SUPER-RESOLUTION DIGITAL IMAGE INTERPOLATION WITH WARPING OF COORDINATE POINT AND BIASING OF SIGNAL AMPLITUDE

Atsushi Shimura and Akira Taguchi

Department of Electrical and Electronic Engineering, Musashi Institute of Technology
1-28-1 Tamazutsumi, Setagaya-ku, Tokyo, Japan
Tel/Fax: +81 3 5707 2161
email: ataguchi@eng.musashi-tech.ac.jp

ABSTRACT

The problem of image interpolation using linear techniques is deal with in this paper. Conventional methods are changed into space-variant ones, by introducing the concept of both the biasing of signal value and the warping of distance among pixels. The biasing creates the peak of uneven signals which is corrspond to the signal details. The warping realizes the edge-preserving interpolation. This paper presents the method which is combined two concepts. The clear interpolated images can be obtained by the proposed method.

1. INTRODUCTION

Digital image interpolation is a problem of prime importance in many fields, ranging from medical imaging to military applications or to consumer electronics. The frequency band of a digital image is decided by the specification of an input device. Thus, it is necessary to enlarge digital images when those are projected onto the high-resolution display. However, the enlarged digital image on the display is usually blurred.

In the idealized world of linear and stationary systems, an optimal approach for interpolation exists: the sinc function, which would permit exact reconstruction of a bandlimited signal. Due to the impossibility of realizing this function physically, different approximations have been devised, yielding to well-known linear and space-invariant techniques such as the bilinear and cubic operators. These techniques can preserve the resolution before enlarged. However, the frequency band of the enlarged image is more wide than that of the original image. Thus, the enlarged image is blurred by using these techniques. The heavy blurred area in the image is the signal including high frequency component in the original image, for example the stepedge signal and the peak signal. Because, the outline and the detail of images correspond to the stepedge signal and the peak signal, respectively. We propose a novel interpolation method which is taken these signals into account.

Recently, several new interpolation methods with warping which can preserve the stepedge are proposed [4],[5]. The warping is as its name suggests to warp image space not to produce blurred images based on the bilinear technique. The method proposed in [5] is less calculated amount than that in [4], and be able to interpolate arbitrary scaling. This warping technique can only preserve the stepedges, however, can't create peaks. We consider the creating peaks based on the concept of the biasing.

In this paper, we propose the peak-preserving interpolation method based on the biasing of signal amplitude, which achieves to increase or decrease the value of the interpolation point. Next, we combine the proposed biasing method and the method of [5]. Two methods have different conceps each other, the prposed method is expected to have the merits of both methods.

2. INTERPOLATION TECHNIQUE WITH BIASING AND WARPING

2.1 Interpolation with biasing of signal amplitude

We introduce the biasing method into the linear interpolation method so as to create the signal peaks. To create peaks is impossible for the conventional method. We consider the one-dimensional (1-D) case. In the case of two-dimensional image, one-dimensional method is used by sequential row-wise and column-wise.

Figure 1 shows the concept of the biasing technique for signal amplitude, example for twice size enlarging. The result of bilinear interpolation shows the dashed line in Fig.1. When the peak exists between x_k and x_{k+1}, the result of bilinear interpolation is smaller than the ideal result. In order to create the peaks, we want to add the suitable value to the result of bilinear interpolation.

Let $f(x_k)$ be the value at point x_k. The interpolated point x is located between x_k and x_{k+1}. The distance between x and x_k or x_{k+1} is given by $s = x - x_k$ or $1 - s = x_{k+1} - x$ ($0 \le s \le 1$). The bilinear interpolation result is calculated by

$$\hat{f}(x) = (1-s)\cdot f(x_k) + s\cdot f(x_{k+1}). \tag{1}$$

When twice size enlarging, $s = 0.5$, since x is the center of x_k and x_{k+1}.

The signal peaks can be created by increasing or decreasing of the result of bilinear interpolation. Let $\bar{f}(x)$ be the result of the proposed interpolation with biasing of the amplitude as

$$\bar{f}(x) = \hat{f}(x) + l\cdot B. \tag{2}$$

where l is constant and B is defined by

$$B = \frac{C}{2} + \frac{C}{2}\cdot(A-1) \times \mathrm{sgn}\left[\{f(x_{k+1}) - f(x_{k-1})\}\times\{f(x_{k+2}) - f(x_k)\}\right] \tag{3}$$

$$A = \frac{\left|f(x_{k+1}) - f(x_{k-1})\right| - \left|f(x_{k+2}) - f(x_k)\right|}{L-1} \tag{4}$$

