definition of the EPC criterion [6] so that the best min delay equalizer is given by $\left(N_l^R\right)^* \mathbf{f}^R$ where $\mathbf{f}^R$ is the eigen vector associated to the largest eigenvalue of $\left(N_l^R\right)^H \left(\mathbf{R}_l - \sigma_n^2\mathbf{I}\right) N_l^R$ while the best max delay equalizer is given by $\left(N_l^L\right)^* \mathbf{f}^L$ where $\mathbf{f}^L$ is the eigen vector associated to the largest eigenvalue of $\left(N_l^L\right)^H \left(\mathbf{R}_l - \sigma_n^2\mathbf{I}\right) N_l^L$. The original algorithm can be summarized as follows :

1. Compute the estimates $\hat{\mathbf{R}}_l$ and $\hat{R}_l$ of $\mathbf{R}_l$ and R_l, respectively.
2. Conduct the EVD of $\hat{\mathbf{R}}_l$. Let λ_i be the i-th largest eigenvalue and $\mathbf{v}_i$ the associated unit-norm eigen vector.
3. Estimate $\widehat{\sigma_n^2}$ as the average of the lowest $(c-1)l - m$ eigenvalues λ_i.
4. Conduct the SVD of $\hat{R}_l - \widehat{\sigma_n^2}\mathbf{J}_l \otimes \mathbf{I}$. Let $\mathbf{v}_i^R$ and $\mathbf{v}_i^L$ be respectively the right and left unit-norm eigen vector associated with the i-th largest singular value. Let $\mathbf{N}_l^R \stackrel{\text{def}}{=} [\mathbf{v}_{l+m}^R, \cdots, \mathbf{v}_{cl}^R]$ and $\mathbf{N}_l^L \stackrel{\text{def}}{=} [\mathbf{v}_{l+m}^L, \cdots, \mathbf{v}_{cl}^L]$.
5. Compute $\mathbf{f}^R$ and/or $\mathbf{f}^L$ as the eigen vector associated to the largest eigenvalue of $\left(N_l^R\right)^H \left(\mathbf{R}_l - \widehat{\sigma_n^2}\mathbf{I}\right) N_l^R$ and/or $\left(N_l^L\right)^H \left(\mathbf{R}_l - \widehat{\sigma_n^2}\mathbf{I}\right) N_l^L$, respectively.
6. Compute the min and/or max delay ZF equalizers as $\left(N_l^R\right)^* \mathbf{f}^R$ and/or $\left(N_l^L\right)^* \mathbf{f}^L$, respectively.

The EPC criterion, as expressed by Step 5, increases heavily the complexity because of the requested EVD.

4. COMPLEXITY REDUCED GRDA ALGORITHM : FIRST SOLUTION

The GRDA algorithm is based on a sufficient and necessary condition for min and/or max delay ZF equalization that implies suppression of ISI but does not prevent the equalizer from restoring the desired symbol with an uncontrolled (possibly low) amplitude. Uselessly, the column span of $(N_l^R)^*$ and $(N_l^L)^*$ include equalizers right orthogonal to $\mathbf{H}_l^T$. These *undesirable* vectors are nothing but those from the (conjugate of the) noise subspace i.e. the kernel of $\mathbf{R}_l - \sigma_n^2\mathbf{I}$. Interestingly, the noise subspace (or equivalently the signal subspace) are obtained in Step. 2 as a by product of the estimation of the noise variance. We denote them by $\mathbf{N}_l$ and $\mathbf{S}_l$, respectively.

Conseqently, we are interested only in min (resp. max) delay equalizers in the column span of $\left(N_l^R\right)^*$ (resp. $\left(N_l^L\right)^*$) but left orthogonal to $\mathbf{N}_l^*$. Such an equalizer is equivalently defined as equal to $\mathbf{S}_l^*\mathbf{f}$ where $\mathbf{f}$ is right orthogonal to $\left(R_l^* - \sigma_n^2\mathbf{J}_l \otimes \mathbf{I}\right)\mathbf{S}_l^*$ (resp. to $\left(R_l^T - \sigma_n^2\mathbf{J}_l^T \otimes \mathbf{I}\right)\mathbf{S}_l^*$). In both cases (min and max delays), the so-computed equalizers span a one dimensional subspace, that of the EPC equalizer. Hence, compared to the original GRDA algorithm, we save the EVD associated to the computation of the EPC solution and the SVD necessary to compute N_l^R and/or N_l^L. Instead, we will have to conduct SVD(s) of $\left(R_l^* - \sigma_n^2\mathbf{J}_l \otimes \mathbf{I}\right)\mathbf{S}_l^*$ and/or $\left(R_l^T - \sigma_n^2\mathbf{J}_l^T \otimes \mathbf{I}\right)\mathbf{S}_l^*$. The modified GRDA algorithm can be described as follows :

