A SEMIDEFINITE PROGRAMMING (SDP) METHOD FOR DESIGNING *L*-BAND FREQUENCY RESPONSE MASKING (FRM) FILTERS FOR MULTIRATE APPLICATIONS

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ABSTRACT

This paper generalizes the modified frequency response masking (MFRM) filter structure for multirate applications. The overall filter is composed of low-delay FIR and IIR filters. A new SDP method is also proposed for designing these MFRM filters. Design results show that the structure and the design method are very useful to reduce the system delay and arithmetic complexity of traditional MFRM filters for multirate application. The application of the low-delay *L*-band MFRM filters to realize sharp-cutoff Discrete Fourier Transform (DFT) filter bank is also studied.

1. INTRODUCTION

The implementation complexity and system delay of a linear-phase finite duration impulse response (FIR) digital filter with sharp-cutoff are usually very large. This is mainly due to the fact that the order of a FIR filter, *N*, is inversely proportional to the width of the transition band. One efficient method to reduce the implementation complexity of sharp-cutoff FIR filters is to employ the Frequency-Response Masking (FRM) technique, which makes use of the transition bands of an up-sampled digital filter to realize the sharp transition band required [2-7]. The structure proposed by Lim [3], as shown in Figure 1-1, is particularly attractive because it supports arbitrary bandwidth. Although linear-phase FRM filters can significantly reduce the implementation complexity, its system delay is also considerably large. Recently, there is an increasing interest in employing and designing low-delay (LD) FIR subfilters to realize the model and masking filters in the FRM structure [4-6]. FRM filters employing an allpass-based (AP) IIR model filter and two linear-phase FIR masking filters were proposed in [7]. The model and masking filters are first designed separately and nonlinear optimization is then applied to search for the optimal solution. Allpass-based model filter significantly reduce the arithmetic complexity and to some extend the system delay. In a recent work [8], the authors studied the design of IIR FRM filters using semi-definite programming (SDP) [10, 11]. It was found that IIR-based FRM filters are very efficient in low-delay and high stopband attenuation applications, where traditional allpass (AP)-based and LD FIR filters are limited by the delay they can achieve and the high filter order. The masking and model filters are designed in turn using the SDP method and model reduction technique, which yields high quality filters and allows linear or convex quadratic constraints be imposed.

One potential application of FRM filters is in the realization of the *L*-th band filters frequently involved in the decimators and interpolators of multirate systems and filter banks (such as the discrete Fourier transform (DFT) filter banks). The main difficulty in such application is that the structure in Figure 1-1 cannot efficiently be decomposed into polyphase components, which are usually moved to the lower rate part of the system using the noble identity. Lim and Yang [1] were the first to recognize this problem and proposed a modified FRM structure for such applications using linear-phase FIR filters as shown in Figure 2-1.

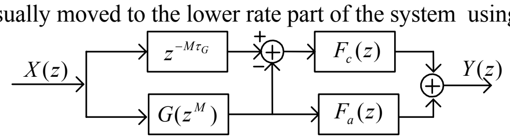

Figure 1-1 The structure of the conventional FRM filter.

In this paper, we extend the modified FRM (MFRM) structure to include low-delay FIR as well as IIR filters for low delay applications. We also proposed a new method based on SDP for designing these modified FRM filters. Design results show that our new structure, which consists of IIR model filters and FIR masking filters, is computationally more efficient than FIR-based FRM filters. To illustrate the application of the new modified FRM filters in multirate applications, a uniform DFT filter bank, derived from the new FRM *L*-band filter, is designed. Design results show that the structure and design method are very useful to reduce the system delay and arithmetic complexity of traditional MFRM filters for multirate application. The rest of the paper is organized as follows: in section 2, we generalize the modified FRM structure to include low-delay FIR filters. The design of the subfilters in the modified FRM structure using a new SDP method is introduced in section 3. Several examples and the new IIR FRM structure are given in section 4. Finally, conclusion is drawn in section 5.

