

NOISE REDUCTION FOR CHAOTIC SIGNALS BASED ON NEW APPROACH OF MEASURING THE SIGNAL DETERMINACY

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ABSTRACT

Noise reduction for noisy chaotic signals in real world situations is a challenging problem due to the lack of an effective way to measure the noise reduction performance. In this paper a new method for measuring the determinacy of chaotic signals based on the local correlation property of the chaotic dynamic system. The proposed method can be used to yield an improved local-projection algorithm. Numerical results are provided to show the efficiency of the new method.

1. INTRODUCTION

One widely used approach for analyzing chaotic dynamic system is phase space reconstruction method [1]. Based on embedding and shadowing properties some noise reduction methods for chaotic signals have been proposed [2][3]. However, these methods are usually effective for computer-produced data but seem inadequate for real world data. One major reason is that there is not an effective way to measure the noise reduction performance. In other words, it is very hard to compare the data before and after noise reduction due to the lack of priori knowledge of the dynamic system.

In real world situations, such as oceanic laser radar system, chaotic signals look more like noise than deterministic signals. Normal noise reduction methods designed for non-chaotic signals could not be applied to chaotic signals due to the fact that they are very similar in terms of their characteristics like waveforms, frequency spectra etc. In fact observations are mixture of chaotic signal and random noises, which makes it very difficult to identify real chaotic systems without the influence of noise. Therefore existing approaches are usually based on the assumption that the chaotic system are known *a priori* or the noise is very small compared to chaotic signals. The assumptions are obviously not true for the real world situations.

Local projection method [5] seems to be an effective technique for noise reduction. However, the proposed approach [5] could not be directly used for real world data because the stop condition is based on SNR which is not measurable for real world signals.

This paper aims to improve the noise reduction performance for chaotic signals by introducing a new way of measuring noise in chaotic signals. Since chaotic systems are deterministic in nature but noise is not, connections among chaotic signal samples must be much stronger than that among the noise signal samples. The proposed approach is based on the connection among signal samples in order to yield a metric for the determinacy of the signals. Because the metric represents the determinacy of the signals, it also gives the relative amount of noise as compared to the chaotic signals.

This paper is organized as follows. Section II gives a brief description on chaotic dynamic systems and conventional way for chaotic system analysis. In Section III a new way for measuring the noise in chaotic signals is proposed. A simple example of Lorenz system is provided to verify the use of D . In Section IV, an improved local-projection algorithm is presented. Finally Section V concludes the paper.

2. CHAOTIC SYSTEMS

Assume that we have an M -dimensional dynamic system, defined by a set of first order differential equations:

$$\dot{\vec{x}}(t) = f[\vec{x}(t)] \quad (1)$$

where $\vec{x}(t) = [x_1(t) \ x_2(t) \ \dots \ x_M(t)] \in R^M$ is the state vector at time t and $f : R^M \mapsto R^M$ describe the dynamics of the system;

The solution of the system can be obtained numerically by integrating the system with time steps t_n . Taking the i th state variable $x_i(t)$ as an example, we defined the variation as follows:

$$\delta x_i(t_{n+1}) = x_i(t_{n+1}) - x_i(t_n) \quad (2)$$

And the increment of the time steps is defined as follows:

$$\Delta_n = t_{n+1} - t_n \quad (3)$$

The increment Δ_n can be constant and defined *a priori* as Δ_0 . However, this fixed step size usually does not yield good estimation of the system. This is because the true system vector varies dynamically, that is, sometimes it change fast, and sometimes it changes slowly with time. It is desired that a small step size be used when the system varied fast, and big step size used when the system changes more slowly. For this reason we use the following factor to describe the variation of a system:

$$\lambda_i(t_n) = \frac{1}{\Delta_n} \ln \left| \frac{\delta x_i(t_n + \Delta_n)}{\delta x_i(t_n)} \right| \quad (4)$$

where $\lambda_i(t_n)$ is called the local expansion (or contraction) rate for the state variable $x_i(t)$ at time t_n . If $\lambda_i(t_n) < 0$ the system variation is shrinking, that is, the system tends to change more slowly. In this case we can increase the time interval from Δ_n to $\Delta_{n+1} = \Delta_n + \delta$. Similarly, if $\lambda_i(t_n) > 0$ the system variation is expanding and the system tends to change faster, which require us to reduce the step-size in order to keep the system variation from expending.

