THE OPTIMAL DETECTOR FOR A WIDEBAND NOISE IMPULSE

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ABSTRACT

In this paper we discuss the problem of wideband noise impulse sequence detection. The structure of the optimal detector has been proposed and error detection probabilities have been estimated.

1. INTRODUCTION

The problems of wideband noise impulse sequence detection and parameter estimation are common for radiophysics. Such signals can be seen in wideband communication systems with a noise-like carrier, in wideband scanning radiometric systems and in acoustic applications.

In this paper we consider the problem of wideband noise impulse detection. We assume that the law of average impulse power variation over HF period is known. Such signal can be defined as follows

$$s(t) = a(t)n_s(t)\ ,$$

where $a^2(t)$ is the law of signal power variation and $n_s(t)$ is a stationary wideband Gaussian noise with zero mean and dispersion equal to one.

Let's define function $a(t)$ by the following equation

$$\begin{cases} a(t) > 0, 0 < t < T_s \\ a(t) = 0, \text{otherwise} \end{cases}.$$

We assume that $a(t)$ is non-increasing within the interval $t \in (0;T_s)$. In case of non-decreasing or convex functions the reasoning will be the same.

The signal spectrum is distributed in the range between $f_{\min}$ and $f_{\max}$ (for wideband signals $f_{\min} << f_{\max}$). Taking into account that the width of the signal autocorrelation function $B_s(t_1,t_2)$ is equal to $\tau_s \approx 1/(2f_{\max})$, the noise impulse can be given by N statistically independent samples $\{s_i, i=\overline{1,N}\}$, where $N = 2T_s f_{\max}$. According to the sampling theorem, the sampling period equals to $T_d = 1/(2f_{\max})$.

Besides the signal at the input, a receiver has its own internal noise $n_r(t)$, including the noise from the antenna. This is a stationary Gaussian process with zero mean and dispersion equal to the average internal noise power. The gain-frequency response of the receiver defines the internal noise spectrum. We assume that the receiver bandwidth is not higher than $f_{\max}$, then the width of the noise autocorrelation function $B_n(\tau)$ is equal to $\tau_n = 1/(2f_{\max})$.

We need to determine the presence of the signal $s(t)$ within the input process observation interval, in order to extract the necessary information from the sum of the signal and the receiver internal noise, given by the following equation

$$x(t) = s(t) + n_r(t)\ .$$

Prior probabilities of impulse presence or absence at a certain time point are equal. Thus, the maximum-likelihood criterion must be taken as the optimum criterion for impulse detection.

2. THE STATIONARY GAUSSIAN SIGNAL

An approach to solving the problem of detecting a stationary stochastic Gaussian signal with autocorrelation function $B_s(\tau)$ against the background of additive Gaussian noise with autocorrelation function $B_n(\tau)$ can be found in [1]. We make the following assumptions: we consider the noise $n_r(t)$ spectrum distribution to be uniform in the band from $-f_{\max}$ to $f_{\max}$, the noise samples with sampling period T_d to be statistically independent and the noise autocorrelation function to be equal to

$$B_n(\tau) = N_0\delta(\tau) = \sigma_n^2 T_d \delta(\tau)\ ,$$

where noise power spectral density equals to

$$N_0 = \sigma_n^2 / (2f_{\max})$$

and the noise dispersion equals to σ_n^2. Proceeding from these assumptions we can define the optimal algorithm of signal detection with respect to the chosen criterion as

$$\frac{1}{N_0}\int_0^{T_s}\int_0^{T_s} h(u,v)x(u)x(v)dudv \geq 2\ln C + \sum_{i=1}^{N} \ln \lambda_i\ , \qquad (1)$$

where $x(t)$ is the input signal of the detector; constant C is defined by the chosen detection criterion, for the maximum likelihood criterion $C=1$; $\lambda_i = 1 + 1/(N_0\mu_i)$, where μ_i is the proper number of the homogeneous integral equation

$$\psi_i(t) = \mu_i \int_0^{T_s} B_s(t-y)\psi_i(y)dy\ , \qquad (2)$$

where $\psi_i(t)$ is the eigen function of this equation and $h(u,v)$ is the solution of the integral equation