2. FRM STRUCTURE IN MULTIRATE SYSTEM

The transfer function of the FRM filter in Figure 1-1 can be written in the following form:

$$H(z) = G(z^M)F_a(z) + G_c(z^M)F_c(z)\ , \tag{2-1}$$

where $G_c(z) = z^{-\tau_G} - G(z)$ and τ_G is the delay of the model filter $G(z)$. $G_c(z)$ is called the complementary filter of $G(z)$. $F_a(z)$ and $F_c(z)$ are called the masking filters. However, when $H(z)$ is used to realize interpolators and decimators in multirate applications, its polyphase components cannot in general be written in simple form of the subfilters. Thus the complexity reduction brings along by the subfilters cannot be fully exploited. The modified FRM structure in [1] was originally proposed to overcome this problem and it is intended to realize zero-phase FIR filters. Here, we shall extend it further to include general FIR filter, which can be utilized to reduce the system delay. The MFRM structure employs a special case of the FRM structure when the model filter is a half-band filter. The periodic model filter of order N and its complementary filter can thus be written in the following forms

$$G(z^M) = \tfrac{1}{2} z^{-M\tau_G} + G_1(z^M) \tag{2-2}$$

$$G_c(z^M) = \tfrac{1}{2} z^{-M\tau_G} - G_1(z^M)\ , \tag{2-3}$$

where τ_G is the delay of the half-band model filter and $G_1(z) = \sum_{m=1}^{N/2} g_{2m-1} z^{-(2m-1)}$. Substitute (2-2) and (2-3) into (2-1), we get the following equivalent FRM filter

$$H(z) = G_1(z^M)F_2(z) + F_1(z) \tag{2-4}$$

where $F_1(z) = \tfrac{1}{2} z^{-M\tau_G}[F_a(z) + F_c(z)]$ and $F_2(z) = F_a(z) - F_c(z)$. Using (2-4), one gets the MFRM filter in

Figure 2-1. We can see that the coefficients of $G_1(z^M)$ are non-zero at an interval of *2M* because $G(z)$ is a half-band filter. $F_1(z)$ and $F_2(z)$ are linear combinations of the two masking filters $F_a(z)$ and $F_c(z)$.

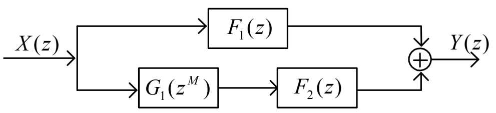

Figure 2-1 The modified FRM (MFRM) filter.

This FRM structure is suitable for designing anti-imaging or anti-aliasing filters with transition bands centred at π/L or π/P, where L or P is the decimation or interpolation factor. However, not all values of *M* will lead to feasible sub-filters. The passband edge $\omega_p^{(G)}$ and stopband edge $\omega_s^{(G)}$ of the model filter $G(z)$ can be computed according to the formulas in [3], and for improper values of *M*, one will end up with $\omega_p^{(G)} < \pi$ and $\omega_s^{(G)} > \pi$. It was shown in [1] that the FRM technique will not produce a feasible design if any one of the following conditions is true:

condition 1: $M\omega_p/\pi$ and $M\omega_s/\pi$ is an integer;

condition 2: $\lfloor M\omega_p/\pi \rfloor \neq \lfloor M\omega_s/\pi \rfloor$,

where $\lfloor x \rfloor$ is the largest integer less than or equal to *x*.

To take full advantage of the computational complexity reduction available in a multirate system, the highest common factor (HCF) of 2*M* and *L* should be *L* so that most parts of the system can be moved to the right side of the down-sampler (or left side of an up-sampler. See also Figure 2-2 for an example). On the other hand, to avoid the two conditions above, *M* should not be an integer multiple of *L*. Because the anti-imaging filter in an *L-fold* interpolator has its cutoff frequency at π/L, we have $\omega_p < \pi/L < \omega_s$ and thus $\lfloor M\omega_p/\pi \rfloor \neq \lfloor M\omega_s/\pi \rfloor$ if *M/L* is an integer. In summary, *2M* should be chosen as an integer multiple of *L,* but *M/L* should not be an integer.

