NEW VERSION OF A MCMC DATA ASSOCIATION ALGORITHM FOR NON-LINEAR OBSERVATION MODEL - APPLICATION TO THE TRACKING PROBLEM WITH FRENCH OTH RADAR NOSTRADAMUS

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ABSTRACT

Over-The-Horizon(OTH) Radars provide a survey of wide areas, using ionospheric reflections of the electromagnetic waves. Most of the time they have to face multipath problems: state estimation has to be done with measurements involving different observation models. To tackle this measurement-to-observation-model association problem, the Monte Carlo Data Association (MCDA) algorithm, and a derivative one, the Iterated Conditional Mode Data Association (ICMDA) have been developed. They only applied in linear context. We propose new versions of these algorithms well adapted to non-linear problems.Our two algorithms are applied, through numerical simulations, to a concrete case: target tracking with the French OTH radar Nostradamus, in clutter environment.

1. INTRODUCTION

Over-The-Horizon (OTH) radars perform long distance survey for wide areas using ionospheric reflections of electromagnetic waves. The French OTH radar NOSTRADAMUS offers two particularities: it is monostatic and it enables elevation angle measurement. In order to handle these characteristics we propose two promising algorithms respectively derived from algorithms called Monte Carlo Data Association (MCDA) algorithm and Iterated Conditional Mode Data Association algorithm (ICMDA) [1], both developed for linear applications.

The final purpose of OTH Radar NOSTRADAMUS is to estimate the target's ground coordinates (ground range, ground range rate, azimuth and elevation angle). It requires a measurement-to-model association: the observation equations depend on the ionospheric reflection layer the measures come from. Furthermore ground coordinates estimation involves non-linear observation equations. MCDA and ICMDA are then inefficient. So, we kept the main idea of the method and built two algorithms taking into account non-linear problems.

In section 2, we point out the problem statement. We describe our algorithms in section 3. Results and comparison with previous algorithms are given in section 4 through numerical simulations.

2. PROBLEM STATEMENT

We consider a linear dynamical target state model

$$x(k+1) = F_k x(k) + v(k) \tag{1}$$

where $x(k)$ is the state (ground range, ground range rate, azimuth, elevation angle: these information are called *earth coordinates*) at time k, $v(k)$ is a sequence of zero-mean, white, Gaussian process noise with known covariance matrix $Q(k)$. $F_k \varepsilon \Re^{n_x \times n_x}$ is the evolution matrix.

At time k, a set of μ_k measurements $\{y_i(k)\}_{i=1}^{\mu_k}$ is detected, called slant coordinates or radar coordinates: it represents the slant range, Doppler, azimuth and elevation angle. Each measurement originates from one of the following models:

$$y_i(k) = \begin{cases} h_1(x(k)) + w_1(k) & \text{for model 1} \\ h_2(x(k)) + w_2(k) & \text{for model 2} \\ \vdots & \vdots \\ h_n(x(k)) + w_n(k) & \text{for model n} \\ \text{clutter} & \text{otherwise} \end{cases} \tag{2}$$

where $y_i(k) \varepsilon \Re^{n_y}$ and $w_i(k)$ is a sequence of zero-mean, white, Gaussian measurement noise with known covariance matrix $R_i(k)$. Furthermore $w_i(k), i = 1...n$ are mutually independent and uncorrelated with $v(k)$. The number of clutter detections in each measurement set is Poisson distributed

$$p_c(q) = \frac{(\lambda V)^q}{q!} \exp(-\lambda V) \quad q = 0, 1, 2... \tag{3}$$

where V is the observation volume and λ is the mean number of clutter detection per volume units. The clutter measurements are uniformly distributed in V. Each model (except for clutter) generates one measurement maximum which is coherent with the OTH-NOSTRADAMUS radar observations: the algorithm can handle cases where several measurements are generated by each model but for sake of simplicity we just restrain to the single measurement per target case.

