SYNCHRONIZATION ALGORITHM FOR LPTV-BASED SPREAD SPECTRUM SIGNALS

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ABSTRACT

Linear Periodic Time-Varying (**LPTV**) filters can be used as a Spread Spectrum (**SS**) technique. A particular case of such spreading LPTV filters are Periodic Clock Changes (**PCCs**). When the spreading operation is realized with PCCs, the synchronization algorithm can also be considered as an LPTV problem. In this paper we will focus on the early-late Delay Lock Loop (**DLL**) algorithm. We show that this technique can be used to achieve synchronization when the spreading operation is realized by a special class of PCCs: matrix interleavers.

1. INTRODUCTION

LPTV filters are filters whose coefficients vary periodically with time [1]. When applied to wide sense stationary signals, LPTV filters can generate wide sense cyclostationary output signals [2]. Moreover, by choosing an apropriate LPTV filter, the output signal is an SS signal. Previous works have shown that an example of such LPTV filters are PCCs [3, 4].

In this paper, a synchronization technique is developped for an SS transmission chain with PCCs. We propose a DLL based synchronization technique, similar to the technique used in DS-CDMA systems [5]. The matrix interleavers are used in our work as spreading PCCs. The rest of the paper is organized as follows. First a short review of LPTV filter theory will be made. Then a transmission chain with PCCs will be introduced and the early-late DLL approach will be adapted to this transmission chain. Finally it will be shown how matrix interleavers can be synchronized with this adapted early-late DLL.

2. REVIEW OF LPTV FILTER THEORY

The fundamental equation used to describe the input-output relation of a discrete-time LPTV filter is [1]:

$$y(n) = \sum_{m=-\infty}^{+\infty} c_n(m)x(n-m) \quad (1)$$

where $x(n)$ and $y(n)$ are the input and output sequences respectively, $c_n(m)$ are the time varying filter coefficients and $c_n(m) = c_{n+N}(m)$, N being the period of the LPTV filter.

According to (1), several equivalent structures for LPTV filters can be developed [1]. The modulator filter equivalent structure (figure 1) is, in our approach, of particular interest. The transfer function of the k^{th} modulator filter, $T_k(\Omega)$, has the following expression:

$$T_k(\Omega) = \frac{1}{N}\sum_{n=0}^{N-1} W^{kn} C_n(\Omega - k\frac{2\pi}{N}) \quad (2)$$

with $W = e^{-j\frac{2\pi}{N}}$ and $C_n(\Omega)$ the Discrete Fourier Transform (**DFT**) of $c_n(m)$. The transfer function $T_k(\Omega)$ is N-periodic with respect to k: $T_k(\Omega) = T_{k+N}(\Omega)$.

For a wide sense stationary input signal $x(n)$, the output signal $y(n)$ of an LPTV filter is a wide sense cyclostationary signal with

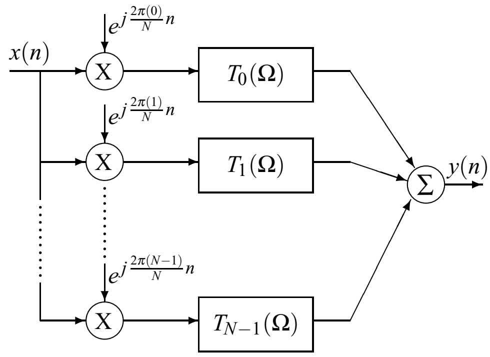

Figure 1: LPTV modulator filter equivalent structure

the same period as the LPTV filter [2]. By considering the modulator filter equivalent structure, the output signal $y(n)$ has a Power Spectral Density (**PSD**) $S_y(\Omega)$:

$$S_y(\Omega) = \sum_{k=0}^{N-1} |T_k(\Omega)|^2 S_x(\Omega - k\frac{2\pi}{N}) \quad (3)$$

where $S_x(\Omega)$ is the PSD of the input signal.