It is possible to find a feasible *M* when *L* is even but there is no suitable *M* for odd *L*. In [1], a solution was proposed to alleviate this problem: a half-band filter (HFB) $G(z)$ whose transition bandwidth is *M* times that of the interpolator filter is first designed. Then the periodic model filter and its complementary filter are obtained by substituting *z* in $G(z^M)$ by $ze^{-j\pi/2M}$ and $ze^{j\pi/2M}$, respectively, to give

$$G_{aI}(z^M) = \tfrac{1}{2}z^{-M\tau_G} \cdot e^{j\tau_G\pi/2} + \sum_{m=1}^{N/2} g_{2m-1} z^{-(2m-1)} e^{j(2m-1)\frac{\pi}{2M}} \quad (2\text{-}3)$$

$$G_{cI}(z^M) = \tfrac{1}{2}z^{-M\tau_G} \cdot e^{-j\tau_G\pi/2} + \sum_{m=1}^{N/2} g_{2m-1} z^{-(2m-1)} e^{-j(2m-1)\frac{\pi}{2M}} \quad (2\text{-}4)$$

Let $G(e^{j\omega M})$, $G_{aI}(e^{j\omega M})$ and $G_{aC}(e^{j\omega M})$ denote the frequency responses of $G(z^M)$, $G_{aI}(z^M)$ and $G_{aC}(z^M)$, respectively. We can see that $G_{aI}(e^{j\omega M})$ and $G_{aC}(e^{j\omega M})$ are respectively the right and left shift version of $G(e^{j\omega M})$ by $\pi/2M$. The two masking filters $F_a(e^{j\omega})$ and $F_c(e^{j\omega})$ can also be obtained by shifting the frequency response of a prototype masking filter to right and left by $\omega_p^{(G)}/M$, respectively. The passband and stopband edges of the prototype masking filter are: $\omega_p^{(F)} = \omega_p - \frac{\omega_p^{(G)}}{M}$ and $\omega_s^{(F)} = \omega_s + \frac{\omega_p^{(G)}}{M}$.

The MFRM structure described can be used to realize uniform DFT filterbanks, which find useful application in the realization of channelizers in software radio base-station. The *k*-th filter of the uniform DFT filter banks can be expressed as [12]

$$H_k(z) = H_0(zW^k) = \sum_{l=0}^{L-1} (z^{-1}W^{-k})^l E_l(z^L) \quad (2\text{-}5)$$

where $H_0(z)$ is a *L-th* band filter, $W = e^{-j2\pi/M}$ and $E_l(z)$ is the polyphase components of $H_0(z)$. If $H_0(z)$ is realized as a MFRM filter, then the MFRM-based decimation uniform DFT filter bank in Figure 2-2 results. The $E_l^{(G_1)}$, $E_l^{(F_1)}$ and $E_l^{(F_2)}$, *l=0,...,L-1*, are the polyphase components of $G_1(z^M)$, $F_1(z)$ and $F_2(z)$, respectively. It can be seen that MFRM filters fit nicely into this structure because all of them are operating at the lowest rate of the system. Moreover, by performing a single DFT (***W***), *L*-uniformly spaced channels can be simultaneously realized with very low arithmetic complexity.

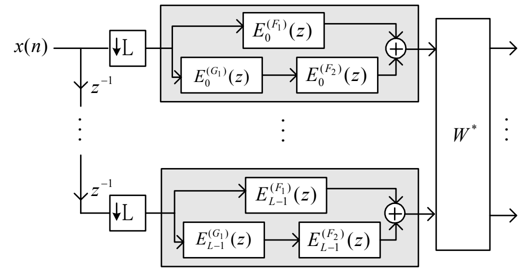

Figure 2-2 The polyphase structure of the FRM DFT filter bank.

3. DESIGNING FRM FILTERS FOR MULTIRATE SYSTEMS WITH SDP

The proposed SDP-based design of the halfband model filters and the FIR masking filters will be described in the following. The halfband model filter is first designed by means of SDP [10]. Then, the two masking filters are designed simultaneously to optimize the overall performance of the filter.

