PERSISTENCE OF EDGE-DETECTING PROPERTIES OF A CNN TO VARIATIONS OF THE CELL OUTPUT FUNCTION

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ABSTRACT

A two-dimensional model of a CNN with local couplings is considered as an image converter the input of which is the profile of initial conditions and the output is the steady-state pattern in the lattice. The properties of an edge-detecting CNN are described. The side effect of checkered pattern formation is discussed. The persistence of the edge-detecting properties of the system with respect to small perturbations of the cell output function is investigated analytically. The consequences of the replacement of the piecewise-linear cell output function by another one are studied numerically.

1. PROBLEM STATEMENT

Over the last decade much research work has been carried out to create efficient analogue image processing systems employing ensembles of coupled nonlinear elements. Due to the inherent parallelism of processes in these systems the number of pixels in the image determines only the complexity of the system and does not affect the processing time. The most widely known type of such systems is CNN (Cellular Neural Network, or Cellular Nonlinear Network) [1]. A CNN is an array of interconnected bistable elements typically (but not necessarily) arranged into a two-dimensional lattice. The image to be processed in such systems is usually introduced as initial conditions, or as some kind of an external force, or in both these ways simultaneously. The result of processing is supposed to be given by the pattern formed in the array in a steady state.

A CNN is described in the most general form by a system of coupled ordinary differential equations, which can be written in the vector form

$$\frac{d\mathbf{x}}{dt} = -\mathbf{x} + \mathbf{A}\mathbf{y}(\mathbf{x}) + \boldsymbol{\gamma} \qquad (1)$$

where $\mathbf{x}$ is the vector of state of the system, $\boldsymbol{\gamma}$ denotes external force, $\mathbf{A}$ is a matrix whose diagonal components are the feedback parameters of individual elements and non-diagonal ones are the coupling coefficients, and $\mathbf{y}(\mathbf{x})$ is a vector function specified componentwise

$$y_i=\Phi(x_i),\ |\Phi(x)|\leq 1$$

where $\Phi(x)$ is a scalar nonlinear function termed as the cell output function. A large number of electronical implementations of the model (1) have been proposed since the time when the CNN paradigm was developed. Most commonly, these implementations employ differential amplifiers providing nonlinearity and RC-cirquits determining the time constant of the system (which is unity in the model).

In almost all of the analytical studies the cell output $\Phi(x)$ is taken in the form of a piecewise-linear function with saturation

$$\Phi_0(x)=(|x+1|-|x-1|)/2 \qquad (2)$$

This representation yields a model quite adequate for most implementations and allows analytical derivation of relevant results. Nevertheless, a question arises, how crucial the type of the nonlinear cell output is for these results to be valid. In particular, their robustness with respect to small perturbations in the cell output function needs to be checked. How results obtained for the model function (2) are affected by a non-small change in the cell output function is also a matter of interest.

As the steady-state pattern formed in the system is considered to be the result of processing, the general problem is to determine the influence of the type of the cell output function upon the laws of pattern formation in systems of the type (1). Any steady-state pattern corresponds to a stable equilibrium state of the system; thus, the number of stable equilibrium states, their position in the phase space and their basins of attraction determine the formation of patterns.

The aim of this paper is to elucidate this problem for a specific type of CNN intended for performing the edges detection operation. It is described by the following system of equations being a special case of the system (1):

$$\begin{aligned}\frac{d}{dt}x_{ij} &= -x_{ij} + \delta_0\Phi(x_{ij}) - \\ &- \delta[\Phi(x_{i+1,j}) + \Phi(x_{i-1,j}) + \Phi(x_{i,j+1}) + \Phi(x_{i,j-1})]\end{aligned} \qquad (3)$$

where i=1÷L, j=1÷M, $L\times M$ is the size of the lattice; x_{ij} are the variables of state; δ_0 and δ are parameters, with δ determining coupling strength. Assume $\delta_0>1$ in order to provide the bistability of the basic element. The boundary conditions are Neumann conditions:

$$x_{0j} = x_{1j},\ x_{L+1,j} = x_{L,j},\ x_{i0} = x_{i1},\ x_{i,M+1} = x_{i,M}$$

The system considered is a two-dimensional lattice with reciprocal local coupling and equal coupling coefficients. The external forcing is zero; the initial image is introduced via initial conditions only.