m_k is the number of non-clutter measurements in $\{y_i(k)\}_{i=1}^{\mu_k}$,

$$Y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_{\mu_k}(k) \end{bmatrix}$$

and ψ_k the measurement-to-model association vector so that $\psi_k(j) = l$ means the j^{th} measure comes from model l. For a 3 measures set and 4 models:

$$\psi_k = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$

means $y_1(k)$ comes from model 2, $y_2(k)$ comes from model 4, $y_3(k)$ comes from clutter. For n models, μ_k measures, m_k

non-clutter measurements, the number of possible association hypothesis is

$$r_k(m_k) = \frac{n!\mu_k!}{m_k!(n-m_k)!(\mu_k-m_k)!} \quad \text{with } m_k = 0,1,2...,\nu_k \tag{4}$$

where $\nu_k = \min(n,\mu_k)$. Thus, the total number of association hypothesis is $\pi_k = \Sigma_{m_k=0}^{\nu_k} r_k(m_k)$. Finally we call $\mathbf{X}_N = \{x_k\}_{k=0}^N$, $\mathbf{Y}_N = \{Y_k\}_{k=0}^N$ and $\Psi_N = \{\psi_k\}_{k=0}^N$.

3. ALGORITHMS

The MCDA gives a Minimum Mean Square Error (MMSE) estimation of Ψ_N knowing $\mathbf{Y}_N$ while the ICMDA gives the Marginal Maximum A Posteriori (MMAP) estimate. Both use Markov Chain Monte Carlo techniques through a Gibbs sampler.

Monte Carlo Data Association Filter:
The MMSE estimation of Ψ_N is

$$\hat{\Psi}_N = \mathbf{E}(\Psi_N|\mathbf{Y}_N) \tag{5}$$

A Monte-Carlo approximation of this relation is

$$\hat{\Psi}_N \approx \frac{1}{P-p_0}\sum_{p=p_0}^{P} \Psi_N^{(p)} \tag{6}$$

where P is the total association random sequence length, p_0 the number of initial discarded samples and $\Psi_N^{(p)} \sim p(\Psi_N|\mathbf{Y}_N)$.

Iterated Conditional Mode Data Association Filter:
MMAP estimation is

$$\hat{\Psi}_N = \arg\max_{\Psi_N} p(\Psi_N|\mathbf{Y}_N) \tag{7}$$

with Monte-Carlo approximation

$$\hat{\Psi}_N \approx \arg \max_{\Psi_N^{(p)};p=1..P} p(\Psi_N^{(p)}|\mathbf{Y}_N) \tag{8}$$

where $\Psi_N^{(p)} \sim p(\Psi_N|\mathbf{Y}_N)$. Both algorithms use the following step sequences:

1. Pick the initial association $\Psi_N^{(0)} = \{\psi_k^{(0)}\}_{k=0}^N$ (randomly or deterministically). Set $p=1$
2. for each $k = 0,1,...,N$ take (9) for the MCDA, (10) for the ICMDA

$$\psi_k^{(p)} \sim p(\psi_k|\mathbf{Y}_N, \Psi_{\neg k}^{(p)}) \tag{9}$$

$$\psi_k^{(p)} = \arg \max_{\psi_k \varepsilon \{\psi_i\}_{i=1}^{\pi_k}} p(\psi_k|\mathbf{Y}_N, \Psi_{\neg k}^{(p)}) \tag{10}$$

3. Set $p = p+1$ and return to item (2).

$\Psi_{\neg k}^{(p)} = \{\Psi_{k-1}^{(p)}, \Psi_{k+1:N}^{(p-1)}\} = \{\psi_0^{(p)}...\psi_{k-1}^{(p)}\psi_{k+1}^{(p-1)}...\psi_N^{(p-1)}\}$. $\{\psi_{k,i}\}_{i=1}^{\pi_k}$ represents the whole set of association hypothesis at time k. Applying Bayesian inference we get: $p(\psi_k|\mathbf{Y}_N, \Psi_{\neg k}^{(p)})$.

$$\begin{aligned} p(\psi_k|\mathbf{Y}_N, \Psi_{\neg k}^{(p)}) &= \frac{p(\mathbf{Y}_N|\Psi_{\neg k}^{(p)}, \psi_k)Pr(\psi_k)}{p(\mathbf{Y}_N|\Psi_{\neg k}^{(p)})} \\ &\propto p(\mathbf{Y}_N|\Psi_{\neg k}^{(p)}, \psi_k)Pr(\psi_k) \end{aligned} \tag{11}$$

One should consider $Pr(\psi_k|\psi_{k-1})$. But as in this paper we focus on the computation of $\mathbf{Y}_N$ conditional probability, we use only $Pr(\psi_k)$.