3.1 Design of linear and low-delay model filters

The transfer function of a FIR HBF with order *N* has the following form

$$G(z) = \sum_{n=0}^{N} g[n] z^{-n} \quad (3\text{-}1)$$

where $g[n]$ is the impulse response of the filter. $g[n]$ is zero when *n* is even except that $g[\tau_d] = 0.5$ where τ_G (even) is the delay of the filter. Thus, $G(z)$ can be written as

$$G(z) = \sum_{m=1}^{N/2} \overline{g}[m] z^{-(2m-1)} + \tfrac{1}{2} z^{-\tau_G} \quad (3\text{-}2)$$

where $\overline{g}[m] = g[2m-1]$. Substitute *z* in (3-2) by $e^{j\omega}$ and letting the frequency response of the desired filter be $G_d(\omega)$, the design problem using the minimax criterion can be formulated as follows:

minimize δ subject to (3-3)

$$W^2(\omega) \,|\, G(e^{j\omega}) - G_d(\omega) \,|^2 < \delta \qquad -\pi \le \omega \le \pi$$

where $W(\omega)$ is a positive weighting function. (3-3) can be further rewritten in the following form

minimize δ subject to (3-4)

$$\delta - \beta_R^2(\omega) - \beta_I^2(\omega) > 0\,, \ -\pi \le \omega \le \pi$$

where $\beta_R(\omega) = W(\omega)[\overline{\mathbf{g}}^T \mathbf{c} + \tfrac{1}{2}\cos(\tau_G\omega) - G_{d\_R}(\omega)]$

$\beta_I(\omega) = W(\omega)[\overline{\mathbf{g}}^T \mathbf{s} + \tfrac{1}{2}\sin(\tau_G\omega) - G_{d\_I}(\omega)]$

$\bar{\mathbf{g}} = [\bar{g}_1 \quad \cdots \quad \bar{g}_{N/2}]^T$

$\mathbf{c} = [\cos(\omega) \quad \cos(3\omega) \quad \cdots \quad \cos((N-1)\omega)]^T$

$\mathbf{s} = [\sin(\omega) \quad \sin(3\omega) \quad \cdots \quad \sin((N-1)\omega)]^T$

$G_{d\_R}(\omega) = real(G_d(\omega))$ and $G_{d\_I}(\omega) = -imag(G_d(\omega))$.

real(x) and *imag(x)* stand respectively for the real and imaginary parts of *x*. Using Schur complement [10], $\delta - \beta_R^2(\omega) - \beta_I^2(\omega) > 0$ can be written as the following linear matrix inequality (LMI)

$$\mathbf{\Lambda}(\bar{\mathbf{g}}, \omega) = \begin{bmatrix} \delta & \beta_R(\omega) & \beta_I(\omega) \\ \beta_R(\omega) & 1 & 0 \\ \beta_I(\omega) & 0 & 1 \end{bmatrix} \geq 0, \tag{3-5}$$

where $-\pi \leq \omega \leq \pi$. Discretizing the frequency variable ω in (3-5) into a dense set of frequencies $\{\omega_i, 1 \leq i \leq K\}$ in the interested band and stacking them together, one gets the LMIs:

$$\mathbf{\Lambda}(\bar{\mathbf{g}}) \succeq 0 \tag{3-6}$$

where $\mathbf{\Lambda}(\bar{\mathbf{g}}) = diag\{\mathbf{\Lambda}(\bar{\mathbf{g}}, \omega_1), \ldots, \mathbf{\Lambda}(\bar{\mathbf{g}}, \omega_K)\}$.

Further, by defining the augmented variable $\mathbf{x} = [\delta \quad \bar{\mathbf{g}}^T]^T$, (3-6) can be formulated into the following standard SDP problem:

$$\text{minimize } \mathbf{c}^T\mathbf{x} \tag{3-7}$$

$$\text{subject to } \mathbf{\Lambda}(\mathbf{x}) \succeq 0, \ \mathbf{c} = [1, 0, \cdots, 0]^T.$$

The advantages of using SDP are that they can be applied to optimally design filters with passband linear phase characteristics using either the least squares or minimax criterion. Also, it is possible to include additional linear constraints and convex quadratic constraints by stacking them together as in (3-6).

