ROBUST SUPER-EXPONENTIAL METHODS FOR DEFLATIONARY BLIND EQUALIZATION OF STATIC SYSTEMS

Mitsuru Kawamoto[1,2], *Kiyotaka Kohno*[1], *and Yujiro Inouye*[1]

[1]Department of Electronic and Control Systems Engineering, Shimane University,
1060 Nishikawatsu, Matsue, Shimane 690-8504 JAPAN.
TEL:+81-852-32-6065, FAX:+81-852-32-8909, e-mail:kawa@ecs.shimane-u.ac.jp,
[2]Bio-Mimetic Control Research Center, RIKEN, Anagahora, Shimoshidami, Moriyama, Nagoya, 463-0003 Japan.

ABSTRACT

The so called "super-exponential" methods (SEM's) are attractive methods for solving blind signal processing problems. The conventional SEM's, however, have such a drawback that they are very affected by Gaussian noise. To overcome this drawback, we propose a new SEM. While the conventional SEM's use the second- and higher-order cumulants of observations, the proposed SEM uses only the higher-order cumulants of observations. Because higher-order cumulants are not affected by Gaussian noise, the proposed SEM is robust to Gaussian noise, which is referred to as a robust super-exponential method (RSEM). To show the validity of the proposed RSEM, some simulation results are presented.

1. INTRODUCTION

The present paper deals with the blind equalization problem of a static system driven by source signals which are spatially independent. To solve this problem, the ideas of the super-exponential methods (SEM's) in [1],[4], and [6] are used. Several researchers (e.g., [1, 4, 5, 6, 10]) have proposed some SEM's until now for solving independent component analysis (ICA), blind source separation (BSS), and blind channel equalization (BCE). One of the attractive properties of the SEM's is that they are computationally efficient and they converge to a desired solution at a super-exponential rate. However, all the SEM's proposed until now have such a drawback that they are very affected by Gaussian noise (this will be shown in Section 4), because they utilize the second-order and the higher-order cumulants of observations.

In the present paper, we propose a new SEM which overcomes the drawback. The proposed SEM utilizes only the higher-order cumulants of observations. Since higher-order cumulants are not affected by Gaussian noise, the proposed SEM becomes robust to Gaussian noise. Simulation results show that the proposed SEM is robust to Gaussian noise and can successfully attain the equalization of the static system.

2. PROBLEM FORMULATION

Throughout the present paper, let us consider the following MIMO static system with n inputs and m outputs:

$$\boldsymbol{y}(t) = \boldsymbol{H}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{1}$$

where $\boldsymbol{y}(t)$ represents an m-column output vector called the *observed signal*, $\boldsymbol{s}(t)$ represents an n-column input vector called the *source signal*, $\boldsymbol{H}$ is an $m \times n$ matrix, $\boldsymbol{n}(t)$ represents an m-column noise vector.

To attain the equalization of the system (1), the following n filters, which are m-input single-output (MISO) systems driven by the observed signals, are used:

$$z_l(t) = \boldsymbol{w}_l^T \boldsymbol{y}(t), \quad l = 1, 2, \cdots, n, \tag{2}$$

where $z_l(t)$ is the output of the l-th filter and $\boldsymbol{w}_l = [w_{l1},\cdots,w_{lm}]^T$ is an m-column vector. Substituting (1) into (2), we obtain

$$\begin{aligned} z_l(t) &= \boldsymbol{w}_l^T \boldsymbol{H}\boldsymbol{s}(t) + \boldsymbol{w}_l^T \boldsymbol{n}(t), \\ &= \boldsymbol{g}_l^T \boldsymbol{s}(t) + \boldsymbol{w}_l^T \boldsymbol{n}(t) \quad l = 1, 2, \cdots, n, \end{aligned} \tag{3}$$