3.1 Proposed conditional probability development in order to handle non-linear equations:

$$p(\psi_k|\mathbf{Y}_N, \Psi_{\neg k}^{(p)}) \propto p(\mathbf{Y}_N|\Psi_{\neg k}^{(p)}, \psi_k)Pr(\psi_k)$$

$$\begin{aligned} p(\mathbf{Y}_N|\Psi_{\neg k}^{(p)}, \psi_k) &= p(\mathbf{Y}_{k-1}|\Psi_{\neg k}^{(p)}, \psi_k)p(Y_k|\mathbf{Y}_{k-1}\Psi_{\neg k}^{(p)}, \psi_k) \\ &\times p(Y_{k+1}|\mathbf{Y}_k, \Psi_{\neg k}^{(p)}, \psi_k)p(\mathbf{Y}_{k+2:N}|\mathbf{Y}_{k+1}, \Psi_{\neg k}^{(p)}, \psi_k) \end{aligned}$$

with $p(\mathbf{Y}_{k-1}|\Psi_{\neg k}^{(p)}, \psi_k) = p(\mathbf{Y}_{k-1}|\Psi_{k-1}^{(p)})$ independent from ψ_k. Furthermore $p(\mathbf{Y}_{k+2:N}|\mathbf{Y}_{k+1}, \Psi_{\neg k}^{(p)}, \psi_k)$ is also independent from ψ_k. Measurement and Dynamical equations give

$$Y_{k+2} = h_{\psi_{k+1}^{(p-1)}}(x_{k+2}) + w_{k+2}$$

$$x_{k+2} = Fx_{k+1} + v_{k+2}$$

Knowing Y_{k+1}, x_{k+1} depends only on $\psi_{k+1}^{(p-1)}$.Thus,

$$\begin{aligned} p(\psi_k|\mathbf{Y}_N, \Psi_{\neg k}^{(p)}) &\propto p(Y_{k+1}|\mathbf{Y}_k, \Psi_{\neg k}^{(p)}, \psi_k) \\ &\quad p(Y_k|\mathbf{Y}_{k-1}\Psi_{\neg k}^{(p)}, \psi_k)Pr(\psi_k) \end{aligned}$$

which becomes

$$\begin{aligned} p(\psi_k|\mathbf{Y}_N, \Psi_{\neg k}^{(p)}) &\propto p(Y_{k+1}|\mathbf{Y}_k, \Psi_k^{(p)}, \psi_{k+1}^{p-1}) \\ &\quad p(Y_k|\mathbf{Y}_{k-1}\Psi_k^{(p)})Pr(\psi_k) \end{aligned} \tag{12}$$

$p(Y_{k+1}|\mathbf{Y}_k, \Psi_k^{(p)}, \psi_{k+1}^{p-1})$ and $p(Y_k|\mathbf{Y}_{k-1}\Psi_k^{(p)})$ can be deduced from two consecutive iterations (at time k and $k+1$) of an extended Kalman algorithm [2].

3.2 Algorithms procedure:

1. Pick the initial association $\Psi_N^{(0)}$ and set $p=1$.
2. initialise the Kalman recursion $\hat{x}_{0|-1}$, $P_{0|-1}$, $k=0$
3. for each data-to-model association hypothesis $\mathscr{H}_i$, $i = 1...\pi_k$ run two consecutive iterations of a Kalman filter: from $\hat{x}_{k-1|k-1}$, $P_{k-1|k-1}$, $\psi_k = \mathscr{H}_i$, $\psi_{k+1}^{(p-1)}$ compute:
   (a) $\hat{x}_{k|k-1}$, $P_{k|k-1}$, $\hat{x}_{k+1|k}$ and $P_{k+1|k}$
   (b) $p(Y_k|\mathbf{Y}_{k-1}\Psi_k^{(p)}) = \mathscr{N}(Y_k; h_{\psi_k}(\hat{x}_{k|k-1}), S_k)$

(c) $p(Y_{k+1}|\mathbf{Y}_k, \Psi_k^{(p)}, \psi_{k+1}^{(p-1)}) =$
$$\mathcal{N}(Y_{k+1}; h_{\psi_{k+1}^{(p-1)}}(\hat{x}_{k+1|k}), S_{k+1})$$
with $S_k = R + H_{\psi_k} P_{k|k-1} H'_{\psi_k}$ and
$S_{k+1} = R + H_{\psi_{k+1}^{(p-1)}} P_{k+1|k} H'_{\psi_{k+1}^{(p-1)}}$