3.2 Designing the masking filters using SDP

The SDP method can also design the two masking filters simultaneously with the knowledge of the model filters. The optimization process works out the care bands and don't care bands automatically. The don't care bands are the bands in which the frequency responses don't affect the overall frequency response much [3].

Let $G(\omega)$ and $G_c(\omega)$ denote the frequency responses of the periodic model filter and complementary periodic model filter, respectively. The frequency response of the overall filer $H(e^{j\omega})$ is

$$H(e^{j\omega}) = G(\omega)\sum_{n=0}^{N_{Fa}} f[n]_a e^{-jn\omega} + G_c(\omega)\sum_{n=0}^{N_{Fc}} f_c[n]e^{-jn\omega} \tag{3-8}$$

where $\sum_{n=0}^{N_{Fa}} f_a[n]e^{-jn\omega} = F_a(e^{j\omega})$ and $\sum_{n=0}^{N_{Fc}} f_c[n]e^{-jn\omega} = F_c(e^{j\omega})$ are respectively the frequency responses of the masking filters $F_a(z)$ and $F_c(z)$. Note, $H(e^{j\omega})$ is a linear function of the impulse responses f_a and f_c of the two masking filters, which are the variables to be determined. Thus, to approximate a desired frequency response $H_d(\omega)$, the design problem, using the minimax criterion, can be formulated as follows and solved by means of SDP:

$$\text{minimize } \delta \text{ subjected to} \tag{3-9}$$

$$W^2(\omega)\,|H(e^{j\omega}) - H_d(\omega)|^2 < \delta$$

$$-\pi \leq \omega \leq \pi$$

where $W(\omega)$ is the positive weighting function. Following a derivation similar to the design of the model filter, (3-9) can be formulated as the standard SDP problem:

$$\text{minimize } \mathbf{c}^T\mathbf{x} \tag{3-10}$$

(3-10)

where $\mathbf{c} = [1 \quad 0 \quad \cdots \quad 0]^T$ $\mathbf{x} = [\delta \quad \mathbf{f}_a^T \quad \mathbf{f}_c^T]^T$

$\mathbf{\Gamma}(\mathbf{f}) = diag\{\Gamma(\mathbf{f}, \omega_1), \ldots, \Gamma(\mathbf{f}, \omega_K)\}$, $\mathbf{f} = [\mathbf{f}_a^T \quad \mathbf{f}_c^T]^T$

$$\Gamma(\mathbf{f}, \omega_i) = \begin{bmatrix} \delta & \alpha_R(\omega_i) & \alpha_I(\omega_i) \\ \alpha_R(\omega_i) & 1 & 0 \\ \alpha_I(\omega_i) & 0 & 1 \end{bmatrix} \geq 0$$

$$\alpha_R(\omega_i) = G_R(\omega_i)\mathbf{f}_a^T\mathbf{c}_1 - G_I(\omega_i)\mathbf{f}_a^T\mathbf{s}_1 + G_{cR}(\omega_i)\mathbf{f}_c^T\mathbf{c}_2 - G_{cI}(\omega_i)\mathbf{f}_c^T\mathbf{s}_2 - H_{dR}(\omega_i)$$

$$\alpha_I(\omega_i) = G_{aI}(\omega_i)\mathbf{f}_a^T\mathbf{c}_1 + G_{aR}(\omega_i)\mathbf{f}_a^T\mathbf{s}_1 + G_{cI}(\omega_i)\mathbf{f}_c^T\mathbf{c}_2 + G_{cR}(\omega_i)\mathbf{f}_c^T\mathbf{s}_2 - H_{dI}(\omega_i)$$

$\mathbf{c}_1 = [1 \quad \cos(\omega_i) \quad \cdots \quad \cos(N_{F_a}\omega_i)]^T$