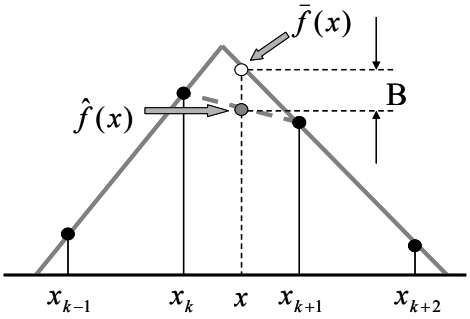
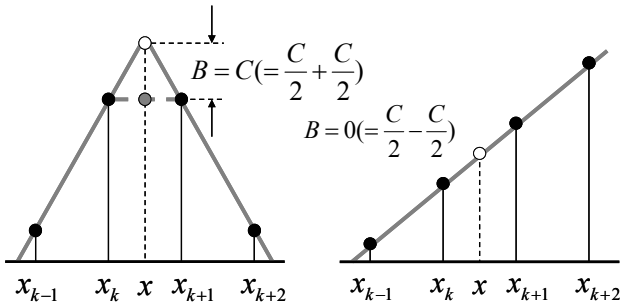


Fig.1 Interpolation with biasing of signal amplitude



(a) Creating Peak (b) No-creating peak

Fig.2 The concept of the biasing of signal amplitude

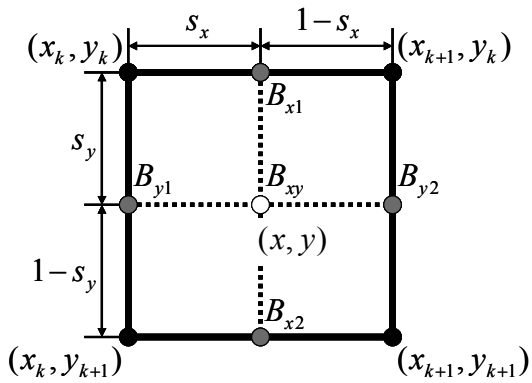


Fig. 3 Interpolation inside four coordinate points (Biasing of signal amplitude)

$$C = \mathrm{sgn}[f(x_k) - f(x_{k-1})] \cdot |C| \tag{5}$$

$$|C| = \min[s \cdot |f(x_k) - f(x_{k-1})|, (1-s) \cdot |f(x_{k+2}) - f(x_{k+1})|] \tag{6}$$

where $L = 256$ for 8bits luminance images, $\mathrm{sgn}[\bullet]$ and $\min[\bullet]$ are the sign function and the minimum operation, respectively.

We explain the concept of the proposed method by using two type signal patterns, one is the peak signal, the other is the monotonous increasing signal as shown in Fig 2. Parameter A of these two signals is 0. Thus,

$$B = \frac{C}{2} - \frac{C}{2} \times \mathrm{sgn}[\{f(x_{k+1}) - f(x_{k-1})\} \times \{f(x_{k+2}) - f(x_k)\}] \tag{7}$$

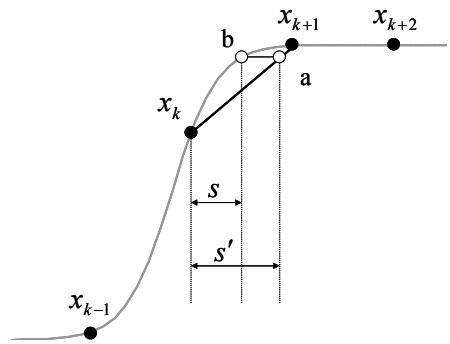


Fig.4 Interpolation with warping of coordinate point

$B = 0$ is desired for the monotonous signal. On the other hand, it is desirable for peak signal that B shows maximum value.

In the case of the peak signal, $B = C/2 + C/2 = C$ (i.e., maximum biasing). In the case of the monotonous increasing signal, $B = C/2 - C/2 = 0$ (i.e., no biasing). We can understand that these are the desired results.

Next, we consider how to interpolate inside four coordinate points. The bilinear interpolated result of (x, y) is given by

$$\hat{f}(x,y) = (1-s_x)(1-s_y)f(x_k, y_k) + s_x(1-s_y)f(x_{k+1}, y_k) + (1-s_x)s_y f(x_k, y_{k+1}) + s_x s_y f(x_{k+1}, y_{k+1}). \tag{8}$$

The biasing value B_{xy} is given by

$$B_{xy} = [(1-s_x)B_{x1} + s_x B_{x2} + (1-s_y)B_{y1} + s_y B_{y2}]/2 \tag{9}$$

Thus, interpolation result is calculated by

$$\bar{f}(x,y) = \hat{f}(x,y) + l \cdot B_{xy} \tag{10}$$

2.2 Interpolation with warping of coordinate point

In this section, first we explain the warping method of [5]. Then, we combine the method of section **2.1** and the method of [5].