(d) normalize $p(\psi_k|\mathbf{Y}_N, \Psi_{\neg k}^{(p)})$ for $i = 1...\pi_k$

4. For the NL-MCDA algorithm: generate a sample $\psi_k^{(p)}$
For the NL-ICMDA algorithm: set $\psi_k^{(p)}$ as the most likely association
5. run a Kalman iteration using $\psi_k^{(p)}$, $\hat{x}_{k-1|k-1}$, $P_{k-1|k-1}$ and store the resulting $\hat{x}_{k|k-1}$, $P_{k|k-1}$, $\hat{x}_{k|k}$ and $P_{k|k}$. Set $k = k+1$ and return to item (3).
6. For the NL-MCDA algorithm if $p = P$ calculate (6) else $p = p+1$, return to item (2).
For the NL-ICMDA algorithm if $\Psi_N^{(p)} = \Psi_N^{(p-1)}$ end, else $p = p+1$, return to item (2).

4. NUMERICAL SIMULATIONS

We first give results on simple models to show the improvement performed by our algorithms. Then come OTH Radar Nostradamus simulations:

1. a comparison with two previous algorithms adapted to non-linear context. This comparison is made in OTH Radar Nostradamus context through simulated data.
2. an example of results we can obtain with the NL-ICMDA algorithm

4.1 Results on simple models

SIM 1 is a set of linear models and SIM 2 is a set of non-linear ones:

$$x_k = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = F. \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix}$$

$$y_k = \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = h_i\left(\begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix}\right) + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}$$

Where h_i is the i^{th} model. The noise covariance is different for SIM 1 and SIM 2 but is the same for all models of the same set.
SIM 1 (linear models):

$$h_i\left(\begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix}\right) = \begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix}$$

SIM 2 (non-linear models):

$$h_i\left(\begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix}\right) = \begin{bmatrix} i \times x_1(k) + 2 \times i \\ \cos(x_2(k)) \end{bmatrix}$$

We take $\lambda V = 3$. All algorithms are initialized with the same $\Psi^{(0)}$ and are performed on the same data sets.
The Root Mean Square Error (RMSE), the average rate of correct data-to-model associations, the relative execution time are given in table 1 for linear models and in table 2 for non-linear ones. NL is put for Non-Linear.

From those results we can point out that the new algorithms are slower than the previous ones but provide as good results when the measurement equation is linear . It is obvious that they are well-adapted to non-linear case.

SIM 1	MCDA	ICMDA	NL-MCDA	NL-ICMDA
RMSE	0.26	0.18	0.24	0.18
correct associations	81.2%	83.3%	81.6%	83.2%
Time	1	0.92	1.9	1.17

Table 1: Results on linear models of sim 1. Computed over 50 independent runs.

SIM 2	MCDA	ICMDA	NL-MCDA	NL-ICMDA
RMSE	120	116	0.47	0.56
correct associations	3%	6%	85%	86%
Time	1	1.21	1.92	1.7

Table 2: Results on non-linear models of sim 2. Computed over 50 independent runs.

We can notice that the NL-MCDA total mean time is nearly twice the MCDA, whereas the NL-ICMDA is only about 1.2 times the ICMDA: for MCDA/NL-MCDA, the total association random sequence length P was given the same value: the new version is actually twice slower than the previous one. Total execution time for the NL-ICMDA is not twice the ICMDA one because it is generally faster to converge: those results show that our algorithms are slower to compute but faster to converge.
We can also notice that ICMDA algorithms results are better than MCDA ones. For these simulations the value of the total association random sequence length P in the MCDA was chosen to give nearly the same computation total time as the ICMDA: if we had set P higher, these results would have been more similar.

4.2 Results on OTH-NOSTRADAMUS radar numerical simulations

For its good performances and its convergence speed we choose to use the NL-ICMDA rather than the NL-MCDA.
We have to consider a target probability detection ($Pd = 0.9$).
We simulate a five-layered ionosphere which turns into five potential propagation models. We assume to know all its true parameters.