$\mathbf{s}_1 = [0 \quad \sin(\omega_i) \quad \cdots \quad \sin(N_{F_a}\omega_i)]^T$

$\mathbf{c}_2 = [1 \quad \cos(\omega_i) \quad \cdots \quad \cos(N_{F_c}\omega_i)]^T$

$\mathbf{s}_2 = [0 \quad \sin(\omega_i) \quad \cdots \quad \sin(N_{F_c}\omega_i)]^T$.

and $\{\omega_i, 1 \leq i \leq K\}$ is the set obtained from digitizing the frequency variable ω over the interested bands. The symbols that have the subscripts of *R* represent the real parts of those variables. Similarly, the symbols that have the subscripts of *I* represent the minus of the imaginary parts of those variables. For example, $H_{dR}(\omega_i)$ is the real part of the frequency response of $H_d(\omega_i)$.

To realize FRM with IIR filters, the low-delay FIR model or masking filters can be converted to their IIR counterparts by means of model reduction [8]. This will reduce the hardware delay (and possible hardware complexity) in implementing the MFRM filters. Alternatively, allpass-based IIR sub-filters can be employed.

4. EXAMPLES

We now present several design examples to illustrate the principle and performance of the proposed filter structure and its SDP-based design method.

Example 1: An *L-fold* decimation filter is designed with *L=10*. The model filter and the two masking filters are all linear phase FIR filters. The width of the transition band is $\Delta\omega = 0.01\pi$, and the maximum ripple in the passband and stopband is 0.01. *M*=25 is found to give a feasible sub-filters design. The order of the model and masking filters are 20, 93 and 93, respectively. The frequency responses of the model and the masking filters are given in Figure 4-1. Note there are don't care bands of the two masking filters, which have an attenuation of less than 45dB. To meet the same frequency specifications, the direct form structure need 186 coefficients, while the FRM structure needs only 103 coefficients. The uniform DFT filter bank based on this *L* band filter is shown in Figure 4-1(d).

Example 2: An *L*-fold decimation filter is designed with the same specifications in example 1, except that the delay of the overall filter is now lowered by 50 samples. From (2-1), we can see that the system delay is mainly determined by the model filter. Therefore, we lower the delay of the model filter by 2 sample, which considerably decreases the system delay by 50 samples, as compared with its conventional linear-phase counterpart. The frequency response of the overall filter and its group delay are shown respectively in Figures 4-2 (a) and (b). On the other hand, the ripple of the group delay is about 1 sample except near the edge of the transition band, which is slightly inferior to the linear-phase case.

Example 3: An *L-fold* decimation FRM filter that consists of an

IIR model filter and two FIR masking filters is designed. The IIR model filter is a halfband filter that composed of an all-pass filter and a delay. It is designed by the SDP method proposed in [9]. The specifications are the same as that of example 1. The order of the all-pass filter is 3 and the orders of the two masking filters are all 93. The design results are shown in Figure 4-3. It can be seen that the IIR FRM filter is more efficient and has lower system delay than its FIR counterpart under the same specification.

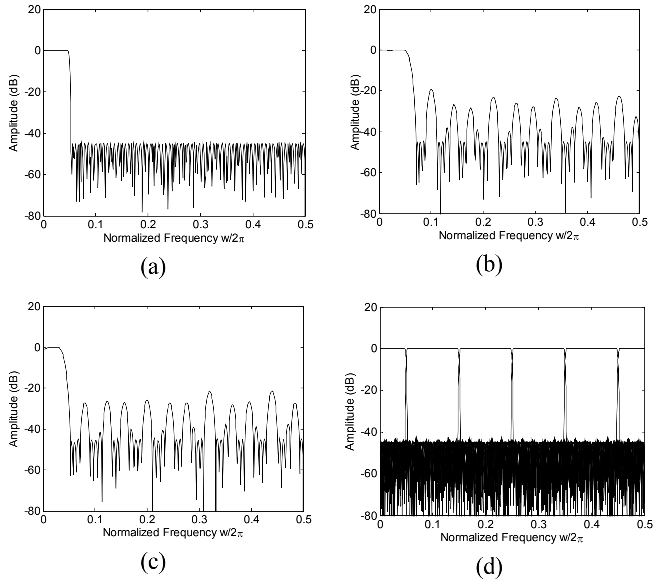

Figure 4-1. Design results of **example 1**: The frequency response of (a) the overall filter $H(z)$, (b) the masking filter $F_a(z)$ and (c) the masking filter $F_c(z)$. (d) The frequency response of the DFT filter bank.