In the warping method of [5], s' is used in Eq.(1) in place of s. s' is defined by

$$s' = s - kAs(s-1) \tag{11}$$

The positive parameter k controls the intensity of the warping. Parameter A is given by Eq.(4). $A = 0$ indicates symmetry. Positive value indicates that the edge is more homogeneous on the right side (see Fig.4), and the pixel to be interpolated belongs to the right objects. Hence pixels on the same side should affect more strongly the interpolated value: accordingly, it should be $s' > s$ if $A > 0$. The opposite holds if $A < 0$.

It is necessary to consider how to calculate the interpolated value inside of the four-coodinate points. However, Ref.[5] doesn't show the method. Thus, the original method in this paper is shown. Figure 5 shows the case of the twice size enlarging. We derive the warping position inside four-points by using the warping results of

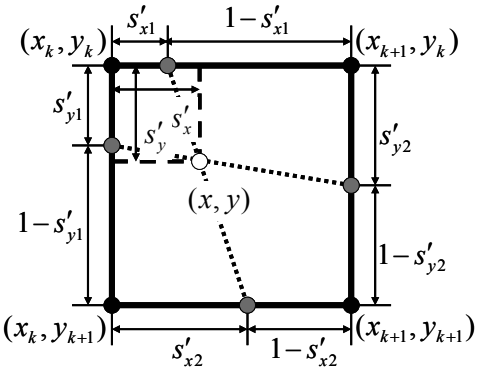
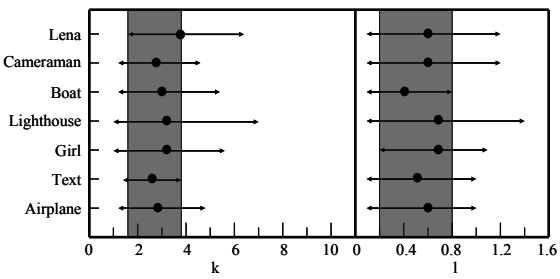
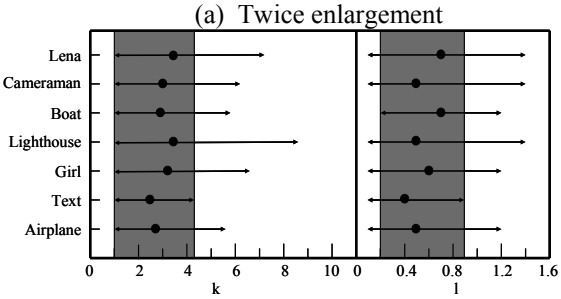


Fig.5 Interpolation inside four coordinate points (warping of coordinate point)



(a) Twice enlargement



(b) Four times enlargemant

Fig.6 Decision of parameters besed on MSE

between coordinate points. The warped point (s'_x, s'_y) is given by

$$s'_x = \frac{2s'_{y1}(s'_{x2} - s'_{x1}) + 4s'_{x1}}{4 - (s'_{y2} - s'_{y1})(s'_{x2} - s'_{x1})} \tag{12}$$

$$s'_y = \frac{2s'_{x1}(s'_{y2} - s'_{y1}) + 4s'_{y1}}{4 - (s'_{y2} - s'_{y1})(s'_{x2} - s'_{x1})} \tag{13}$$

Interpolation result of inside of four-points: $\hat{f}(x, y)$ is defined by

$$\hat{f}(x, y) = (1 - s_x')(1 - s_y')f(x_k, y_k) + s_x'(1 - s_y')f(x_{k+1}, y_k) + (1 - s_x')s_y' f(x_k, y_{k+1}) + s_x' s_y' f(x_{k+1}, y_{k+1}). \tag{14}$$

2.3 Interpolation with warping of coordinate point and biasing of signal amplitude

In this section, we combine the biasing method and the warping method. First, we calculate the warped coordinate points by using

Table 1 Comparing the performance of each method

(a) Twice enlargement

MSE	Bilinear	Cubic	CW [5]	Proposed (opt.)	Proposed (fix.)
Lena	84.2	73.8	70.7	68.1	68.7
Cameraman	199.6	175.2	168.1	162.9	163.2
Boat	78.7	66.9	62.4	61.4	61.6
Lighthouse	296.5	271.8	268.6	258.9	260.1
Girl	36.3	29.5	30.6	28.0	28.1
Text	173.1	144.0	136.4	129.6	130.9
Airplane	140.7	119.1	115.7	109.3	109.4

