Echo Cancellation Using a Variable Step-Size NLMS Algorithm

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Abstract

In this work, an echo cancellation scheme using a variable step-size Normalized Least Mean-Square (VSS-NLMS) adaptive algorithm is proposed. This work shows that the use of the VSS-NLMS algorithm will eliminate much of the trade-off between residual error and speed of convergence existing with the fixed step-size NLMS algorithm and therefore resulting in an improved performance.

1 Introduction

Because of its simplicity, the Least Mean Square (LMS) algorithm [1] is the most popular adaptive algorithm. However, the LMS algorithm suffers from slow and data-dependent convergence behavior. The NLMS algorithm [1]-[2], an equally simple, but more robust variant of the LMS algorithm, exhibits a better balance between simplicity and performance than the LMS algorithm, and has been given more attention in real time applications.

A very serious problem, however, encountered in both the LMS and the NLMS algorithms, is the choice of the step size parameter that is a trade-off between the steady-state excess error and the speed of convergence. To remedy this problem, several works have discussed variable step-size LMS algorithms [3]-[6]. In the same context, a study on the improvement of the performance of the NLMS algorithm is worth investigating. In [8], a variable step-size NLMS algorithm (VSS-NLMS) is proposed, where the convergence and steady-state analysis of the VSS-NLMS algorithm is detailed. In this work, echo cancellation using the variable step-size NLMS (VSS-NLMS) algorithm in [8] is presented.

2 Proposed VSS-NLMS Echo Canceller

Given the input vector $\mathbf{x}_k$, the Euclidean norm of the input vector $\|\mathbf{x}_k\|^2$, the NLMS algorithm with fixed step size, μ, for adjusting the adaptive echo canceller's coefficients at time instant k is defined as follows:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu e_k \frac{\mathbf{x}_k}{\|\mathbf{x}_k\|^2}\ , \tag{1}$$

where the error e_k is defined as $e_k = d_k + n_k - \mathbf{x}_k^T \mathbf{w}_k$, d_k is the desired value and n_k is the additive noise. In this work, the fixed step size μ in (1) is made variable and is updated according to the following recursion [7]:

$$\mu_k = \mu_{k-1} - \frac{\rho}{2}\frac{\partial e_k^2}{\partial \mu_{k-1}}, \tag{2}$$

which can be transformed, after substituting Equation (1), to the form:

$$\mu_k = \mu_{k-1} + \rho e_k e_{k-1} \frac{\mathbf{x}_k^T \mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2}\ , \tag{3}$$

where the parameter ρ is a small positive constant that controls the adaptive behavior of the step-size sequence μ_k and T denotes transpose operation. Accordingly, the coefficients of the VSS-NLMS echo canceller will be updated according to a variable step-size NLMS (VSS-NLMS) algorithm given by [8]:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu_k e_k \frac{\mathbf{x}_k}{\|\mathbf{x}_k\|^2}\ , \tag{4}$$

where the variable step size parameter is confined to the following limits [8]:

$$\mu_k = \begin{cases} \mu_{max} & \text{if} \ \ \mu_k > \mu_{max} \\ \mu_{min} & \text{if} \ \ \mu_k < \mu_{min} \\ \mu_k & \text{otherwise,} \end{cases} \tag{5}$$

and μ_{min}, μ_{max} are chosen to satisfy the convergence requirements of the NLMS algorithm with fixed step size, that is $0 < \mu_{min} < \mu_{max} < 2$.

3 Performance Analysis

Let $\mathbf{w}_k^*$ denote the optimal coefficient vector being tracked, and assume it is time varying according to $\mathbf{w}_{k+1}^* = \mathbf{w}_k^* + \mathbf{c}_k$, where $\mathbf{c}_k$ is the disturbance process that is a zero-mean white process with covariance matrix $\sigma_c^2\mathbf{I}$. Also, let ζ_k be the optimum estimation error process defined as:

$$\zeta_k = d_k - \mathbf{x}_k^T\mathbf{w}_k^*. \tag{6}$$

Finally, let $\mathbf{v}_k = \mathbf{w}_k - \mathbf{w}_k^*$ denote the coefficient misalignment vector. Then it is straight forward to see that:

$$E[e_k^2] = \xi_{min} + \sigma_x^2 tr[\mathbf{G}_k] \ , \tag{7}$$

where $\xi_{min} = E[\zeta_k^2]$, $\mathbf{G}_k = E[\mathbf{v}_k\mathbf{v}_k^T]$ and $tr[\ ]$ are the minimum mean-square-error (MSE), the second moment matrix of the misalignment vector and the trace operator, respectively.