$$\int_0^{T_s} B_s(t-v)h(u,v)du + N_0 h(t,v) = B_s(t-v) \qquad (3)$$

$$0 < t < T_s, 0 < v < T_s$$

As the signal is wideband, we can assume that

$$B_s = N_s \partial(\phi), \qquad (4)$$

where N_s is the signal power spectral density. Substitution of equation (4) into (2), yields $\mu_i = 1/N_s$, and $\psi_i(t)$ is any given orthogonal function system. The next substitution (4) into (3) gives us

$$h(t,v) = \frac{N_s}{N_s + N_0}\delta(t-v),$$

which enables us to rewrite the optimal algorithm of signal detection (1) as follows:

$$\frac{1}{N_0}\int_0^{T_s}\frac{N_s}{N_s+N_0}x^2(t)dt \ge \sum_{i=1}^{N}\ln(1+{N_s}/{N_0}) \qquad (5)$$

So, the optimal detector can be implemented as a second-order nonlinear filter and an inertia-free threshold device.

3. THE INTERMITTENT SIGNAL

Let's consider the autocorrelation function for the intermittent signal $s(t)$

$$B_s(t_1,t_2) = \langle a(t_1)n_s(t_1)a(t_2)n_s(t_2)\rangle$$

$$B_s(t_1,t_2) = a(t_1)a(t_2)T_d\delta(t_1-t_2), \qquad (4`)$$

and the eigen functions

$$\psi_i(t) = \mu_i\int_0^{Ts} a(t)a(y)T_\partial\delta(t-y)\psi_i(y)dy = \mu_i a^2(t)T_d\psi_i(t),$$

so $\psi_i(t)$ is any given orthogonal sequence of functions in the interval T_s. Let's consider the eigen functions according to the sampling theorem as

$$\psi_i(t) = \frac{\sin\left(2\pi f_{\max}\left(t - {i}/{2f_{\max}}\right)\right)}{2\pi f_{\max}\left(t-{i}/{2f_{\max}}\right)},$$

then $\mu_i = 1/\left(a_i^2 T_d\right)$ and $\lambda_i = 1 + a_i^2 T_d/N_0$, where $a_i^2 = a^2(iT_d)$.

Substitution (4`) in (3) gives us

$$\int_0^{Ts} a(t)a(u)T_d\delta(t-u)h(u,v)du + N_0 h(t,v) = a(t)a(н)T_d\delta(t-н)$$

and $h(t,v) = \dfrac{a(t)a(v)\delta(t-v)}{a^2(t)+\sigma_n^2}$.

Equation (5) can be further converted into equation (5`) as a result of the following calculations:

$$\frac{1}{N_0}\int_0^{T_s}\int_0^{T_s}\frac{a(t)a(v)\delta(t-v)}{a^2(t)+\sigma_n^2}x(t)x(v)dtdv \ge \sum_{i=1}^{N}\ln\left(1+{a_i^2}/{\sigma_n^2}\right)$$

$$\frac{1}{N_0}\int_0^{T_s}\frac{a^2(t)x^2(t)dt}{a^2(t)+\sigma_n^2} \ge \sum_{i=1}^{N}\ln\left(1+{a_i^2}/{\sigma_n^2}\right)$$

$$\frac{1}{T_d}\int_0^{T_s}\frac{\frac{a^2(t)}{\sigma_n^2}\frac{x^2(t)}{\sigma_n^2}}{1+\frac{a^2(t)}{\sigma_n^2}}dt \ge \frac{1}{T_d}\int_0^{T_s}\ln\left(1+\frac{a^2(t)}{\sigma_n^2}\right)dt = L \qquad (5`)$$

where L is the threshold value. With all the assumptions that we have made equation (5`) defines the optimum signal detector as a number of blocks connected in series. The first block is a squarer, the second one is a linear filter and the last one is an inertia-free threshold device. The filter and the threshold device characteristics are uniquely determined by N samples of the process

$$q(t) = a^2(t)/\sigma_n^2,\ \{q_i\} = \{a_i^2/\sigma_n^2\},$$

where $N = 2T_s f_{\max}$ as we mentioned before.