From the above analysis, it can be seen that step size can be adjusted to match the variation of the system. In other words, we can use the following sampling time moments for doing integration:

$$t_0 = \Delta_0, t_1 = t_0 + \Delta_1, t_2 = t_1 + \Delta_2, \dots \quad (5)$$

If Integration of equation (1) is performed using the samples at the time moments above, the variation of the system can be reduced below a pre-assigned value. In this case the chaotic system is called *being locked*.

There are many ways to determine the step size $\{\Delta_n\}$. We use a the following approach:

Firstly, integrate the system with fixed time interval Δ_0 and get a solution $x_i(t)$. Then calculate the $\{\Delta_n\}$ with following formulas:

$$\begin{cases} \lambda_i(t_n) = \frac{1}{\Delta_n} \ln \left| \frac{\delta x_i(t_n + \Delta_n)}{\delta x_i(t_n)} \right| \\ \Delta_{n+1}^i = \Delta_n^i \{1 - \tanh[k\lambda_i(t_n)]\} \\ \Delta_{n+1} = \min_i \Delta_{n+1}^i \end{cases} \quad (6)$$

where k is a constant factor which defines the sensitivity of step-size change to the value of $\lambda_i(t_n)$.

3. MEASUREMENT OF DETERMINACY

In this section we will propose a new approach for measure the determinacy of signals mixed with noise. The idea is to make use of local correlation property of chaotic signals. On one hand, chaotic signals are deterministic rather than random, and their short-term behavior can be predicted locally, although the long-term behavior cannot be predicted. In fact, the behavior of a chaotic system is strongly related to the behavior of very short time ago. On the other hand, white noise does not have such property, and two samples of a white noise at any different time instances are independent. In other words, correlations between two adjacent samples also reflect the determinacy of the chaotic system.

We may use the correlation index between two adjacent sample values to measure the determinacy of the chaotic signal. However, due to noisy behavior the signal samples usually vary too much and the correlation index of signal samples may also behave like a noise sequence, which does not yield useful information regarding the determinacy. For this reason we use the calculated step size Δ_n by Equation (6) instead. The scenario is that for deterministic system Δ_n and Δ_{n+1} are nearly equal during a very short period of time. However for noise signals, Δ_{n+1} could be very different from Δ_n due to the lack of correlation. Fig. 1 gives the relationship between Δ_{n+1} and Δ_n for Lorenz system. It is seen that Δ_{n+1} is almost the same as Δ_n when there is no noise, and Δ_{n+1} could be very different from Δ_n in noisy environment. Therefore we use the correlation index between two adjacent Δ_n and Δ_{n+1} , given by

$$\begin{aligned} r(\Delta_i, \Delta_{i+1}) &= \frac{E\{\Delta_i \Delta_{i+1}\} - m_{\Delta_i} m_{\Delta_{i+1}}}{E\{[\Delta_i - m_{\Delta_i}]^2\}} \\ &= \frac{E\{\Delta_i \Delta_{i+1}\} - m_{\Delta_i} m_{\Delta_{i+1}}}{E\{\Delta_i^2\} - (m_{\Delta_i})^2} \end{aligned} \quad (7)$$

where $E\{\cdot\}$ denotes statistical expectation and $m_{\Delta_i} = E\{\Delta_i\}$.