A. Mean Behavior of the Weight Vector:

In what follows, the derivations of the mean behavior of the weight vector for the VSS-NLMS algorithm are presented. Starting with the error, e_k, the expression in (5), can be re-written as:

$$e_k = \zeta_k - \mathbf{v}_k^T\mathbf{x}_k, \tag{8}$$

and using the independence assumption, the uncorrelatedness of μ_k with $\mathbf{x}_k$, ζ_k, and the fact that $\mathbf{c}_k$ is zero mean [3], the recursion for the mean behavior of the weight vector is given by:

$$E[\mathbf{v}_{k+1}] = \left[\mathbf{I} - E[\mu_k]E[\frac{\mathbf{x}_k\mathbf{x}_k^T}{\|\mathbf{x}_k\|^2}]\right]E[\mathbf{v}_k], \tag{9}$$

where it is easy to show that the convergence of the algorithm in the mean takes place if the average value of the step size parameter is confined in the following range:

$$0 < E[\mu_k] < 2. \tag{10}$$

B. Mean-Square Behavior of the Weight Vector:

In this section, the mean-square behavior of the weight vector of the VSS-NLMS algorithm is presented. Following similar procedure as above, the second moment matrix of the coefficient misalignment vector is given by:

$$\begin{aligned}\mathbf{G}_{k+1} \quad &= \quad \left[1 - \frac{2}{N}E[\mu_k] + \frac{2}{N}E[\mu_k^2]\right]\mathbf{G}_k \\ &\quad + \left[\frac{\sigma_{e_k}^2}{N\sigma_x^2}E[\mu_k^2] + \sigma_c^2\right]\mathbf{I},\end{aligned} \tag{11}$$

where it is easy to notice that $\mathbf{G}_k$ is a diagonal matrix and all its elements are equal. Notice also that $\sigma_{e_k}^2 = E[e_k^2]$. Similarly, the following expressions for the mean and mean-square behavior of the step-size sequence μ_k, respectively, are obtained as:

$$\begin{aligned}E[\mu_k] \quad &= \quad \left[1 - \rho(\sigma_{e_{k-1}}^2 + \frac{2\sigma_x^2}{N}tr[\mathbf{G}_{k-1}])\right]E[\mu_{k-1}] \\ &\quad +\rho\frac{\sigma_x^2}{N}tr[\mathbf{G}_{k-1}],\end{aligned} \tag{12}$$

and

$$\begin{aligned}E[\mu_k^2] \quad &= \quad \left[1 - \frac{2\rho}{N}(N\sigma_{e_{k-1}}^2 + 2\sigma_x^2 tr[\mathbf{G}_{k-1}])\right]E[\mu_{k-1}^2] \\ &\quad +\frac{2\rho}{N}E[\mu_{k-1}]\sigma_x^2 tr[\mathbf{G}_{k-1}] + \rho^2 tr\left[\frac{1}{N}(\sigma_{e_k}^2\mathbf{I}\right. \\ &\quad \left. +2\sigma_x^2\mathbf{G}_k)(\sigma_{e_{k-1}}^2\mathbf{I} + 2\sigma_x^2\mathbf{G}_{k-1})\right].\end{aligned} \tag{13}$$

4 Simulation results

In this section, the performance of an echo cancellation system using the proposed variable step-size NLMS (VSS-NLMS) algorithm is compared to that using the NLMS algorithm with fixed step size. The transfer function of the echo path is modeled as $(0 < a < 1)$:

$$H(z) = 1 + az^{-1} + \cdots a^{N-1}z^{-\{N-1\}}. \tag{14}$$

Two models for the impulse response of the echo path was used: The first model, assumed for short echo path, is significant up to the first 100 samples, while the second model, assumed for relatively long echo path, is significant up to the first 500 samples. The coefficient a in both cases is chosen in such a way that the power level of the impulse response will be attenuated by $60\ dB$ at 100 and 500 samples respectively. Two different analysis are considered for each of these models. One uses Gaussian input signals while the other uses real-time speech signals. The speech signal is sampled at sampling frequency Fs=11025Hz, and digitized at 8 bits/sample. The amplitude values of the input vector in both examples are in the range -1, +1.

The initial step size for the VSS-NLMS algorithm is $\mu_0 = 0.04$ while for the NLMS algorithm $\mu = 0.04$.Also, the value of the step size adaptation constant ρ used for the VSS-NLMS algorithm is $\rho = 8\text{x}10^{-4}$. Time averaging of 50 independent runs was used, and the measure of performance used is the echo return loss enhancement (ERLE) defined as [6]:

$$ERLE = 10log_{10}\frac{E[y_k^{*2}]}{E[\{y_k^* - y_k\}^2]} \ \ dB \ ,$$

where $y_k^* = \mathbf{x}_k^T\mathbf{w}^*$ is the true echo, $\mathbf{w}^*$ is the impulse response of the echo path, and $y_k = \mathbf{x}_k^T\mathbf{w}$ is the simulated one.