The paper is organised as follows. First, available results on the system (3) with the cell output function (2) are provided. Next, analytical results for modified systems are presented. Finally, numerical experiments are described.

2. MAIN RESULTS

In [2] a CNN model of the general form (1) with the cell output function (2) was examined for existence of stable equilibrium points. The results of [2] follow that for δ and δ_0 satisfying the condition

$$\delta_0-4\delta>1 \tag{4}$$

the phase space of the system contains $2^{L\times M}$ stable equilibrium points, each uniquely determined by its set of signs of the variables of state. If this set of signs is defined by an $L\times M$ matrix $\mathbf{Y}=(y_{ij})$ whose elements can take on values (-1) or 1 (a sign-matrix), then the corresponding equilibrium point is given by

$$u^{\mathbf{Y}}_{ij}=\delta_0 y_{ij}-\delta(y_{i+1,j}+y_{i-1,j}+y_{i,j+1}+y_{i,j-1}) \tag{5}$$

As it follows from (4) and (5), $sign(u^{\mathbf{Y}}_{ij})=y_{ij}$, each of these equilibrium points corresponds to a certain steady-state pattern that can set in as a result of evolution of the system. Note, that a pattern with any sign-matrix can be realized in a steady state, depending on the initial conditions.

Moreover, any point in the phase space with coordinates of the form

$$x_{ij}=ay_{ij} \tag{6}$$

where $a>1$ belongs to the basin of attraction of the equilibrium point $u^{\mathbf{Y}}_{ij}$. Thus, the result of transformation of a two-level input pattern (6) has the same sign-matrix as the input pattern. However, according to (5), the pattern undergoes a transformation that, for $\delta>0$, can be characterized as elimination of halftones and detection of the edges of areas with opposite signs in the input pattern.

As it was demonstrated in [3] by numerical simulation, the transformation of arbitrary input patterns by the system (3) with the cell output function (2) may be more complicated than it is for two-level patterns (6). Depending on the values of parameters and the specific type of input pattern, the steady-state sign-matrix either follows the input one, as it does for input patterns (6) (in this case the transformation again leads to detection of edges of opposite-sign regions), or has also sections filled with small-scale pattern which appears as a chessboard or stripes (see Fig. 1 *a*, *b*; hereinafter it is referred to as checkered pattern). This pattern is typically formed in the regions neighboured by the lines of loss of smoothness in the input pattern, where itself or its gradient makes a jump (lines of fracture), as well as in the neighbourhood of the zero level of input pattern (if the latter has alternating sign). With respect to the edges-detection operation, formation of checkered pattern is a corrupting effect, though for other applications (such as detection of lines of fracture) it may appear to be useful.

Experiments reveal [3], that the sign-matrix of the steady-state pattern in this system at given initial conditions is not affected by variation of the parameters δ_0 and δ, if the value of the parameter α defined by

$$\alpha=4\delta/(\delta_0-1) \tag{7}$$

remains constant. The effects of checkered pattern formation are the stronger, the greater α is. For $\delta_0>1$ and $\delta>0$ the condition (4) is equivalent to $0<\alpha<1$. At given α, these effects are the stronger, the less the absolute module of input pattern in the region of a line of fracture is or the less steep the slope of the input pattern near the zero level is. The preferable orientation for the formation of the checkered pattern is that along the lattice diagonal. According to (7), at given δ, greater values of δ_0 correspond to lower α, so in order to suppress checkered pattern formation, δ_0 must be increased.

The first modification of the cell output function considered in this paper consists in adding to it small non-identical perturbing terms for each individual element. Let the individual cell output function of the element (i,j) be written as follows:

$$\Phi_{ij}(x)=\Phi_0(x)+\varphi_{ij}(x) \tag{8}$$

where $|\varphi_{ij}(x)|<\varepsilon$ for $1<|x|<R$,

$\varepsilon>0$ and $R>1$ are parameters governing the maximal allowed magnitude of perturbations. The exact type of added terms $\varphi_{ij}(x)$ is not specified. For this form of modification analytical treatment is carried out.