(b) Four times enlargement

MSE	Bilinear	Cubic	CW [5]	Proposed (opt.)	Proposed (fix.)
Lena	197.9	178.3	176.7	170.9	171.8
Cameraman	455.1	418.5	410.9	405.2	405.2
Boat	226.9	198.4	205.7	189.8	191.4
Lighthouse	553.5	522.3	516.5	509.8	510.5
Girl	112.7	98.0	100.8	96.5	96.9
Text	496.6	423.5	412.3	397.7	399.8
Airplane	378.8	343.7	345.0	337.2	337.6

Eq.(11)-(14). Next, we calculated the biasing value of signal amplitude after replacing s with s' in Eq. (2)-(6), (8)-(10).

3. DECISION OF PARAMETERS

3.1 Decision of parameters based on MSE

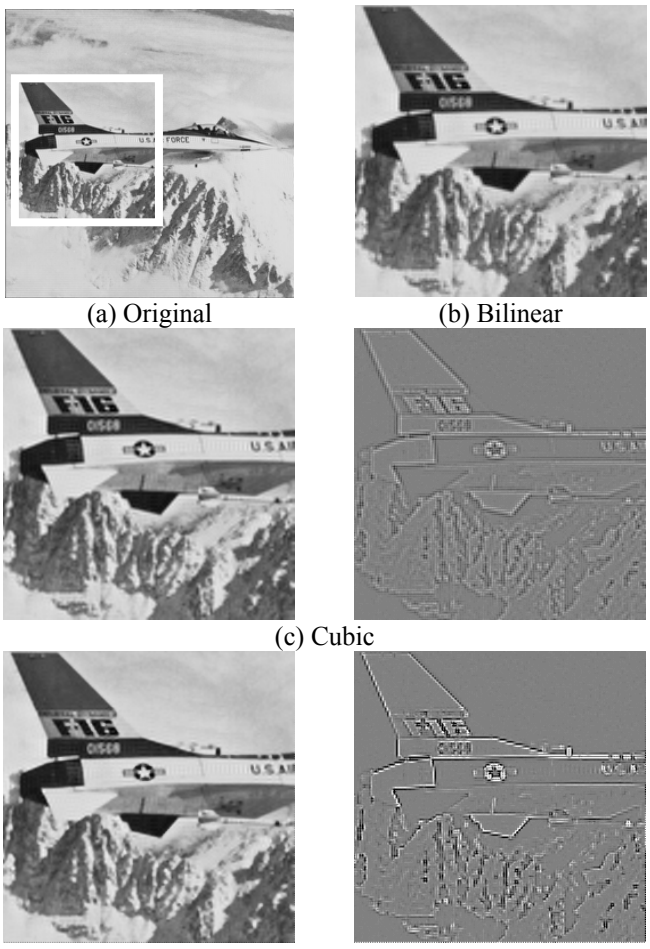
It is necessary to decide the parameter l of Eq.(2) and k of Eq.(11). Two parameters are derived based on the mean square error (MSE). The size of original images is 256x256, the 1/2 size image is calculated by low-pass filtering and down sampling. The 1/4 size image is also obtained. These images are enlarged to the original size, and calculate MSEs between original image and enlarged image for various k or l. Figure 6 shows the parameter k and l of minimum MSE (black circle in the figure). Moreover, we show the suitable region of k and l. The regions are defined between $\pm 5\%$ from minimum MSE. We can understand that $k = 2 \sim 4$ and $l = 0.3 \sim 0.7$ are suitable for all test images. Consequently, we can decide $k = 3.0$ and $l = 0.5$ based on MSE. These values of two parameters are robust for a lot of images, since these values are center of suitable regions.

Table 1 shows MSEs of the proposed method with (k,l)=(3.0,0.5) (Proposed(fix.)), the proposed method with optimal parameters (Proposed(opt.)), the method of [5] (CW), the bilinear interpolation (Bilinear) and the cubic interpolation (Cubic). Proposed(fix.) and Proposed(opt.) show the same performance on MSE for both twice and four times enlargement. Moreover, the performance of Proposed(fix.) is superior to those of CW and Cubic.