where $\boldsymbol{g}_l = [g_{l1},g_{l2},\cdots,g_{ln}]^T := \boldsymbol{H}^T \boldsymbol{w}_l$ is an n-column vector. The blind equalization problem considered in the present paper can be formulated as follows: Find a filter $\boldsymbol{w}_l$ denoted by $\tilde{\boldsymbol{w}}_l$ satisfying the following condition, without the knowledge of $\boldsymbol{H}$, even if the Gaussian noise $\boldsymbol{n}(t)$ is added to the observed signal $\boldsymbol{y}(t)$,

$$\tilde{\boldsymbol{g}}_l = \boldsymbol{H}^T \tilde{\boldsymbol{w}}_l = \tilde{\boldsymbol{\delta}}_l, \quad l = 1, 2, \cdots, n, \tag{4}$$

where $\tilde{\boldsymbol{\delta}}_l$ is an n-column vector whose elements $\tilde{\delta}_{lr}$ ($r = 1, 2, \cdots, n$) are zero expect for p_lth element, that is,

$$\tilde{\delta}_{lr} = d_l \delta(r - p_l), \quad r = 1, 2, \cdots, n, \tag{5}$$

Here, $\delta(t)$ is the Kronecker delta function, d_l is a number standing for a scale change, and each p_l is equal to one of the integers $1, 2, \cdots, n$, and all p_l's are distinct, which means that $\{p_1,p_2,\cdots,p_n\}$ is a permutation of the integers $1, 2, \cdots, n$.

To solve the blind equalization problem, we put the following assumptions on the system and the source signals.

A1) The matrix $\boldsymbol{H}$ in (1) is an $m \times n$ ($m \geq n$) matrix and has full column rank.

A2) The input sequence $\{\boldsymbol{s}(t)\}$ is a zero-mean, non-Gaussian vector process whose element processes $\{s_i(t)\}$, $i = 1, 2, \cdots, n$, be mutually independent. Moreover, each element process $\{s_i(t)\}$ is an independent and identically distributed (i.i.d.) process with nonzero variance $\sigma_{s_i}^2 \neq 0$ and nonzero $(p+1)$st-order cumulants, κ_i defined as

$$\kappa_i = \text{cum}\{\underbrace{s_i(t), s_i(t), \cdots, s_i(t)}_{p+1}\} \neq 0, \tag{6}$$

where i=1,2,$\cdots$,n and $p \geq 2$.

A3) The noise signal sequence $\{\boldsymbol{n}(t)\}$ is a zero-mean, Gaussian vector stationary process whose element processes $\{n_i(t)\}$, $i = 1,2,\cdots,m$, are mutually statistically independent.

A4) The two vector sequences $\{\boldsymbol{n}(t)\}$ and $\{\boldsymbol{s}(t)\}$ are mutually statistically independent.

It is assumed for the sake of simplicity in the present paper that all the signals and all the systems are real-valued.

3. SUPER-EXPONENTIAL METHODS

3.1 Two-step iteration procedure for vector $\boldsymbol{g}_l$

To find the solutions in (4), the following two-step iterative procedure with respect to the elements g_{lj} $j = 1, 2, \cdots, n$ of the vector $\boldsymbol{g}_l$ is used:

$$g_{lj}^{[1]} = \frac{\kappa_j}{a_j \gamma_j} g_{lj}^p, \qquad j = 1, 2, \cdots, n, \tag{7}$$

$$g_{lj}^{[2]} = g_{lj}^{[1]} / \sqrt{\sigma_{z_l}^2}, \qquad j = 1, 2, \cdots, n, \tag{8}$$

where $(\cdot)^{[1]}$ and $(\cdot)^{[2]}$ stand for the result of the first step and the result of the second step per iteration, p is nonnegative integer, a_j denotes a positive number (in subsection 3.2, it will be shown how we choose the values of a_j's), γ_j denotes the fourth-order cumulant of $s_j(t)$, that is, γ_j is equal to κ_j in the case of $p = 3$, and $\sigma_{z_l}^2$ denotes the variance of the output signal $z_l(t)$.