The other modification consisting in replacement of the function (2) by the following one

$$\Phi_1(x)=4x/(4+x^2) \tag{9}$$

is studied numerically. This function appears in the model (3), if the bistable element is implemented as an oscillator with inverted frequency feedback control. Both functions $\Phi_0(x)$ and $\Phi_1(x)$ have equal slope in the vicinity of the zero argument: $\Phi_0'(0)=\Phi_1'(0)=1$. The distinction of $\Phi_1(x)$ is the presence of decreasing branches tending to zero at infinity.

In [4] non-identity of elements in the form similar to (8) was introduced into the general model (1). The results obtained in [2] for the model (1) with the basic non-linear function (2) were generalized to the modified model. This generalization was applied to the analysis of a one-dimensional edges detecting CNN. In this paper the two-dimensional case is considered.

For a system with small arbitrary perturbations of cell outputs (7) steady-state regimes are no longer represented by stable equilibrium states in the general case. Other types of attractors such as oscillatory limit cycles and even chaotic attractors may appear instead. Nevertheless, the location of these attractors in the phase space under certain conditions may be estimated. The summary result reads as follows (cf. (4)-(6)).

If the condition

$$\begin{gathered}\delta_0(1-\varepsilon)-4\delta(1+\varepsilon)>1\\ \delta_0(1+\varepsilon)+4\delta(1+\varepsilon)<R\end{gathered} \tag{10}$$

is fulfilled, then $2^{L\times M}$ attractors exist in the phase space of the system, each uniquely determined by its set of signs of the variables of state. If this set of signs is defined by an $L\times M$ matrix $\mathbf{Y}=(y_{ij})$ whose elements can take on values (-1) or 1, then the corresponding attractor is located within the hypercube

