HOUSEHOLDER BASED LOW-RANK SPACE-TIME PROCESSOR FOR ANTI-JAMMING NAVIGATION RECEIVERS

S. Werner, M. With and V. Koivunen

Signal Processing Laboratory, SMARAD CoE
Helsinki Univ. of Technology
P.O. Box 3000, FIN-02015 HUT, Finland
Email: {Stefan.Werner, Matias.With, Visa.Koivunen}@hut.fi

ABSTRACT

This paper addresses the problem of mitigating intentional jamming in navigation receivers using smart antennas. For example, GPS receivers provide high precision location information needed in many military and civilian applications. The signal powers employed are so low that navigation receivers are a lucrative target for intentional jamming. Consequently, it is important to use advanced receiver structures that can deal with different types of jamming. Space-time processing methods used in the receiver provide sufficient number of degrees of freedom for cancelling large number of wideband jammers. In this paper a low rank method stemming from multistage Wiener filtering (MSWF) is developed. A Householder transform based method for generating unitary blocking matrices is proposed. The computation of blocking matrices is efficient and the actual blocking operation has low complexity, too. The reliable performance of the method is demonstrated in anti-jamming GPS reception where a good number of both narrowband and wideband jammers are cancelled.

1. INTRODUCTION

High precision satellite navigation systems such as GPS and Galileo [3, 1] are a key component in many modern civilian and military applications. For example, navigation systems are used to guide missiles into target, give real-time information about the location of aircrafts and their tactical situation. In general, situation awareness requires precise information on location of own and adversary troops in ground-based, air and naval operations.

Satellite navigation systems typically apply the principles of spread spectrum communications to navigation. The navigation systems must be able to operate in the presence of interference and intentional jamming. Significant power differences among close-by jammers and distant satellite signal sources make the navigation receiver vulnerable. The despreading gain of the GPS signal, for example, is inadequate for anti-jamming protection. In addition, the jamming signal can be wideband and, therefore, not further suppressed by the despreading at the receiver. Consequently, receivers used in military applications must be robust in the face of jamming, interference and noise. This robustness may be achieved through antenna design, signal processing at the receiver and careful signal design. The receiver needs to have enough degrees of freedom to suppress jammers and simultaneously reliably receive the desired signal.

In this paper we propose an anti-jamming processor for navigation purposes. The method suppresses jammers using both spatial and time-domain processing. Smart antenna systems change their radiation pattern adaptively depending on the signal environment. A numerically stable and low complexity method stemming from multistage nested Wiener filters and Householder transformations [4, 2] is introduced. The Householder transform based processing allows for generating unitary blocking matrices at low computational cost. Moreover, the blocking operation itself becomes computationally efficient since the blocking operation is carried out by a single reflection. Furthermore, dimensions of the data signals are properly reduced at each stage.

This paper is organized as follows. First the core operation of a GPS receiver is explained. Computationally efficient methods based on Householder transformations and multistage Wiener filtering (MSWF) are described then. Finally, some illustrating examples on anti-jamming are presented.

This work was made possible by the grants from the Finnish Defence Forces Research Centre and Academy of Finland, SMARAD Center of Excellence program.

2. OVERVIEW OF THE NAVIGATION SYSTEM

The Global Positioning System (GPS) was developed in 1970's in order to allow military aircrafts, land vehicles and vessels navigate with high precision. More recent European Galileo system will follow similar design principles and use very similar signal structures. In the examples of this paper, we consider GPS system since the parameters of Galileo are not fully fixed yet.

The transmitted GPS signals are direct sequence spread spectrum, DS-SS, signals. The spreading is currently done with two different type of codes, C/A code and P (precision) code. The GPS satellites transmit continuously and employ the same frequency bands but different codes, which means the multiple access technique is CDMA. Currently two different carriers are used, L1 and L2. Also, new carrier frequencies are being deployed and new codes intended for military use introduced. A GPS user demodulates the signals from the minimum of 4 satellites and uses the signals to calculate the distance to the satellites and consequently the position of the user.

3. SPACE-TIME PROCESSOR FOR ANTI-JAMMING

In conventional receivers strong jamming signals can be mitigated by careful antenna design and by using antenna arrays. The performance of such anti-jamming receivers deteriorates significantly if the jammer experiences multipath propagation or the jammers are broadband. This leads to ambiguities such as slightly delayed versions of the signal appear to arrive from many directions.

In space-time receivers for jamming mitigation it is important to have enough delay and sufficient number of taps that the delay spread of jammer multipath is encompassed and broadband jammers can be handled. If there is multipath in the jamming signal or the jammer is wideband, it consumes more degrees of freedom from the array than narrowband jammers. Space-time processing adjusts both spatial antenna weights and time domain filter tap coefficients simultaneously. In practice it means that the dimensions of the matrices needed in solving the unknowns become high. Typical receiver algorithm requires matrix inversion which increases the computational complexity so much that the algorithms may not be feasible in practical receivers.