Example 1: Analysis with real-time Speech Signals

The experiments under this consideration use real-time speech signal digitized as described above. Figure 1 displays the digitized speech signal and its frequency spectrum. Figure 2 depicts the ERLE, using this signal, for the VSS-NLMS algorithm and the NLMS algorithm with fixed step size for the case when both algorithms track an echo path of length, $N = 100$. The same scenario for for longer echo path of length $N = 500$

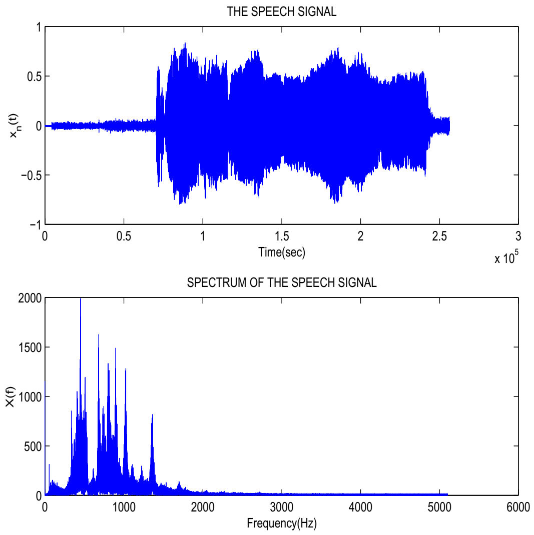

Figure 1: Digitized speech signal and its frequency spectrum.

is shown in Fig. 3. The results show that the VSS-NLMS algorithm converges faster, and at the same time, achieves considerably higher ERLE than the NLMS algorithm with fixed step size. A gain of about $7.5\ dB$ and $1.5\ dB$ of ERLE, for $N = 500$ and $N = 100$ respectively, can be observed with the proposed algorithm over the performance of the NLMS algorithm with fixed step size. It is also note worthy to point out that for a real time application, echo paths can typically be few thousands taps long. The ERLE of the traditional NLMS algorithm is very low compared to that of the VSS-NLMS algorithm as can be observed from Fig.3.

Example 2: Analysis with Gaussian input Signals

Figure 4 depicts the ERLE, using Gaussian input Signals, for the VSS-NLMS algorithm and the NLMS algorithm with fixed step size for the case when both algorithms track an echo path of length, $N = 100$. The same scenario for for longer echo path of length $N = 500$ is shown in Fig. 5. The results also show that the VSS-NLMS algorithm converges faster, and at the same time, achieves considerably higher ERLE than the NLMS algorithm with fixed step size. A gain of about $6\ dB$ and $4\ dB$ of ERLE, for $N = 100$ and $N = 500$ respectively, can be observed with the proposed algorithm over the performance of the NLMS algorithm with fixed step size.

The step-size behavior of the VSS-NLMS and NLMS algorithm for echo path lengths of $N = 100$ and $N = 500$, respectively, are depicted in Figures 6 and 7. It can be observed from these figures that the step-size sequence increases very quickly immediately after initialization, and therefore the VSS-NLMS algorithm is able to converge faster than the NLMS algorithm as shown earlier. Also the step-size reduces to a lower value as

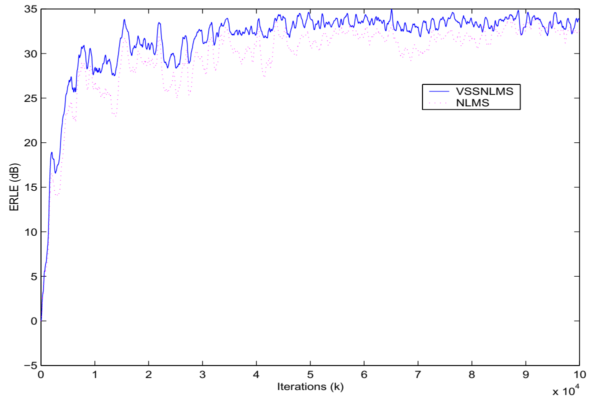

Figure 2: ERLE, using real-time speech signals, for the VSS-NLMS algorithm and the NLMS algorithm with fixed step size for echo path length $N = 100$.

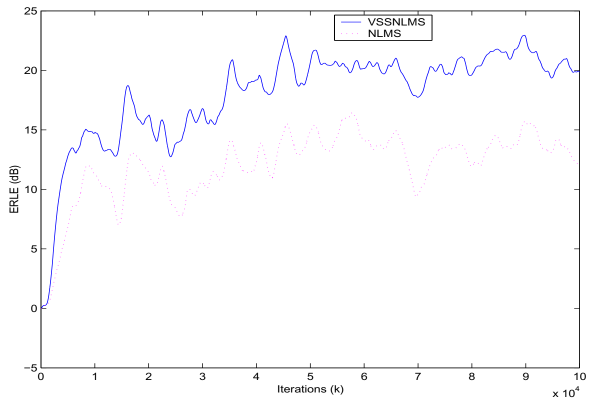

Figure 3: ERLE, using real-time speech signals, for the VSS-NLMS algorithm and the NLMS algorithm with fixed step size for echo path length $N = 500$.

the steady-state is approached, and therefore the proposed VSS-NLMS algorithm is able to converge to a lower steady-state error, and consequently better ERLE, than the NLMS algorithm with fixed step size.