The above two-step procedure becomes one cycle of iterations in the super-exponential method [1, 4, 5, 6, 10]. In the conventional methods (e.g., [1, 4, 5, 6, 10]), the denominator of the right-hand side of (7) was set to 1 or the variance of $s_j(t)$, whereas we consider the fourth-order cumulant of $s_j(t)$, i.e., γ_j. Therefore (7) becomes insensitive to Gaussian noise. This is a *novel key point* of our proposed super-exponential method.

Let $g_{lj}(k)$ denotes the value obtained in the k-th cycle of the iterations of two steps (7) and (8). The important fact of the two-step procedure is that the n values $g_{lj}(k)$ $(j = 1, 2, \cdots, n)$ converge to zero except for only one of the values as the iteration number k approaches infinity, that is, $k \to \infty$. This will be shown in the following theorem.

Theorem 1 *Let $g_{lj}(0)$ be an initial value for iterations of two steps (7) and (8) for each $j = 1, 2, \cdots, n$. Let α_j be a non-negative scalar defined as*

$$\alpha_j = |\frac{\kappa_j}{a_j \gamma_j}|^{\frac{1}{p-1}}. \tag{9}$$

Let j_0 be $j_0 = \arg\max_{j \in \{1,2,\cdots,n\}} \alpha_j |g_{lj}(0)|$. Suppose the index j_0 is unique, that is, $\alpha_{j_0}|g_{lj_0}(0)| > \alpha_j |g_{lj}(0)|$ for any other $j \in \{1, 2, \cdots, n\}$, then as $k \to \infty$, it follows

$$\lim_{k \to \infty} |g_{lj}(k)| = \begin{cases} 0 & \text{for } j \neq j_0, \\ \tilde{d}_j \neq 0 & \text{for } j = j_0, \end{cases} \tag{10}$$

where $\tilde{d}_j$ is a scalar positive constant.

Proof: The proof is omitted for page limit, but will be found in a forthcoming paper.

3.2 Two-step iterative procedure for equalizer vector $\boldsymbol{w}_l$

In (7) and (8), since the parameters g_{lj}'s include the unknown parameters h_{ij}'s, the two-step procedure cannot be handled directly. Therefore, by solving the following weighted least squares problem, we derive an algorithm with respect to $\boldsymbol{w}_l$ so that the two steps (7) and (8) can be handled indirectly.

$$\min_{\boldsymbol{w}_l} (\boldsymbol{H}^T \boldsymbol{w}_l - \boldsymbol{g}_l)^T \boldsymbol{\Lambda} (\boldsymbol{H}^T \boldsymbol{w}_l - \boldsymbol{g}_l), \quad l = 1, 2, \cdots, n \tag{11}$$

Here, $\boldsymbol{\Lambda}$ is a diagonal matrix with positive diagonal elements. The solutions are known to be given by

$$\boldsymbol{w}_l = (\boldsymbol{H} \boldsymbol{\Lambda} \boldsymbol{H}^T)^{\dagger} \boldsymbol{H} \boldsymbol{\Lambda} \boldsymbol{g}_l, \qquad l = 1, 2, \cdots, n, \tag{12}$$

where $\dagger$ denotes the pseudo-inverse operation of a matrix. In the conventional methods [1, 4, 5, 6, 10], the positive diagonal elements of $\boldsymbol{\Lambda}$ are set to 1 or the variances of the source signals. This means that $\boldsymbol{H}\boldsymbol{\Lambda}\boldsymbol{H}^T$ is calculated by the second-order statistics of the observed signal $\boldsymbol{y}(t)$. We consider that this is the reason why the conventional methods are sensitive to Gaussian noise.