3.2 Decision of parameters considering to vision

In section 3.1, we decided the parameters of proposed method based on MSE. We compare the proposed method of fixed parameters to the cubic interpolation on Airplane in Fig.8. Figure 8 shows the enlarged images and the

differential images from the bilinear interpolation. The differential images show the degree of the enhancement of edge and detail parts. The clearness of enlarged image of $(k,l)=(3.0,0.5)$ is insatisfactory.

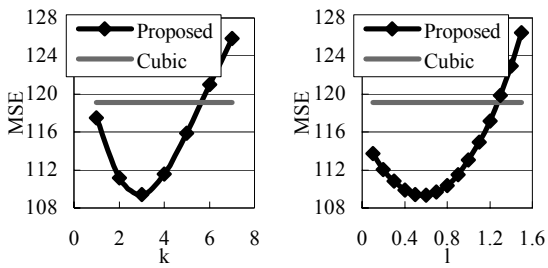


(a) Original (b) Bilinear

(c) Cubic

(d) $k=3.0,\quad l=0.5$

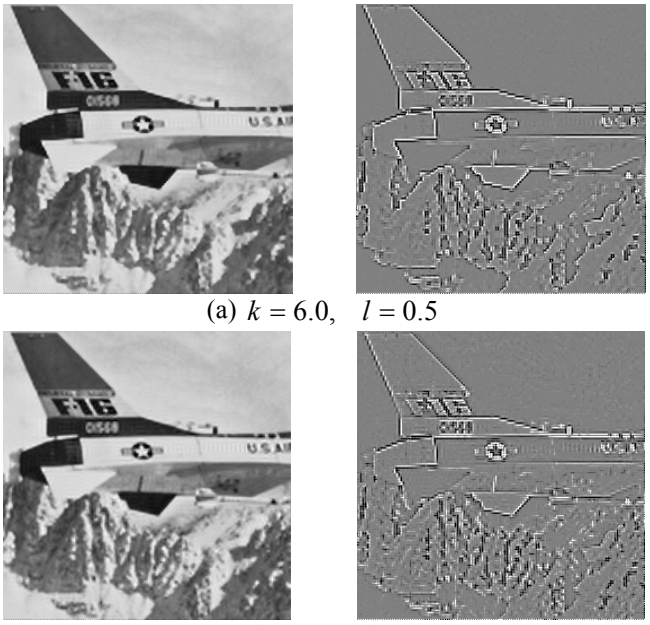
Fig.7 Enlarged images of the cubic interpolation and the proposed method
(Left : Interpolation result, Right : Differential image)



(a) MSEs for various k (b) MSEs for various l

Fig.8 MSEs for various k or l

The parameters k and l in the proposed method decide the biasing or warping rate. We set k or l larger, the enlarged image is shown as more clearly. The MSE of the proposed method with $(k,l)=(6.0,0.5)$ or $(k.l)=(3.0,1.3)$ is approximately equal to the MSE of the cubic interpolation from Fig.8. Figure 9 shows the enlarged images and the defferential images of $(k,l)=(6.0,0.5)$ and $(k,l)=(3.0,1.3)$. Both results are more clearly compared to the proposed method with $(k,l)=(3.0,0.5)$. Especially, the proposed method with $(k,l)=(3.0,1.3)$ shows excellent in the detali parts.



(a) $k=6.0,\quad l=0.5$

(b) $k=3.0,\quad l=1.3$

Fig.9 Enlarged images for various k or l

4. CONCLUSIONS

The technique, which has been presented in this paper permits to improve the linear interpolation operators, such as bilinear and cubic, using easily techniques. In this way, two weak points (preserving edges and creating peaks) of linear interpolation are resolved by the warping and biasing techniques. Furthermore, we study the decision of parameters of the proposed method. The proposed interpolation shows excellent results.

REFERENCES

[1] W.K. Pratt, Digital Image Processing, New York: Wiley, 1991

[2] S. Carrato, G. Ramponi and S. Marsi, "A Simple Edge-Sensitive Image Interpolation Filter," IEEE, International Conf. on Image Processing, pp.711-714, Lausanne, Switzerland, Sept. 1996.

[3] A. Taguchi and T. Kimura, "Edge-Preserving Interpolation by using the Fuzzy Technique," Nonlinear Image Processing and Pattern Analysis XII, SPIE 4304, 2001.

[4] N. Arad and C. Gotsman, "Enhancement by Image-Dependent Warping," IEEE Trans. Image Processing, vol.8, pp. 1063-1074, Aug. 1999.

[5] G. Ramponi, "Warped Distance for Space-Variant Linear Image Interpolation," IEEE Trans. Image Processing, vol.8, pp. 629-639, May 1999.