Among LPTV filters we find PCCs. A PCC is described by an N-periodic function $f(n)$, $f : \mathscr{Z} \to \mathscr{Z}$,so that, for a given input $x(n)$, the output $y(n)$ of the PCC is expressed as:

$$y(n) = x(n - f(n)) \quad (4)$$

Following equation (1), the equivalent LPTV filter of the above PCC is defined by the time-varying coefficients $c_n(m) = \delta(m - f(n))$, where $\delta(n)$ is the Kronecker function. Now equation (2) becomes:

$$T_k(\Omega) = \frac{1}{N}\sum_{n=0}^{N-1} W^{kn}(W^k e^{j\Omega})^{-f(n)} \quad (5)$$

In the following we will study a transmission chain for SS communications where the spreading operation is achieved thanks to a PCC.

3. SYNCHRONIZATION ISSUES FOR A TRANSMISSION CHAIN WITH PCCS

We consider an SS digital transmission chain (figure 2). The input signal $x(n)$ is passed first through a pulse shaping filter with transfer function $H_E(\Omega)$. Then, an emission PCC, PCC_E, with an N-periodic function $f(n)$, is applied. Here we consider that the chosen PCC_E is perfectly invertible. The PCC_E output signal has a much larger bandwidth (spectral spreading) than the input signal. The emitted signal passes through a baseband transmission channel. At the reception the despreading operation is realized by the inverse PCC, PCC_R, with an N-periodic function $g(n)$. Then a matched filter with transfer function $H_R(\Omega)$ is used in order to recover the original transmitted signal.

Figure 2: Transmission chain with PCCs

In the case of an ideal channel, with the impulse response $\delta(n)$, the input signal $x(n)$ can be perfectly recovered at reception. However, in real channels, there is always some transmission delay. In the presence of an unknown delay n_0, a synchronization technique must be used to synchronize PCC_R on the received delayed signal.

In order to develop a synchronization technique for the above defined transmission chain, we express first the received power as a function of the channel delay n_0. We consider the two PCCs as a part of the channel so that the equivalent channel (figure 3) is also an N-periodic PCC with the function $h_{n_0}(n)$:

$$h_{n_0}(n) = g(n) + n_0 + f(n - g(n) - n_0) \tag{6}$$

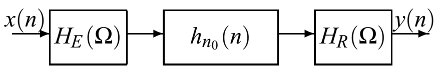

Figure 3: Transmission chain with equivalent PCC channel

Next, we consider the LPTV representation of the equivalent channel with the modulator filter transfer functions $T_k^{(n_0)}(\Omega)$ (5). The limited band of $H_E(\Omega)$ and $H_R(\Omega)$ allows us to reduce the number of modulator filters that appear in the input-output relation for the SS transmission chain. Hence we choose the bandwidth of the emission and reception filters so that $B = \nu \frac{\pi}{N}$, where ν is an integer number and N is the period of the equivalent PCC. Since the modulator filters $T_k^{(n_0)}(\Omega)$ are N-periodic with respect to k, we can change the sum in (3) to another interval of length N. So the input-output relation can be written as:

$$S_y^{(n_0)}(\Omega) = |H_R(\Omega)|^2 \sum_{k=-\nu}^{\nu} \left| T_k^{(n_0)}(\Omega) \right|^2 S_E\left(\Omega - k\frac{2\pi}{N}\right) \tag{7}$$

where $S_E(\Omega) = |H_E(\Omega)|^2 S_x(\Omega)$, $S_x(\Omega)$ and $S_y^{(n_0)}(\Omega)$ are the input and output signal PSDs, respectively.

The received signal power $P_y(n_0)$ is:

$$P_y(n_0) = \frac{1}{2\pi} \int_0^{2\pi} S_y^{(n_0)}(\Omega) d\Omega \tag{8}$$

Using (5), (6) and (7) one can show that the received signal power $P_y(n_0)$ is an N-periodic function with respect to n_0.

We are now able to propose a synchronization technique for the transmission chain with PCCs. Our approach is similar to the one used in DS-CDMA systems [5] and consists of two parts: acquisition and tracking.

The acquisition step is realized at the beginning of the transmission in order to reduce the timing uncertainty to some known interval $(-D/2, D/2)$. D will be specified latter in this paper. When the spreading operation is done with PCCs, the acquisition step can be done using the "sliding correlator" method [5].

To be able to use this method we choose the emission PCC so that the received power $P_y(n_0)$ has larger values in the interval $(-D/2, D/2)$ than outside this interval:

i. $P_y(n_0) > P_y(m_0), \forall |n_0| < D/2$ and $D/2 < |m_0| < N/2$.