In what follows, we shall show that the weighted least squares approach in (11) can be applied to a set of fourth-order cumulants of the observe signals $y_i(t)$ $(i = 1, 2, \cdots, m)$, if we choose appropriately a diagonal matrix $\boldsymbol{\Lambda}$ in (11). To this end, we introduce fourth-order cumulants matrices of m-vector random process $\{\boldsymbol{y}(t)\}$ [8], which constitute a set of $m \times m$ matrices $\boldsymbol{C}_{\boldsymbol{y},i,j}^{(4)}$ $(i, j = 1, 2, \cdots, m)$ defined by

$$\boldsymbol{C}_{\boldsymbol{y},i,j}^{(4)} = [\mathrm{cum}\{y_p(t), y_q(t), y_i(t), y_j(t)\}]_{p,q}, \tag{13}$$

where $[x]_{p,q}$ denotes the (p, q)th element of the matrix $\boldsymbol{C}_{\boldsymbol{y},i,j}^{(4)}$. Then we consider an $m \times m$ matrix $\tilde{\boldsymbol{R}}$ expressed by

$$\tilde{\boldsymbol{R}} = \textstyle\sum_{i,j=1}^{m} \beta_{ij} \boldsymbol{C}_{\boldsymbol{y},i,j}^{(4)}, \tag{14}$$

where β_{ij}'s are either 1 or 0, which represent *design parameters*. It is shown by a simple calculation that (14) becomes $\tilde{\boldsymbol{R}} = \boldsymbol{H}\tilde{\boldsymbol{\Lambda}}\boldsymbol{H}^T$, where $\tilde{\boldsymbol{\Lambda}}$ is a diagonal matrix defined by

$$\tilde{\boldsymbol{\Lambda}} := \mathrm{diag}\{\tilde{a}_1 \gamma_1, \tilde{a}_2 \gamma_2, \cdots, \tilde{a}_n \gamma_n\} \tag{15}$$

$$\tilde{a}_r := \textstyle\sum_{i,j=1}^{m} \beta_{ij} h_{ir} h_{jr}, \qquad r = 1, 2, \cdots, n, \tag{16}$$

and $\mathrm{diag}\{\cdots\}$ denotes a diagonal matrix with diagonal elements built from its arguments.

Here we note that the diagonal matrix $\tilde{\boldsymbol{\Lambda}}$ is not positive semi-definite but the diagonal matrix $\hat{\boldsymbol{\Lambda}}$ defined by

$$\hat{\boldsymbol{\Lambda}} := \mathrm{diag}\{|\tilde{a}_1 \gamma_1|, |\tilde{a}_2 \gamma_2|, \cdots, |\tilde{a}_n \gamma_n|\} \tag{17}$$

is positive semi-definite. It is clear from the definitions (15) and (17) that there exists a sign matrix $\dot{\boldsymbol{I}}$ such that $\tilde{\boldsymbol{\Lambda}} := \hat{\boldsymbol{\Lambda}}\dot{\boldsymbol{I}}$, where the sign matrix $\dot{\boldsymbol{I}}$ is defined as a diagonal matrix whose diagonal elements are either +1 or −1.

Remark 1 *If $\hat{\boldsymbol{\Lambda}}$ is full rank, then, by putting $\boldsymbol{\Lambda}$ in (11) to $\hat{\boldsymbol{\Lambda}}$, the solution $(\boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{H}^T)^{\dagger}\boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{g}_l$ can be obtained from (11). However, $\boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{H}^T$ cannot be calculated from (14), that is, $\tilde{\boldsymbol{R}} \neq \boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{H}^T$.*

Here, we show the following theorem.

Theorem 2 *If $\boldsymbol{H}$ is of full column rank and both $\hat{\boldsymbol{\Lambda}}$ and $\tilde{\boldsymbol{\Lambda}}$ are of full rank, then*

$$(\boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{H}^T)^\dagger\boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{g}_l = (\boldsymbol{H}\tilde{\boldsymbol{\Lambda}}\boldsymbol{H}^T)^\dagger\boldsymbol{H}\tilde{\boldsymbol{\Lambda}}\boldsymbol{g}_l, \qquad (18)$$

where $\tilde{\boldsymbol{\Lambda}} := \hat{\boldsymbol{\Lambda}}\dot{\boldsymbol{I}}$.

Proof: Let the left-hand and the right-hand sides of (18) be denoted by $\hat{\boldsymbol{w}}_l$ and $\tilde{\boldsymbol{w}}_l$, respectively.