4. ERROR ESTIMATION

Let's estimate the probabilities of wrong decisions, intrinsic to the given algorithm.

First we consider the case of a pause in impulse series transmission, that is $x(t) = n_r(t)$.

The optimal receiver output signal compared with the threshold L can be written as follows

$$U_{0n} = \frac{1}{T_d}\int_0^{T_s}\frac{q(t)\,{n_r^2(t)}/{\sigma_n^2}}{1+q(t)}dt = \sum_{i=1}^{N}\frac{q_i}{1+q_i}({n_i^2}/{\sigma_n^2}),$$

where n_i are samples of the process $n_r(t)$.

The stationarity of the Gaussian process $n_r(t)$ defines the probability distribution of value

$$y(t) = {n_r^2(t)}/{\sigma_n^2} \text{ as } W(y) = \frac{1}{\sqrt{2\pi y}}\exp\left(-\frac{y}{2}\right),$$

where $y > 0$ with moment coefficients $m(y) = 1$, $m(y^2) = 3$ and $D_y = 2$ [2]. The relationships between the moments of linear combination of statistically independent stochastic variables allow us to write the following:

$$M[U_{0n}] = \frac{1}{T_d}\int_0^{Ts}\frac{q(t)}{1+q(t)}dt = m_n \text{ and}$$

$$D[U_{0n}] = \frac{2}{T_d}\int_0^{Ts}\left(\frac{q(t)}{q(t)+1}\right)^2 dt = D_n .$$

On the other hand according to the law of mean [2], taking into consideration that

$$q(t)\big/\left(1+q(t)\right) \geq 0$$

and the fact that it is not increasing, the output receiver signal can be given as

$$U_{0n} = \frac{Q}{1+Q}\frac{1}{T_d}\int_0^{T_0} n^2(t)\big/\sigma_n^2\, dt = \frac{Q}{1+Q}\sum_{i=1}^{N'}\left[n_i^2\big/\sigma_n^2\right]^2 ,$$

where $N` = T_0/T_d$, $T_0 < T_s$ and

$$Q = \max\left\{a^2(t)/\sigma_n^2\right\} = a^2(0)/\sigma_n^2 .$$

As the samples n_i/σ_n are independent, standardized and normally distributed, the sum of their squares from 1 to $N`$ has $\chi^2$–distribution, with mean value equal to $N`$ and dispersion equal to $2N`$. Weighting coefficients consideration gives us $M[U_{0n}] = N`Q(1+Q)$. Let's equate mean value expressions:

$$\frac{Q}{1+Q}N' = \frac{1}{T_d}\int_0^{T_s}\frac{q(t)}{1+q(t)}dt \rightarrow N' = \frac{1+Q}{Q}\frac{1}{T_d}\int_0^{T_s}\frac{q(t)}{1+q(t)}dt .$$

χ^2-distribution with degrees of freedom number $N` \geq 30$ to high precision can be approximated by Gaussian distribution with the same mean and dispersion. For example, the impulse input signal model

$$q(t) = Qe^{-2t/\tau}$$

gives the following equations

$$m_n = g^2 \ln(1+Q),$$

$$D_n = 2g^2\left(\ln(1+Q) - \frac{Q}{1+Q}\right),$$

$$N` = g^2 \ln(1+Q)(1+Q)/Q ,$$

where $g^2 = f_{\max}\tau_{eff}$ and τ_{eff} is efficient impulse duration evaluated from the impulse signal energy proportion

$$E_s = \frac{a^2(0)\tau_{eff}}{2} .$$

The threshold value

$$L = g^2\int_1^{1+Q}\frac{\ln x}{x-1}dx .$$

Obtained $W_n(U_{0n})$ distribution characteristics enable us to find probability of false detection

$$\alpha = \int_L^{\infty} W_n(U_{0n})dU_{0n} = 1 - \int_{-\infty}^{L} W_n(U_{0n})dU_{0n} = 1 - F(t_\alpha) \quad (6)$$

where Laplace integral

$$F(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{t^2}{2}\right)dt \text{ and } t_\alpha = \frac{L - m_n}{\sqrt{D_n}} .$$