The tracking step is used next, in order to obtain the exact channel delay and to maintain the synchronization for the whole transmission duration [5]. The tracking step for PCCs can be implemented with an early-late DLL (figure 4). The reception PCCs in the two branches, $PCC_R^{(early)}$ and $PCC_R^{(late)}$, are delayed relatively to each other by an amount of D samples. After $PCC_R^{(early)}$ and $PCC_R^{(late)}$, the received signal is filtered and then the resulting samples are squared. Summing over a sufficiently large number of samples, we find the received power in each branch. Next, by using a delay estimator based on the difference of the received powers in the two branches, we can obtain an estimation $\widetilde{n}_0$ of the unknown channel delay n_0. Finally, this estimation is used to delay the $PCC_R^{(early)}$ and $PCC_R^{(late)}$ by the same amount $\widetilde{n}_0$. When the estimation is correct, the received powers in the two branches are equal.

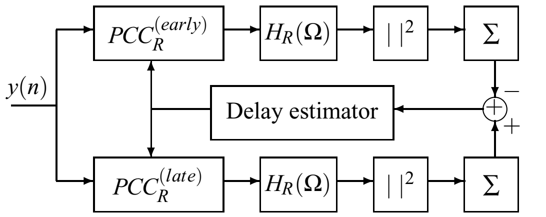

Figure 4: Early-late delay locked loop for PCCs

To build the delay estimator, we impose for the received power $P_y(n_0)$ to have also a linear variation in the interval $(-D, D)$:

ii. $P_y(n_0) = a|n_0| + b, \forall |n_0| < D$, where $a < 0$, $b > 0$ are real numbers.

With this property a delay estimator can be realized only by using the sign of the difference between the received powers in the two branches. However, with this approach the acquisition time can be quite long. A better approach would be to use a delay estimator that immediately estimates the channel delay n_0. Hence, with the second property (ii), the proposed delay estimator to use in the early-late DLL is:

$$\widetilde{n}_0 = \frac{P_y(n_0 + D/2) - P_y(n_0 - D/2)}{2a} \tag{9}$$

where $P_y(n_0 + D/2)$ and $P_y(n_0 - D/2)$ are the received powers estimated in the branches of $PCC_R^{(late)}$ and $PCC_R^{(early)}$, respectively and a is a constant which depends on PCC_E and PCC_R parameters. Choosing $P_y(n_0 - D/2)$ and $P_y(n_0 + D/2)$ allows us to let the channel delay n_0 take values in the largest possible interval $(-D/2, D/2)$.

In the next section the above developments will be specified for the case of matrix interleavers.

4. DELAY ESTIMATOR USED IN THE DLL SYNCHRONIZATION TECHNIQUE FOR MATRIX INTERLEAVERS

In this part we will show that matrix interleavers verify the above stated properties (i and ii). The channel delay estimator (9) is then calculated for this particular case. A description of the SS transmission system, together with the performances in terms of BER can be found in [6].

4.1 Numerical results for matrix interleavers

The matrix interleaver performs block interleaving by filling a matrix with the input samples, row by row, and then sending the matrix contents to the output, column by column. When matrix interleavers are used in the transmission chain (figure 2), the variations of the received signal power $P_y(n_0)$ are depicted in figure 5. This curve has been established with a ± 1 binary input signal and a symbol duration of 10 samples. The shaping (emission) filter is a square root raised cosine FIR filter (61 coefficients, rolloff 0.5) and the emission matrix interleaver has $P = 20$ lines and $Q = 15$ columns.

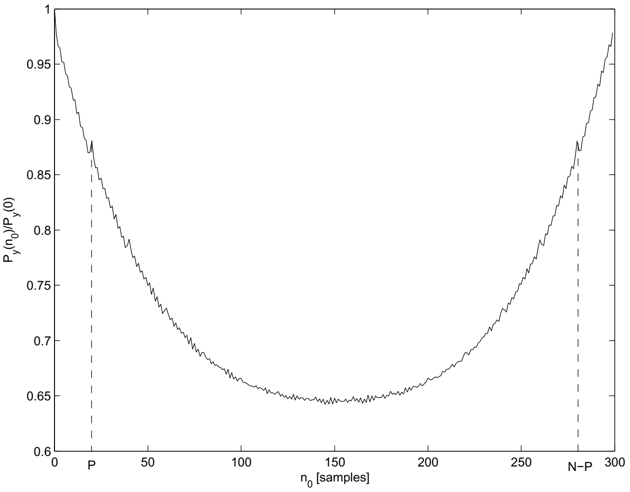

Figure 5: Received power $P_y(n_0)$ as a function of the channel delay n_0 for one period ($P = 20$, $Q = 15$, $N = 300$)