Since $\boldsymbol{H}$ has full column rank, using a property of the pseudo-inverse operation ([3], p. 433), we obtain

$$\begin{aligned}\hat{\boldsymbol{w}}_l &:= (\boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{H}^T)^\dagger\boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{g}_l = \boldsymbol{H}^{T\dagger}(\boldsymbol{H}\hat{\boldsymbol{\Lambda}})^\dagger\boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{g}_l\\ &= \boldsymbol{H}^{T\dagger}\hat{\boldsymbol{\Lambda}}^{-1}\boldsymbol{H}^\dagger\boldsymbol{H}\hat{\boldsymbol{\Lambda}}\boldsymbol{g}_l = \boldsymbol{H}^{T\dagger}\boldsymbol{g}_l, \qquad (19)\end{aligned}$$

where the fourth equality comes from the fact that $\boldsymbol{H}^\dagger\boldsymbol{H} = \boldsymbol{I}$ because $\boldsymbol{H}$ is of full column rank. From (19) and $\boldsymbol{H}^\dagger\boldsymbol{H} = \boldsymbol{I}$, we have

$$\begin{aligned}\boldsymbol{H}^{T\dagger}\boldsymbol{g}_l &= \boldsymbol{H}^{T\dagger}\tilde{\boldsymbol{\Lambda}}^{-1}\boldsymbol{H}^\dagger\boldsymbol{H}\tilde{\boldsymbol{\Lambda}}\boldsymbol{g}_l\\ &= (\boldsymbol{H}\tilde{\boldsymbol{\Lambda}}\boldsymbol{H}^T)^\dagger\boldsymbol{H}\tilde{\boldsymbol{\Lambda}}\boldsymbol{g}_l = \tilde{\boldsymbol{w}}_l, \qquad (20)\end{aligned}$$

where the last equality comes from the definition of $\tilde{\boldsymbol{w}}_l$. The reverse, which $\hat{\boldsymbol{w}}_l$ can be derived from $\tilde{\boldsymbol{w}}_l = (\boldsymbol{H}\tilde{\boldsymbol{\Lambda}}\boldsymbol{H}^T)^\dagger\boldsymbol{H}\tilde{\boldsymbol{\Lambda}}\boldsymbol{g}_l$, can also be shown in the same way. Therefore, both $\hat{\boldsymbol{w}}_l$ and $\tilde{\boldsymbol{w}}_l$ are identical.

Remark 2 *If $\boldsymbol{H}$ is not of full column rank, Theorem 2 does not hold. Because, in such a case, $\boldsymbol{H}^T\boldsymbol{H}$ does not become a nonsingular matrix. Moreover, it can be seen from (19) and (20) that $\hat{\boldsymbol{w}}_l$ and $\tilde{\boldsymbol{w}}_l$ are respectively irrelevant to $\hat{\boldsymbol{\Lambda}}$ and $\tilde{\boldsymbol{\Lambda}}$, that is, Theorem 2 holds for any pair of full rank diagonal matrices. In fact, $\hat{\boldsymbol{w}}_l = \tilde{\boldsymbol{w}}_l = \boldsymbol{H}^{T\dagger}\boldsymbol{g}_l$ shown in (19) and (20) attains the zero minimum value of the weighted least squares function in (11) for any diagonal positive definite matrix. In general, the right-hand side of (18) is always expressed by the fourth-order cumulants or fourth- and higher-order cumulants of $\{\boldsymbol{y}(t)\}$.*

From Theorem 2 and Remark 2, it is seen that the right-hand side of (12) can be given by the right-hand side of (18) under the condition that the diagonal matrix $\tilde{\boldsymbol{\Lambda}}$ ($=\hat{\boldsymbol{\Lambda}}\dot{\boldsymbol{I}}$) is of full rank. This condition, however, will be satisfied by the following theorem.