In order to use Equation (7) we have to calculate the parameters in the equation. As the values of the time steps can be obtained using Equation (6), the parameters can be estimated based on $\{\Delta_i, \text{ for } i=1,2,\dots,N\}$ as follows:

$$E\{\Delta_i \Delta_{i+1}\} \approx \frac{1}{N-1} \sum_{i=1}^{N-1} \Delta_i \Delta_{i+1} = a \quad (8)$$

$$m_{\Delta_i} = E\{\Delta_i\} \approx \frac{1}{N-1} \sum_{i=1}^{N-1} \Delta_i = b \quad (9)$$

$$m_{\Delta_{i+1}} = E\{\Delta_{i+1}\} \approx \frac{1}{N-1} \sum_{i=2}^N \Delta_i = c \quad (10)$$

$$E\{\Delta_i^2\} \approx \frac{1}{N-1} \sum_{i=2}^N \Delta_i^2 = d \quad (11)$$

Hence the correlation index between two adjacent time steps can be estimated as follows:

$$r(\Delta_i, \Delta_{i+1}) \approx \frac{a - bc}{d - b^2} = D \quad (12)$$

It is expected that D be close to 1 for deterministic signals and 0 for random noise.

4. IMPROVED LOCAL-PROJECTION METHOD.

A local-projection method for chaotic signals was proposed to implement noise reduction [5]. The main idea of the local-projection method can be described as follows. Firstly reconstruct the noisy signal into an appropriate phase space. Pure chaotic signal should form an attractor in phase space. Because of the noise, signal orbits in phase space will depart from the original chaotic attractor. According to the shadowing property an accurate trajectory always exists together with an inaccurate trajectory. Local-projection method linearizes the chaotic system locally so that the system can be treated as linear system locally. Then wavelet transform is used to adjust the orbit in phase space so that it is closer to the real chaotic attractor's orbit. The local-projection algorithm consists of three steps [5].

Step 1: Overlapping process. The noisy time series is reconstructed according to time delay embedding property so that Neighbor fields with least overlap on the whole orbit are obtained.

Step 2: Projecting process. Projection of points in each neighbor fields which yields a new state vector, based on which a new time series is constructed. The new time series should be more *clean*.

Step 3: Iteration process. Repeat step 1 and step 2 until a stop condition is met. The output time series of the process is the final data after noise reduction process.

In [4], signal's Signal Noise Ratio (SNR) is used for the stop condition. If SNR can be improved, the algorithm will not stop. However, for real world signals, SNR is hard to be measured and the SNR cannot be used as the stop condition.

As discussed in Section III parameter D gives a metric for the determinative. Hence we could evaluate the noise reduction performance by comparing the values of D before and after the noise reduction filtering, and a stop condition can be established based on the comparison. For every iteration of the algorithm in [4], we calculate the following factor:

$$E = \frac{|D(Y) - D(X)|}{D(X)} \times 100\% \quad (13)$$

where $D(\cdot)$ calculates input vector's parameter based on (12), X is the time series before filtering, Y is the filtered time series after an iteration using the local-projection algorithm. Clearly E gives the improvement in terms of signal determinacy. The stop condition can be defined as:

$$E > \varepsilon \quad (14)$$

where ε is a pre-assigned small positive constant.

5. NUMERICAL EXPERIMENTS

Two experiments have been done to verify the effectiveness of the proposed approach. Firstly the improved local projection algorithm is simulated for Henon system based on computer produced data. Then the improved local-projection algorithm is simulated using real world data collected from oceanic laser radar for submarine detection.

5.1 Local-projection Algorithm for Computer Produced Data

Fig. 2. shows the result of local-projection algorithm applied to Henon system. In Fig. 2(a), 40 percent color noise is mixed into the chaotic signal. Fig. 2(b) shows the filtered signal after 5 iteration steps. The improvement of SNR is about 6.56dB. It is seen that local-projection algorithm is effective for computer produced data.