In figure 5 the received power has an approximately linear variation in the interval $(0,P)$ and $(N-P,N)$ and smaller values outside these intervals. Since the received power is N-periodic, we can say that the linear variation is in the interval $(-P,P)$ (for matrix interleavers we consider $D = P$). So matrix interleavers satisfy both conditions needed to use the synchronization method proposed in section 3.

4.2 Analytical expression for the delay estimator

We propose a general expression for the delay estimator in the case of an arbitrary chosen matrix interleaver. Our developments are based on the linear variation of the received power $P_y(n_0)$ in the interval $(-P,P)$. First, with some additional assumptions, we find an analytical expression for $P_y(n_0)$ in two points: $n_0 = 1$ and $n_0 = P-1$. Second, by using the knowledge of these two points, the slope of $P_y(n_0)$ is deduced for $n_0 \in (-P,P)$. Finally, based on the slope knowledge, the delay estimator is calculated.

The matrix interleaver with P lines and Q columns can be analytically described by a PCC of period $N = PQ$ with the function $f(n)$:

$$f(n) = -(Q-1)[n]_P + (P-1)\left[\frac{n-[n]_P}{P}\right]_Q \tag{10}$$

where $[X]_Y$ is the remainder of the Euclidian division of X by Y. The above defined PCC is PCC_E of figure 2.

The inverse interleaver is also an N-periodic PCC, PCC_R, described by a function $g(n)$ with an expression obtained from equation (10) by interchanging P and Q:

$$g(n) = -(P-1)[n]_Q + (Q-1)\left[\frac{n-[n]_Q}{Q}\right]_P \tag{11}$$

Consequently, the analytical expression for the N-periodic function $h_{n_0}(n)$ (6) is (see appendix):

$$h_{n_0}(n) = \begin{cases} n_0 Q & \text{if } n \geq n_0 Q \\ n_0 Q + 1 - N & \text{if } n < n_0 Q,\ [n]_Q \neq 0 \\ n_0 Q + 1 - Q & \text{if } n < n_0 Q,\ [n]_Q = 0 \end{cases} \tag{12}$$

for $0 < n_0 < P$ and

$$h_{n_0}(n) = \begin{cases} n_0' Q & \text{if } n < n_0' Q \\ n_0' Q - 1 + N & \text{if } n \geq n_0' Q,\ [n]_Q \neq Q-1 \\ n_0' Q - 1 + Q & \text{if } n \geq n_0' Q,\ [n]_Q = Q-1 \end{cases} \tag{13}$$

where $n_0' = n_0 - N + P$ and for $N - P < n_0 < N$.

Using (12) and the modulator filter representation (5) of the equivalent PCC, the transfer functions for each modulator filter have the following expressions for $0 < n_0 < P$:

$$\begin{aligned} \left|T_0^{(n_0)}(\Omega)\right| &= \Big|1 - \frac{n_0}{P} + \frac{n_0}{N}e^{j(Q-1)\Omega} + \\ &\quad + (Q-1)\frac{n_0}{N}e^{j(N-1)\Omega}\Big| \end{aligned} \tag{14}$$

if $k \neq 0$ and $[k]_P = 0$ we have

$$\left|T_k^{(n_0)}(\Omega)\right| = \frac{n_0}{N}\left|e^{j(Q-1)\Omega} - e^{j(N-1)\Omega}\right| \tag{15}$$

and if $k \neq 0$ and $[k]_P \neq 0$:

$$\begin{aligned} \left|T_k^{(n_0)}(\Omega)\right| &= \frac{1}{N}\left|\frac{\sin(\frac{\pi}{P}kn_0)}{\sin(\frac{\pi}{N}k)\sin(\frac{\pi}{P}k)}\right|\Big|1 - W^{kQ} + \\ &\quad + \left(W^k - 1\right)W^{k(Q-1)}e^{j(Q-1)\Omega} + \\ &\quad + \left(W^{k(Q-1)} - 1\right)e^{j(N-1)\Omega}\Big| \end{aligned} \tag{16}$$

Using (13), similar relations can be found for $N - P < n_0 < N$.