- Steps 1- 3 of the original GRDA algorithm.
- Let $\mathbf{S}_l \stackrel{\text{def}}{=} [\mathbf{v}_1, \cdots, \mathbf{v}_{l+m}]$.
- Compute $\mathbf{f}^R$ and/or $\mathbf{f}^L$ as the right singular vectors associated to the lowest singular value of, respectively, $\left(R_l^* - \sigma_n^2\mathbf{J}_l \otimes \mathbf{I}\right)\mathbf{S}_l^*$ and/or $\left(R_l^T - \sigma_n^2\mathbf{J}_l^T \otimes \mathbf{I}\right)\mathbf{S}_l^*$.
- Compute the min and/or max delay ZF equalizer as $\mathbf{S}_l^*\mathbf{f}^R$ and/or $\mathbf{S}_l^*\mathbf{f}^L$, respectively.

Unfortunately, when the channel order is assumed to be larger than the exact order, say $m' > m$, one can easily show that $\left(R_l^* - \sigma_n^2\mathbf{J}_l \otimes \mathbf{I}\right)\mathbf{S}_l^*$ has $m' - m$ zero columns and so, no more has a unique right singular vector. A similar result holds for maximum delay equalizers. Hence, this modified algorithm is not robust to channel order over estimation. In the following section, we present a different modification to the GRDA algorithm so that robustness (to order over estimation) is maintained and complexity is reduced (but is larger than that of the solution of Sec. 4).

5. COMPLEXITY REDUCED GRDA ALGORITHM : SECOND SOLUTION

This time, we only rewrite step 5 of the original GRDA algorithm. This step is intended to estimate the (absolute) amplitude of the restored (equalized) symbol , the constant u in (1). In the original GRDA algorithm, because $|u| = ||\mathbf{H}_l^T\mathbf{g}||$, $|u|^2$ is blindly estimated [8, 6] by $\mathbf{g}^T(\mathbf{R}_l - \sigma_n^2\mathbf{I})\mathbf{g}^*$. Alternatively, we write $u = \mathbf{g}^T(0)\mathbf{h}(0)$ and use [3]

$$\sigma_s^2\mathbf{h}(0)\mathbf{h}^H(0) = \gamma(0) - \sigma_n^2\mathbf{I} - \begin{bmatrix} \gamma^H(1) \\ \vdots \\ \gamma^H(m) \end{bmatrix}^H \mathbf{R}_m^\dagger \begin{bmatrix} \gamma^H(1) \\ \vdots \\ \gamma^H(m) \end{bmatrix}$$

where $\mathbf{R}_m^\dagger$ stands for the pseudo-inverse of $\mathbf{R}_m$. Consequently, $|u|^2$ equals, up to a multiplicative constant,

$$\mathbf{g}^T(0)\left(\gamma(0) - \sigma_n^2\mathbf{I} - \begin{bmatrix} \gamma^H(1) \\ \vdots \\ \gamma^H(m) \end{bmatrix}^H \mathbf{R}_m^\dagger \begin{bmatrix} \gamma^H(1) \\ \vdots \\ \gamma^H(m) \end{bmatrix}\right)\mathbf{g}^*(0)$$

Notice that the computation of the pseudo-inverse of $\mathbf{R}_m$ does not require a supplementary EVD. In fact, only a unique EVD of $\mathbf{R}_m$ is necessary to both estimate the noise variance and the pseudo-inverse

$$\mathbf{R}_m^\dagger = \begin{bmatrix} \mathbf{v}_1^H \\ \vdots \\ \mathbf{v}_{l+m}^H \end{bmatrix}^H \begin{bmatrix} \frac{1}{\lambda_1 - \sigma_n^2} & & \\ & \ddots & \\ & & \frac{1}{\lambda_{l+m} - \sigma_n^2} \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^H \\ \vdots \\ \mathbf{v}_{l+m}^H \end{bmatrix}.$$

Step 5 can now be replaced by the following : Compute $\mathbf{f}^R$ as the eigen vector associated to the largest eigenvalue of

$$\left(\underline{N_l^R}\right)^H \left(\gamma(0) - \begin{bmatrix} \gamma^H(1) \\ \vdots \\ \gamma^H(m) \end{bmatrix}^H \mathbf{R}_m^\dagger \begin{bmatrix} \gamma^H(1) \\ \vdots \\ \gamma^H(m) \end{bmatrix}\right) \underline{N_l^R}$$

where $\underline{N_l^R}$ is formed by the first c rows of N_l^R. The resulting algorithm is less computation consuming since the required EVD is that of a $((c-1)l - m + 1)$-dim matrix instead of a cl-dim matrix, when the EVD of an N-dim matrix has a computational load of $O(N^3)$.