$$u^{\mathbf{Y}}_{ij}-4\varepsilon<x_{ij}<u^{\mathbf{Y}}_{ij}+4\varepsilon$$

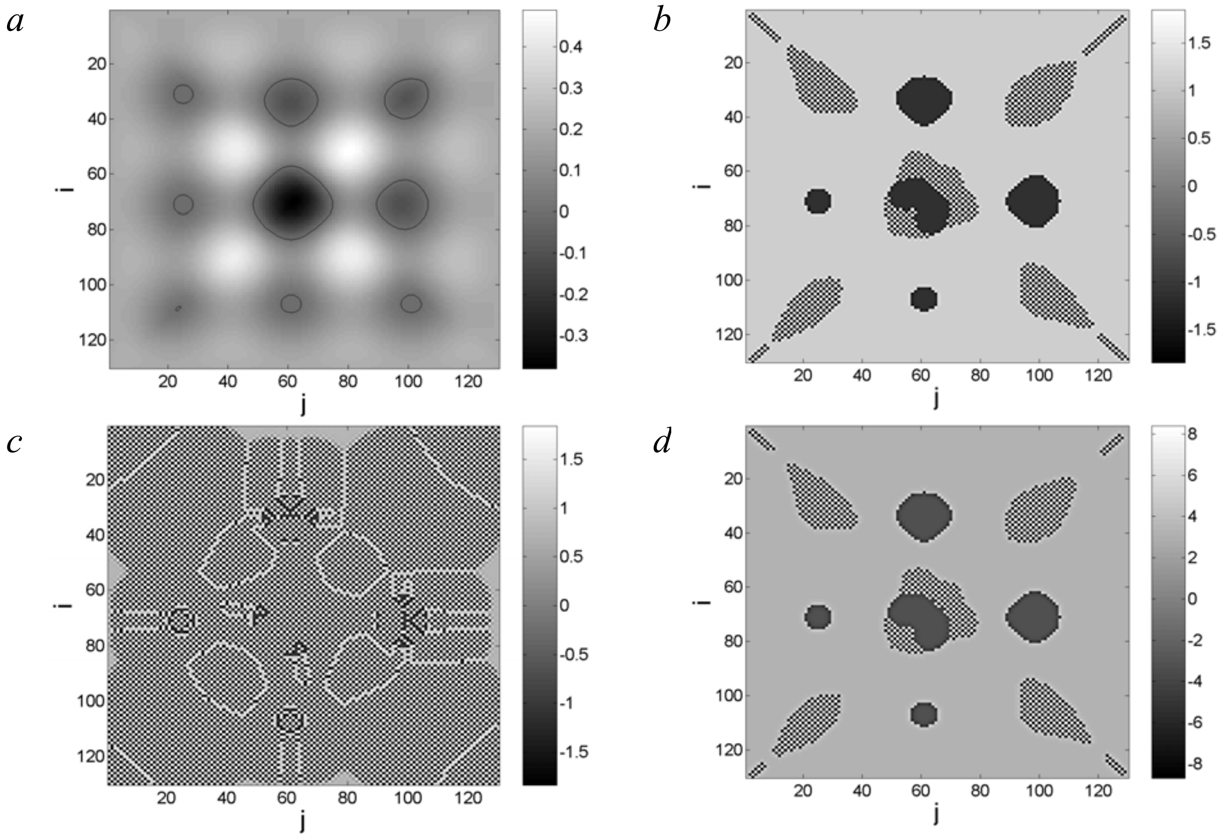

Figure 1: Transformation of a sample image; *a* – initial conditions with zero level line marked; *b*, *c*, *d* – steady-state patterns (α=0.7 everywhere; *b*: $\Phi(x)=\Phi_0(x)$, δ_0=1.5; *c*: $\Phi(x)=\Phi_1(x)$, δ_0=1.5; *d*: $\Phi(x)=\Phi_1(x)$, δ_0=10).

where $u^{\mathbf{Y}}_{ij}$ is defined by (5). Moreover, any point in the phase space with coordinates of the form (6) where $1<a<R$ belongs to its basin of attraction. Thus, the result of transformation of a two-level input pattern (6) in this system agrees with the result in the system with the undisturbed cell output function (2) up to the precision 4ε.

Based on this inference and the expression (5), the condition of reliable detection of edges can be written as follows:

$$\delta>\varepsilon(\delta_0+4\delta) \qquad (11)$$

The system of inequalities (10,11) defines a non-empty region on the plane of parameters (δ_0,δ), provided

$$R>(1+\varepsilon)/(1-9\varepsilon). \qquad (12)$$

When the cell output function is changed to (9), the sign-matrix of the steady-state pattern at given initial conditions depends on both δ_0 and δ, so the quantity α defined in (7) loses its role of a controlling parameter. Furthermore, the condition $0<\alpha<1$ (which is equivalent to (4)) is no longer sufficient for existence of $2^{L\times M}$ attractors. In other words, it is not sufficient for possibility for any sign-matrix to be formed in the steady state. There exist values of δ_0 and δ satisfying (4) for which checkered pattern spreads over the whole lattice for any initial conditions (Fig. 1 *c*). Generally, replacement of the function (2) in the model by (9) leads to intensification of the effect of the checkered pattern emergence [3].

Numerical experiments reveal, however, that the formation of checkered pattern can be suppressed by increase of δ_0. Moreover, at given value of α, a threshold for the parameter δ_0 exists, so that for any δ_0 exceeding this threshold the steady-state matrix of signs for any input pattern agrees with the one obtained in the system with piecewise-linear cell output (2) at the same α (see Fig. 1 *d*, cf. Fig. 1 *b*). So, for large enough δ_0, the sign-matrix of the steady-state pattern is again determined by the value of α and is invariant with respect to variations of δ_0 and δ preserving α.

3. CONCLUSIONS

Thus, our studies approve, that the type (2) of the cell output function is not crucial for performing the edges detection operation. First, it is proven, that small perturbations (8) of this function do not corrupt reliable detection of edges, provided that the condition (12) is fulfilled and the parameters δ_0 and δ are chosen according to (10,11). Finally, it is shown numerically, that replacement of the saturated function (2) by another feasible one (9) does not affect the qualitative type of image transformation in this system, provided that δ_0 exceeds a threshold depending on δ.

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