We make the following assumptions:

1. the band of emission and reception filters $B = \nu\frac{\pi}{N}$ is chosen much larger than $\frac{\pi}{N}$. This assumption is made in order to have a linear variation of the received power $P_y(n_0)$ for $n_0 \in (-P,P)$;
2. the power spectral density of the input signal is constant in the band of the emission and reception filters: $S_x(\Omega) = S_x$ for $\Omega \in (-B,B)$;
3. in equation (7) the terms that correspond to the index $|k| \geq P$, can be neglected, because the emission and reception filters are band limited and so, as k increases, the influence of $\left|T_k^{(n_0)}(\Omega)\right|$ becomes smaller and smaller.

Hence, it is possible to obtain a mathematically tractable expression for $P_y(n_0)$. From (16), by making the observation that $\left|T_k^{(1)}(\Omega)\right| = \left|T_k^{(P-1)}(\Omega)\right|$, we can find an analytical expression for $P_y(n_0)$ in the points $n_0 = 1$ and $n_0 = P-1$:

$$P_y(n_0) = \frac{S_x}{2\pi}\int_{2\pi}\left|H(\Omega)T_0^{(n_0)}(\Omega)\right|^2 d\Omega + cst \tag{17}$$

where $H(\Omega) = H_E(\Omega)H_R(\Omega)$, cst has the same value for $n_0 = 1$ and $n_0 = P-1$. cst represents the influence of all the other terms for $k \neq 0$.

With (14) we obtain by using the Parseval theorem:

$$\begin{aligned} &\frac{1}{2\pi}\int_{2\pi}\left|H(\Omega)T_0^{(n_0)}(\Omega)\right|^2 d\Omega = \\ &\left(\left(1-\tfrac{n_0}{P}\right)^2 + \left(\tfrac{n_0}{N}\right)^2 + \left((Q-1)\tfrac{n_0}{N}\right)^2\right)\tfrac{1}{2\pi}\int_{2\pi}|H(\Omega)|^2 d\Omega \end{aligned} \tag{18}$$

With (18) and using the equality:

$$\frac{S_x}{2\pi}\int_{2\pi}|H(\Omega)|^2 d\Omega = P_y(0) \tag{19}$$

we can rewrite equation (17) as:

$$P_y(n_0) = \left(1 - 2\frac{n_0}{P} + 2\left(Q^2 - Q + 1\right)\left(\frac{n_0}{N}\right)^2\right)P_y(0) + cst \tag{20}$$

where $n_0 = 1$ or $n_0 = P-1$.

This relation allows us to compute the slope of the linear approximation for the received power $P_y(n_0)$:

$$a = \frac{P_y(P-1) - P_y(1)}{P-2} = -\frac{2}{N}\left(1 - \frac{1}{Q}\right) P_y(0) \qquad (21)$$

so that the received power $P_y(n_0)$ can be approximated by:

$$P_y(n_0) \approx \left(-\frac{2}{N}\left(1-\frac{1}{Q}\right) n_0 + 1\right) P_y(0) \qquad (22)$$

where $0 \le n_0 < P$ and $P_y(0)$ is a known constant.

When $N - P < n_0 \le N$ one can show that $P_y(n_0) \approx \left(-\frac{2}{N}\left(1-\frac{1}{Q}\right)(N-n_0)+1\right)P_y(0)$. Since $P_y(n_0)$ is an N-periodic function with respect to n_0, we can conclude that the relation (22) is true for $n_0 \in (-P, P)$.

Hence the equation (21) makes it possible to define the delay estimator for matrix interleavers as:

$$\widetilde{n}_0 = \frac{P_y(n_0 + P/2) - P_y(n_0 - P/2)}{P_y(0)} \frac{N}{4} \frac{Q}{Q-1} \qquad (23)$$

The equation (23) represents the delay estimator to use in the early-late DLL in order to realize the acquisition step for the synchronization of matrix interleavers.