Theorem 3 *Let $\boldsymbol{H}$ be of full column rank and γ_i ($i = 1, 2, \cdots, n$) be nonzero for all i. Suppose that $\beta_{ij} = 1$ for $i = j$ and $\beta_{ij} = 0$ for $i \neq j$ (see (14)), then the diagonal matrix $\tilde{\boldsymbol{\Lambda}}$ in (15) becomes full rank.*

Proof: The proof is omitted for page limit, but will be found in a forthcoming paper.

Note that if β_{ij} in (14) is 1 for $i = j$ and 0 for $i \neq j$, then $\tilde{a}_r$'s of $\tilde{\boldsymbol{\Lambda}}$ in (15) become

$$\tilde{a}_r = \textstyle\sum_{i=1}^m h_{ir}^2 \qquad r = 1, 2, \cdots, n. \qquad (21)$$

For the time being, in the present paper, we consider (14) with $\beta_{ij} = 1$ for $i = j$ and $\beta_{ij} = 0$ for $i \neq j$. As for $\boldsymbol{H}\tilde{\boldsymbol{\Lambda}}\boldsymbol{g}_l$, by using (7) with $a_j = \tilde{a}_j$ in (21), it can be calculated by $\boldsymbol{d}_l := [d_{l1}, d_{l2}, \cdots, d_{lm}]^T$, where d_{lj} is given by $\mathrm{cum}\{\underbrace{z_l(t), z_l(t), \cdots, z_l(t)}_{p}, y_j(t)\}$. Then (12) can be expressed as

$$\boldsymbol{w}_l^{[1]} := \tilde{\boldsymbol{R}}^\dagger\boldsymbol{d}_l, \qquad l = 1, 2, \cdots, n, \qquad (22)$$

Since the second step (8) is a normalization of $\boldsymbol{g}_l$, it is easily shown that the second step reduces to

$$\boldsymbol{w}_l^{[2]} := \boldsymbol{w}_l^{[1]}/\sqrt{\sigma_{z_l}^2}. \; l = 1, 2, \cdots, n, \qquad (23)$$

Therefore, (22) and (23) are our proposed two steps to modify $\boldsymbol{w}_l$.

3.3 The proposed SEM

For now, there are two approaches to multichannel (or MIMO) blind equalization, a *concurrent blind equalization approach* and a *deflationary blind equalization approach*. The former is to equalize all the channels concurrently, while the latter equalize sequentially (or iteratively with respect to source signals) the channels one by one. It is well known that iterative algorithms based on the former approach converge to a desired solution when they starts in a neighborhood of the desired solution while iterative algorithms based on the latter approach converge to a desired solution globally (or regardless of their initialization) [1]. The latter approach is employed in this paper. Let l denote the number of the channels (or the sources) equalized. At first, set $l = 1$, then $\tilde{\boldsymbol{w}}_1$ is calculated by the two steps (22) and (23) such that $\boldsymbol{H}^T\tilde{\boldsymbol{w}}_1 = \tilde{\boldsymbol{\delta}}_1 = [0,\cdots,0,1(p_1\text{th element}),0,\cdots,0]^T$. The contribution signals $c_{ip_1}(t) = h_{ip_1}s_{p_1}(t)$ ($i = 1, 2, \cdots, m$) are calculated next by using the output signal $z_1(t) = \tilde{\boldsymbol{w}}_1^T\boldsymbol{y}(t)$. Then, by calculating $y_i(t) - c_{ip_1}(t)$ for $i = 1, 2\cdots, m$, we remove the contribution signals from the outputs in order to define the outputs of a multichannel system with $n-1$ inputs and m outputs. The number of inputs becomes deflated by one. The procedures mentioned above are continued until $l = n$. Therefore, the proposed RSEM is summarized as shown in Table 1.

Table 1: The proposed method.