6. SIMULATIONS

A series of simulations has been conducted to compare the modified GRDA algorithm to the original one. In the simulations, the channel, taken from [10], is driven by unit-variance i.i.d. 4-QAM symbols and corrupted by AWG noise. The SNR is defined as $\frac{\mathbf{E}\left[||\mathbf{x}(t)||^2\right]}{\mathbf{E}\left[||\mathbf{n}(t)||^2\right]} = \frac{\sigma_n^2||\mathbf{h}||^2}{c\ \sigma_n^2}$. The simulation results are averaged over 100 Monte Carlo runs. Because maximum delay equalization is not relevant when the channel order is unknown or over estimated, only no-delay equalizers are considered.

An estimated ZF equalizer is tested with a sequence $s(0), \cdots, s(N-1)$ of randomly generated source symbols (in the simulations, $N = 200$). The equalizer outputs $z(0), \cdots, z(N-1)$ are compared to the transmitted symbols in terms of the MSE defined as

$$\frac{1}{\sigma_s^2 N}\min_\beta ||\mathbf{s}_N(N-1-d) - \beta\mathbf{z}_N(N-1)||^2$$

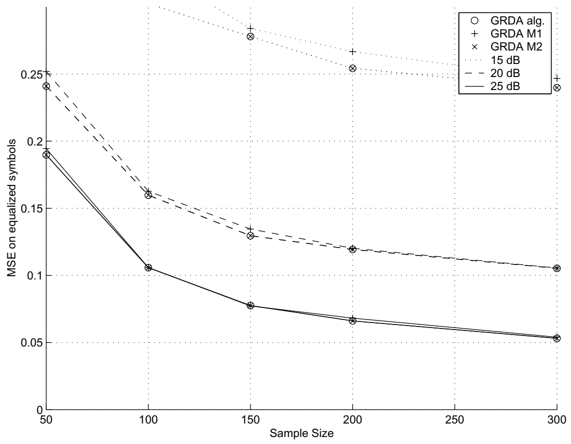
and proved to be equal to [7]

$$\frac{1}{\sigma_s^2 N}\left[||\mathbf{s}_N(N-1)||^2 - \left(\frac{\mathbf{s}_N^H(N-1)\mathbf{z}_N(N-1)}{||\mathbf{z}_N(N-1)||}\right)^2\right]$$

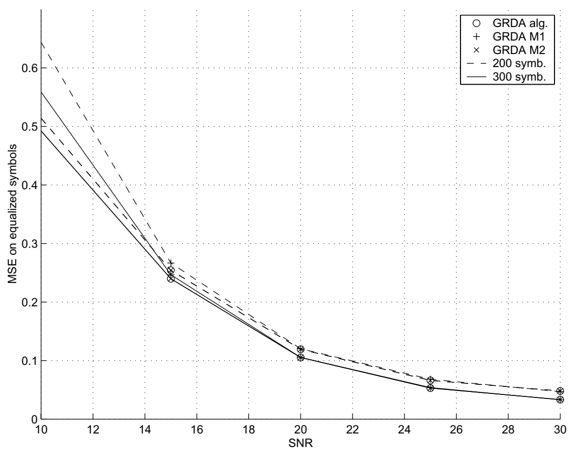
Results, presented in Fig. 2, confirm that the modifications made to the original algorithm do not result in any degradation in performances. It is shown that whenever the original algorithm shows practical (low enough) equalization errors, the modified algorithms reach the same error. Degradation observed at low SNR for the algorithm of Sec. 4 are meaningless since the equalization error is then unpractical anyway.

7. CONCLUSION

Zero-Forcing equalizers of the same length and delay form a linear subspace. Its full characterization is is made possible by the GRDA algorithm for no-delay equalization. A selection step is then required to select the *best* equalizer within the subspace. This is a computation consuming step that requires an Eigen Vector Decomposition. We show that we can suppress this EVD or reduce its dimension and still efficiently estimate the same *optimal* equalizer.



(a)



(b)

Fig. 2. Algorithms comparison. M1 and M2 refer to the algorithms in Sec. 4 and Sec. 5, respectively.

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