4.3 Comparison between the received power and its linear approximation

Finally we show by numerical simulations that there is a good concordance between the slope of the received power and the slope of its linear approximation (figure 6). The same numerical parameters

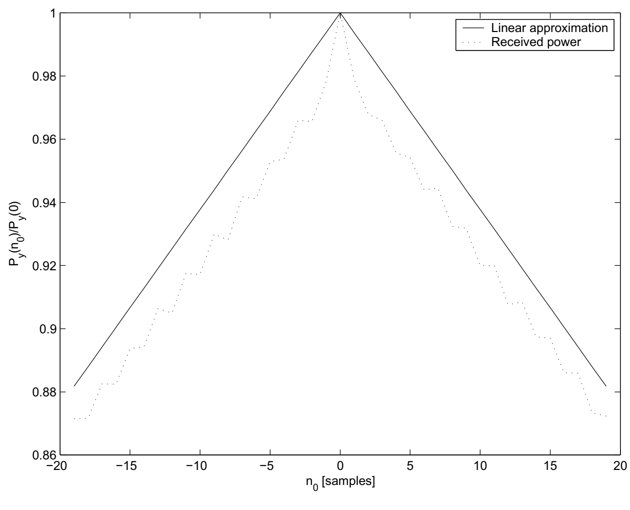

Figure 6: The received power and its linear approximation for $n_0 \in (-P, P)$, $P = 20$

for the transmission chain as in subsection 4.1 are considered. The offset between the two curves doesn't impact the results because the channel delay estimator $\widetilde{n}_0$ depends only on the slope of the curves.

5. CONCLUSIONS

In this paper we have proposed a synchronization technique for an SS transmission chain with PCCs. We have shown that the SS transmission chain with matrix interleavers can be synchronized by using a classical early-late DLL. This synchronization technique for matrix interleavers is possible due to the linear variation of the received power as a function of the channel delay. We have also calculated an analytical expression for the DLL delay estimator using an arbitrary matrix interleaver.

Now it would be interesting to evaluate the performances of such SS technique in a multiuser context. LPTV filters can be found that satisfy the same synchronization conditions together with a good spectral spreading and small multi-user interference. This issue will be the topic of our future work.

6. APPENDIX

We search $h_{n_0}(n)$ for $0 < n_0 < P$. Since $h_0(n) = 0$, the functions $f(n)$ and $g(n)$ are linked by:

$$g(n) = -f(n - g(n)) \qquad (24)$$

$h_{n_0}(n)$ being an N-periodic function we consider $0 \le n \le N-1$.

If $[n]_P \ge n_0$:

$$\begin{aligned} f(n - n_0) &= -(Q-1)([n]_P - n_0) + (P-1)\left[\frac{n-[n]_P}{P}\right]_Q \\ &= f(n) + (Q-1)n_0 \end{aligned} \qquad (25)$$

Replacing n by $n - g(n)$ in (25) and using (24) we obtain (6):

$$h_{n_0}(n) = n_0 Q, \ [n - g(n)]_P \ge n_0 \qquad (26)$$

We can write $n - g(n)$ as:

$$\begin{aligned} n - g(n) &= \frac{n-[n]_Q}{Q} Q + [n]_Q - g(n) \\ &= [n]_Q P + \frac{n-[n]_Q}{Q} \end{aligned} \qquad (27)$$

and so $[n - g(n)]_P \ge n_0$ can be written as $n \ge n_0 Q$.

If $[n]_P < n_0$:

$$\begin{aligned} f(n-n_0) &= -(Q-1)([n]_P - n_0 + P) + \\ &\quad +(P-1)\left[\frac{n-[n]_P}{P} - 1\right]_Q \\ &= f(n) + n_0(Q-1) - P(Q-1) + \\ &\quad + \begin{cases} (P-1)(Q-1), \text{ if } n = [n]_P \\ -(P-1) \text{ otherwise} \end{cases} \end{aligned} \qquad (28)$$

Replacing n by $n - g(n)$ in (28) and using (24) we obtain (6):

$$h_{n_0}(n) = \begin{cases} n_0 Q + 1 - Q, \text{ if } n - g(n) = [n-g(n)]_P \\ n_0 Q + 1 - N \text{ otherwise} \end{cases}, \ n < n_0 Q \qquad (29)$$

The identity $n - g(n) = [n - g(n)]_P$ is equivalent with $[n]_Q = 0$.

Similar steps can be followed to find $h_{n_0}(n)$ for $N - P < n_0 < N$ with the notation $n'_0 = n_0 - N + P$.

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