Step	Contents
1	Set $l = 1$ (where l denotes the number of the channels equalized).
2	Choose random initial value $\boldsymbol{w}_l(0)$, where $\boldsymbol{w}_l(0)$ denotes the initial value of $\boldsymbol{w}_l$. Set k in $\boldsymbol{w}_l(k)$ to 0 (k denotes the iteration number).
3	Calculate (2).
4	Calculate $\boldsymbol{w}_l(k)$ using (22) and (23).
5	After $k \to \infty$, $\boldsymbol{d}_l$ is calculated by using $\tilde{z}_l(t)$ which is estimated by using $\boldsymbol{w}_l(\infty)$.
6	Calculate $\tilde{\boldsymbol{y}}(t) = \boldsymbol{y}_l(t) - (\boldsymbol{d}_l/\kappa_{\tilde{z}})\tilde{z}_l(t)$, where $\boldsymbol{y}_l(t)$ is the observed signal in number l and $\kappa_{\tilde{z}}$ is the $(p+1)$st-order cumulant of $\tilde{z}_l(t)$.
7	If the subscript l of $\boldsymbol{y}_l(t)$ is less then n, then set $\boldsymbol{y}(t) = \tilde{\boldsymbol{y}}(t)$, $l = l + 1$, and the procedures (Step 2 to Step 6) are continued until $l = n$.

The procedure from Step 5 to Step 7 are implemented for making it possible to obtain all the solutions in (4). In Step 6, the calculation of $\boldsymbol{y}_l(t) - (\boldsymbol{d}_l/\kappa_{\tilde{z}})\tilde{z}_l(t)$ is equivalent to the calculations of $y_i(t) - c_{ip_1}(t)$ $(i = 1,2,\cdots,n)$ mentioned above. (On the details of Step 6, see [5].)

4. SIMULATION RESULTS

To demonstrate the validity of the proposed SEM, many computer simulations were conducted. One of the results is shown in this section. We considered a two-input and three-output system, that is, $\boldsymbol{H}$ in (1) was set to

$$\boldsymbol{H} = \begin{bmatrix} 1.0 & 0.6 \\ 0.7 & 1.0 \\ 0.2 & 0.5 \end{bmatrix}. \quad (24)$$

Two source signals $s_1(t)$ and $s_2(t)$ were sub-Gaussian and super-Gaussian, respectively, in which $s_1(t)$ takes one of two values, -1 and 1 with equal probability 1/2, $s_2(t)$ takes one of three values, -2, 0 and 2 with probability 1/8, 6/8, and 1/8, respectively, and they are zero-mean and unit variance. The parameter p in (6) was set to $p = 3$, that is, κ_j's $(j = 1,2)$ in (7) were the fourth-order cumulants γ_j's of the source signals. These values were set to $\gamma_1 = -2$ and $\gamma_2 = 1$. Three independent Gaussian noises (with identical variance σ_w^2) were added to the three outputs $y_i(t)$'s at various SNR levels. The SNR is, for convenience, defined as SNR := $10\log_{10}(\sigma_i^2/\sigma_w^2)$, where σ_i^2's are the variances of $s_i(t)$'s and are equal to 1. As a measure of performance, we used the *multichannel intersymbol interference* (MISI) defined in the logarithmic (dB) scale by

$$\mathrm{MISI} = 10\log_{10}\Big[\sum_{l=1}^{n}\frac{|\sum_{j=1}^{n}|g_{lj}|^2 - |g_{l\cdot}|^2_{max}|}{|g_{l\cdot}|^2_{max}} + \sum_{j=1}^{n}\frac{|\sum_{l=1}^{n}|g_{lj}|^2 - |g_{\cdot j}|^2_{max}|}{|g_{\cdot j}|^2_{max}}\Big], \quad (25)$$

where $|g_{l\cdot}|^2_{max}|$ and $|g_{\cdot j}|^2_{max}$ are respectively defined by $|g_{l\cdot}|^2_{max}| := \max_{j=1,\cdots,n}|g_{lj}|^2$ and $|g_{\cdot j}|^2_{max} := \max_{l=1,\cdots,n}|g_{lj}|^2$. As a conventional method, the method proposed in [5] was used for comparison.