Let's assume that

$$x(t) = a(t)n_s(t) + n_r(t)$$

and estimate the probability of signal omission. Reasoning will be quite analogous to what we did for the false detection probability. The output signal of the optimal receiver is

$$U_{0s} = \frac{1}{T_d}\int_0^{T_s}\frac{q(t)}{1+q(t)}\left[\sqrt{q(t)}n_s(t) + n_r(t)\big/\sigma_n\right]^2 dt = U_{1s} + U_n$$

it is the sum of the signal U_{1s} and the noise U_n parts, which are independent from each other. The integral of the independent noise components $n_r(t)$ and $n_s(t)$ product is practically a negligible quantity, thus U_{0s} mean value is

$$M[U_{0s}] = M[U_{1s}] + M[U_n] = \frac{M\left[n_s^2\right]}{T_d}\int_0^{T_s}\frac{q^2(t)}{1+q(t)}dt + m_n$$

$$M[U_{0s}] = \frac{1}{T_d}\int_0^{T_s}\frac{q^2(t)}{1+q(t)}dt + m_n = m_{s1} + m_n$$

and dispersion is

$$D[U_{0s}] = \frac{2}{T_d}\int_0^{T_s}\frac{q^4(t)}{(1+q(t))^2}dt + D_n$$

Let's estimate the degree of freedom number for χ^2-distribution. The signal component can be written as

$$U_{1s} = \frac{1}{T_d}\int_0^{T_s}\frac{q^2(t)}{1+q(t)}n_s^2(t)dt = \frac{Q^2}{1+Q}\frac{1}{T_d}\int_0^{T_1}n_s^2(t)dt = \frac{Q^2}{1+Q}\sum_{i=1}^{N``}n_{si}^2$$

where $N`` = T_1/T_d$ and $T_1 < T_s$.

The signal component mean value is

$$M[U_{1s}] = \frac{Q^2 N``}{1+Q},$$

it gives us

$$N`` = \frac{1+Q}{Q^2}\int_0^{T_s}\frac{q^2(t)}{1+q(t)}dt.$$

The input signal model gives us

$$m_s = g^2(Q - \ln(1+Q)) + g^2\ln(1+Q) = g^2 Q$$

$$D_s = 2g^2\left(Q^2/2 - 2Q + 4\ln(1+Q) - Q/(1+Q)\right)$$

$$N`` = g^2(Q - \ln(1+Q))(1+Q)/Q = g^2(1+Q) - N`$$

$$(N`` + N`) = g^2(1+Q)$$

The output signal χ^2-distribution $W_s(U_{0s})$ is to high precision approximated by Gaussian distribution with the same mean and dispersion values for the case of $(N``+N`) \geq 30$, where $N`` \geq 0$. The false decision probability of signal omission is

$$\beta = \int_{-\infty}^{L} W_s(U_{0s})dU_{0s} = F(t_\beta),\ t_\beta = \frac{L - m_s}{\sqrt{D_s}}. \qquad (7)$$

The total probability of false detection is equal to

$$P_{err} = (\alpha + \beta)/2 = \left(F(t_\alpha) + F(-t_\beta)\right)/2. \qquad (8)$$

If we compare the result (8) with the total probability of the stationary noise false detection for the optimal algorithm (5), we will see the parity of the values.

5. CONCLUSION

The optimal detector of a wideband noise impulse with the given law of noise power variation is defined by (5`) as a series of a squarer, a linear filter and an inertia-free threshold device. The filter and the threshold device characteristics are uniquely determined by *N* samples of process $a^2(t)/\sigma_n^2$.

The probabilities of false detection are defined by expressions (6) and (7). These functions have the argument which is the ratio of the signal impulse peak power to the average receiver noise power $Q = a^2(0)/\sigma_n^2$ and the parameter $g = \sqrt{\tau_{eff} f_{\max}}$.

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