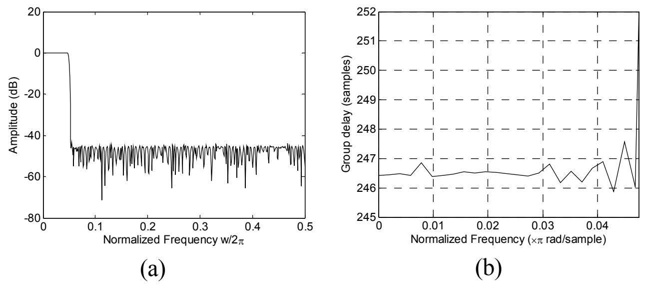

Figure 4-2. The design results of example2: (a) The frequency response of the overall filter $H(z)$. (b) The group delay of the overall filter $H(z)$.

Example 4: A low-delay FIR *L*-fold decimation MFRM filter with *L* and odd number is designed. It is similar to the case when *L* is even. Due to page limitation, the detailed formulation is omitted. *L* and *M* are set to 5. The transition bandwidth and ripples in passband and stopband are 0.01π and 0.01, respectively. The order of the model and masking filters are set respectively to 80 and 21. The delay of the model filter is 32, which leads to a lower overall delay of 170.5 samples. The delay of this odd *L* decimation filter is 40 samples lower than that of its conventional linear-phase counterpart. Moreover, it only needs 103 coefficients, as compared with 186 for the direct form FIR decimator. Figure 4-4 gives the frequency response of the designed filter.

5. CONCLUSION

A modified frequency response masking (MFRM) filter structure for multirate applications using low-delay FIR and IIR filters is presented. A new SDP method is also presented for designing these MFRM filters. Design results show that the structure and design method are very useful to reduce the system delay and arithmetic complexity of traditional MFRM filters. The application of the low-delay *L*-band MFRM filters to realize sharp-cutoff Discrete Fourier Transform (DFT) filter bank is also illustrated by a design example.

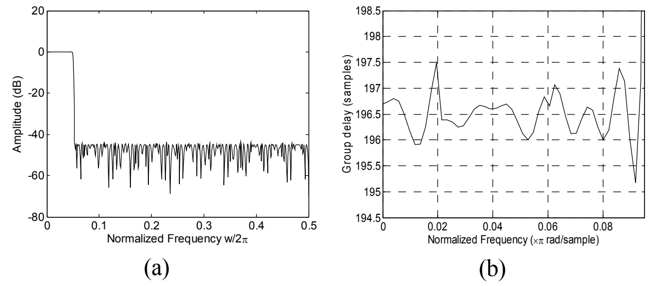

Figure 4-3. The design results of example3: (a) The frequency response of the overall filter $H(z)$. (b) The group delay of the overall filter $H(z)$.

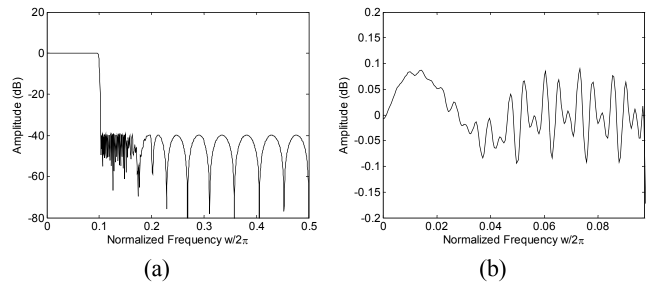

Figure 4-4. The design results of example4: The frequency response of (a) the overall filter $H(z)$. (b) the overall filter in passband.

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