5 Conclusion

In this work, an echo cancellation scheme using a simple and robust variable step-size NLMS (VSS-NLMS) algorithm is presented. The step-size adaptation is controlled by a gradient algorithm designed to minimize the squared estimation error. Simulation results showed that the VSS-NLMS algorithm achieved better ERLE than the NLMS algorithm with fixed step size. Lastly, we should mention that the proposed VSS-NLMS algorithm requires $(N + 40)$ extra multiplications/divisions and N extra additions in comparison to the NLMS algorithm.

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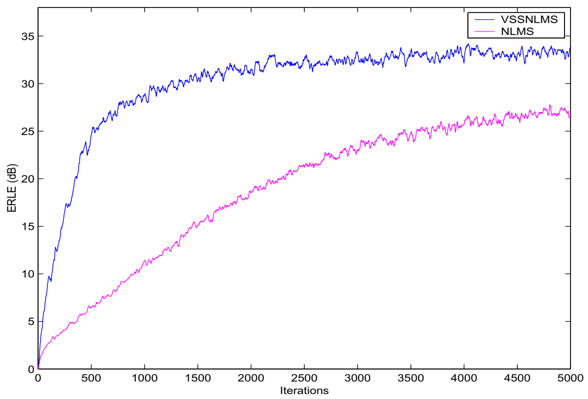

Figure 4: ERLE, using Gaussian input signals, for the VSS-NLMS algorithm and the NLMS algorithm with fixed step size for echo path length $N = 100$.

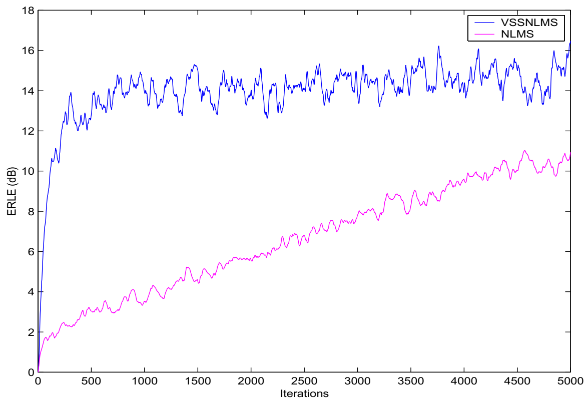

Figure 5: ERLE, using Gaussian input signals, for the VSS-NLMS algorithm and the NLMS algorithm with fixed step size for echo path length $N = 500$.

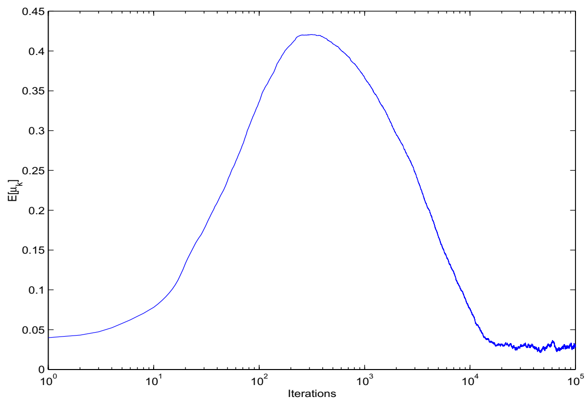

Figure 6: Mean behavior of μ_k for the VSS-NLMS algorithm for echo path length $N = 100$.

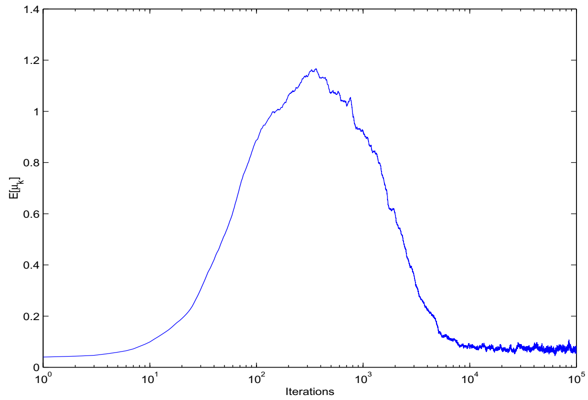

Figure 7: Mean behavior of μ_k for the VSS-NLMS algorithm for echo path length $N = 500$.

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