Figure 1 shows the results of performances for the proposed SEM and the conventional SEM when the SNR level was taken to be 0[dB] $(\sigma_w^2 = 1)$, 2.5[dB], 5[dB], 10[dB], 15[dB], and ∞ [dB] $(\sigma_w^2 = 0)$, in which each MISI shown in Fig.1 was the average of the performance results obtained by 10 independent Monte Carlo runs. In each Monte Carlo run, the number of the integers k's in Step 4 (see Table 1) was 10, in which $\tilde{\boldsymbol{R}}$ and $\boldsymbol{d}_l$ were estimated by 10,000 data samples. It can be seen from Fig.1 that our proposed SEM is robust to Gaussian noise and implement successfully the equalization of the system.

5. CONCLUSIONS

We have proposed a deflationary SEM for solving blind equalization problem, in which the solutions of the problem, $\tilde{\boldsymbol{w}}_l$'s satisfying (4) are found one by one. The proposed SEM is not sensitive to Gaussian noise, which is referred to as a robust super-exponential method (RSEM). This is a novel property of the proposed method, whereas the conventional methods do not posses it. It was shown from the simulation results that the proposed RSEM was robust to Gaussian noise and could successfully solve the blind equalization problem.

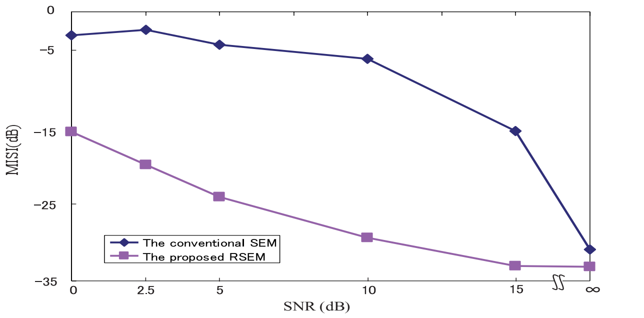

Figure 1: The performances for the proposed SEM and the conventional SEM.

REFERENCES

[1] Y. Inouye and K. Tanebe, "Super-exponential algorithms for multichannel blind deconvolution," *IEEE Trans. Signal Processing*, vol. 48, no. 3, pp. 881-888, Mar. 2000.

[2] M. Kawamoto and Y. Inouye, "A deflation algorithm for the blind source-factor separation of MIMO-FIR systems driven by colored sources," *IEEE Signal Processing Letters*, vol. 10, no. 11, pp. 343-346, Nov. 2003.

[3] P. Lancaster and M. Tismenetsky, *The Theory of Matrices*, second edition, Academic Press, INC., 1985.

[4] M. Martone, "An adaptive algorithm for antenna array Low-Rank processing in cellular TDMA base stations," *IEEE Trans. Commun.*, vol. 46, no. 5, pp. 627-643, May 1998.

[5] M. Martone, "Fast adaptive super-exponential multistage beamforming cellular base-station transceivers with antenna arrays," *IEEE Trans. Vehicular Tech.*, vol. 48, no. 4, pp. 1017-1028, Jul. 1999.

[6] O. Shalvi and E. Weinstein, "Super-exponential methods for blind deconvolution," *IEEE Trans. Inform. Theory*, vol. 39, no. 2, pp. 504-519, Mar. 1993.

[7] C. Simon, P. Loubaton, and C. Jutten, "Separation of a class of convolutive mixtures: a contrast function approach," *Signal Processing*, vol. 81, pp.883-887, 2001.

[8] L. Tong, Y. Inouye, and R.-w. Liu, "Waveform-preserving blind estimation of multiple independent sources," *IEEE Trans. Signal Processing*, vol. 41, no. 7, pp. 2461-2470, Jul. 1993.

[9] J. K. Tugnait, "Identification and deconvolution of multichannel non-Gaussian processes using higher order statistics and inverse filter criteria," *IEEE Trans. Signal Processing*, vol. 45, no. 3, pp. 658-672, Mar. 1997.

[10] K. L. Yeung and S. F. Yau, "A cumulant-based super-exponential algorithm for blind deconvolution of multi-input multi-output systems," *Signal Processing*, vol. 67, pp. 141-162, 1998.