Comparison with previous algorithms usable in non-linear context.
We compare NL-ICMDA results to those of MPDA [3] and EMDA [4]. Like the NL-ICMDA the EMDA estimates Ψ_N. It is based on an EM algorithm. The MPDA is an online algorithm which directly estimates ground coordinates $\hat{X}_N = E(X_N|Y_N)$. This expression is developed on every possible data-to-model association.
The three algorithms are applied to OTH Radar Nostradamus through numerical simulations. Results are given in Table 3.
Remark: because of the structure of MPDA it is a nonsense to give the correct association percentage.

The main interest of the MPDA is its low computational

	MPDA	EMDA	NL-ICMDA
RMSE	858.4	347.7	333.11
Time	1	4.99	4.62

Table 3: RMSE on ground range(m) and relative total mean time computed over 50 independent runs.

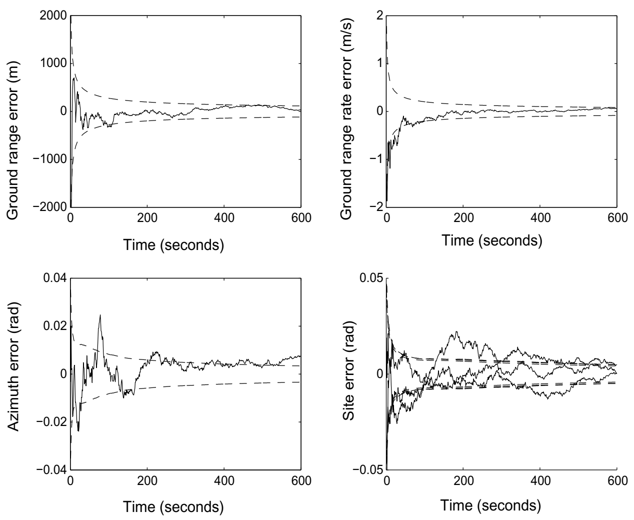

Figure 1: State estimation errors in multipath environment. Estimates are based on the association (propagation model/measurement) given by the NL-ICMDA. $Pd = 0.9$, the target comes toward the radar. Its initial position is: azimuth 120 degrees, ground range 1350 km. Its ground range rate is 120 m/s. Errors are solid, $\pm 3\sigma$ confidence intervals are dashed. There are three elevation angle measurements (called sites) because in this example there are three paths (each one has the same ground range, ground range rate and azimuth but a different elevation angle measurement).

complexity. Better results are obtained with the EMDA and the NL-ICMDA.
NL-ICMDA algorithm has higher performances than the EMDA. Computational time of one iteration of the NL-ICMDA algorithm is slower than one of the EMDA algorithm, but it needs less iteration to converge so that final mean time is in favor of the NL-ICMDA.

Example of a single target tracking using the NL-ICMDA algorithm
Fig.1 shows the results of a simulation with multipath propagation.
When considering a known ionosphere, the NL-ICMDA gives about 90% of correct associations and so the state estimation algorithm provides accurate estimates of the target's state (ground range and rate, azimuth and elevation angle). These performances are not achievable with a simple ICMDA.

5. CONCLUSION

In this paper we proposed two algorithms: NL-MCDA and NL-ICMDA. The method is the same than the MCDA and ICMDA: MCMC approximation to estimate data-to-model association, but their development enables to tackle non-linear problems. They have a higher computational complexity per iteration but other tested algorithms converge slower (MCDA, ICMDA, EMDA) or are less precise (particularly the MPDA algorithm).
NL-ICMDA is thus the algorithm to use to track with the ionosphere dependant OTH-NOSTRADAMUS radar.
From a practical point of view, it may remain two drawbacks:

1. it is not an on-line algorithm
2. it requires to calculate $p(\psi_k|\mathbf{Y}_N, \Psi^{(p)}_{\neg k})$ under all the possible association hypothesis: when we are in a dense clutter environment it generates a large hypothesis set

The first point depends on data acquisition cadence. If it is inferior to the computation time, one has to find a sub-optimal version: a very simple one consists in computing only one iteration of the Kalman filter in the third step of the algorithm. Another one is to generate only the most likely association hypothesis. This latter solution may also be a solution of the second drawback. According to this point of view the computation and the choices of $Pr(\psi_k|\psi_{k-1})$ become important steps of the algorithm, depending on the model to be used.
Future works will focus on developing an algorithm dealing with a non-linear measurement equation with unknown parameters. Indeed, measurement equations depend on the ionospheric characteristics which can only be estimated.

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