5.2 Improved Local-projection Algorithm for Real World Data

We apply the improved local-projection algorithm described in Section IV to oceanic laser radar signal. We assume that no prior knowledge of the underlying chaotic dynamic system is available. So it is not very convenient to compare the attractors' shape in phase space to estimate the algorithm's efficiency. In order to show the effectiveness of using (14) as the stop condition, we calculated the relation Δ_n vs. Δ_{n+1} . Fig. 4(a) gives the Δ_n vs. Δ_{n+1} figure of the noisy signal. Fig. 4(b) gives the Δ_n vs. Δ_{n+1} figure of signal after 5 iterations. It is clear that the processed data is characterized by being much concentrated to the line $\Delta_n = \Delta_{n+1}$. Hence the proposed algorithm should be effective to the real world data.

6. CONCLUSION

In this paper the problem of noise reduction for chaotic signal is addressed. A new parameter is defined which can effectively measure the determinacy of chaotic signals. The parameter can be used to construct stop condition for existing noise reduction techniques. As an example, an improved local projection algorithm is proposed and tested with real world data. The results show that the parameter gives a good measurement of the signal determinacy and thus also the noise reduction performance.

The proposed parameter may also be used for other noise reduction techniques. However more analysis on its characteristics should be performed which may yield more useful information on the chaotic signals.

7. REFERENCE

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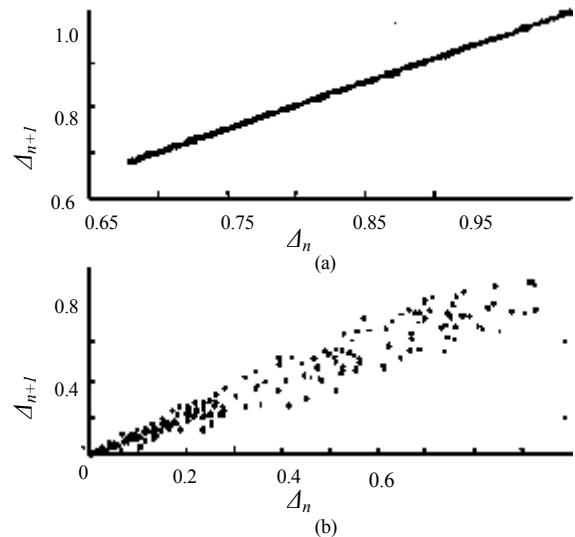


Fig. 1: Comparison of values of Δ_n with or without noise, Original dynamic system is Lorenz system. The figure is plotted as Δ_n vs. Δ_{n+1} . (a) without noise; (b) with white of uniform distribution on $[0, 1/70]$.

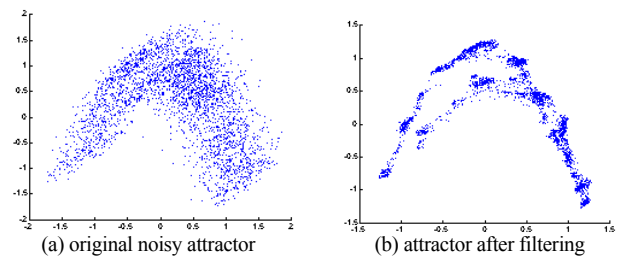


Fig. 2: Local-projection algorithm applied to Henon system (40% color noise is added)

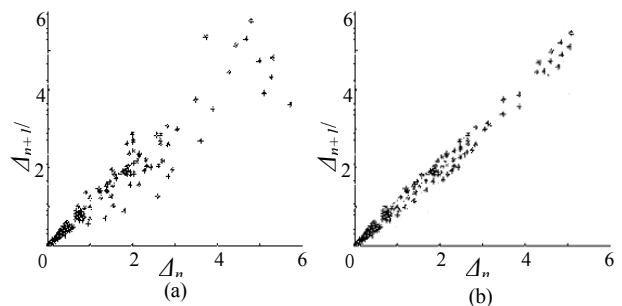


Fig. 3: Δ_n vs. Δ_{n+1} for oceanic laser radar signal