3.1 Multistage nested Wiener Filtering

Recently, Multistage Nested Wiener Filters (MSWF) [4, 5] have been proposed for anti-jamming GPS receivers. This technique drives the output power of the preprocessor down to noise level by placing nulls in the directions of strong narrowband jammers as well as broadband jammers. The method is appealing since it does not require matrix inversion and needs only small sample support. The underlying idea is to decompose the Wiener filter needed in finding the MMSE space-time coefficients into a nested chain of scalar Wiener filters.

At every stage of the decomposition two subspaces are formed; one in the direction of the correlation vector and the other orthogonal to this direction. The data orthogonal to the correlation vector is then passed to the next stage for further decomposition. This data is obtained by multiplying data vector by a *blocking matrix* that spans the nullspace of normalized cross-correlation vector. The dimension of the data vector is reduced in this operation. Scalar data in the direction of correlation vector serves as the desired signal, and the output of next stage processed with a scalar wiener filter as its estimate.

The algorithm may be summarized as follows. The Wiener filter weights that minimize the MSE are given by

$$\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p} \tag{1}$$

where $\mathbf{R}$ is the covariance matrix of input vector $\mathbf{x}$ and $\mathbf{p}$ is the cross-correlation between input vector $\mathbf{x}$ and desired signal d. The decomposition to nested chain of scalar Wiener filters proceeds the following way [4]:

- *initialization:* $d_0(n) = d(n)$ and $\mathbf{x}_0(n) = \mathbf{x}(n)$
- *Forward Recursion:* For $k = 1, 2, \ldots, D$:

$$\mathbf{p}_k = \frac{E\{d^*_{k-1}(n)\mathbf{x}_{k-1}(n)\}}{\|E\{d^*_{k-1}(n)\mathbf{x}_{k-1}(n)\}\|} \tag{2}$$

$$d_k(n) = \mathbf{p}_k^H \mathbf{x}_{k-1}(n) \tag{3}$$

$$\mathbf{B}_k = null\{\mathbf{p}_k\} \tag{4}$$

$$\mathbf{x}_k(n) = \mathbf{B}_k \mathbf{x}_{k-1}(n) \tag{5}$$

- *Set:* $e_D(n) = d_D(n)$
- *Backward Recursion:* For $k = D, D-1, \ldots, 1$

$$w_k = \frac{E\{d^*_{k-1}(n)e_k(n)\}}{E\{|e_k(n)|^2\}} \tag{6}$$

$$e_{k-1}(n) = d_{k-1}(n) - w_k^* e_k(n) \tag{7}$$

3.2 Novel Householder Based MSWF

This section proposes a novel cross-correlation based Householder Multistage Wiener Filter (HMSWF) that improves the performance and numerical stability as well as lowers the computational complexity of the conventional MSWF. Instead of decomposing the input signal in two subspaces using cross-correlation vector and blocking matrix as the conventional MSWF structure, HMSWF makes the division using only one square Householder transformation matrix, $\mathbf{Q}_k$, in each stage. The main advantage of the HMSWF is that it provides unitary blocking matrices with low computational complexity: both the creation of the blocking matrix and the blocking operation are computationally simple (see Sections 3.3–3.4).

Let $\mathbf{Q}_k$ be the $(M-k+1)\times(M-k+1)$ Householder transformation matrix for the k^{th} stage constructed such that

$$\mathbf{Q}_k\mathbf{p}_k = -\frac{p_{k,1}}{|p_{k,1}|}\mathbf{e}_1, \tag{8}$$

where $\mathbf{e}_1 = [1, 0, \ldots, 0]^T$ is a $(M-k)\times 1$ unit vector and $p_{k,1}$ is the first element of the normalized cross-correlation vector $\mathbf{p}_k$. If matrix $\mathbf{Q}_k$ is partitioned as

$$\mathbf{Q}_k = \left[\begin{array}{c} \mathbf{q}_U^T \\ \mathbf{Q}_L \end{array}\right], \tag{9}$$

where $\mathbf{q}_U \in \mathbb{C}^{(M-k)\times 1}$ and $\mathbf{Q}_L \in \mathbb{C}^{(M-k-1)\times(M-k)}$, it can be easily verified by using equations (8) and (9) that $\mathbf{Q}_L$ is a valid blocking matrix: $\mathbf{Q}_L\mathbf{p}_i = \mathbf{0}$. In other words, the HMSWF could be seen as a conventional MSWF using a unitary blocking matrix $\mathbf{Q}_L$. We stress that the implementation using Householder reflections, provides a solution of much lower complexity.

The HMSWF algorithm multiplies the input signal $\mathbf{x}_k(n)$ at each stage with $\mathbf{Q}_k$. The first element of the resulting vector is a scaled scalar component of $\mathbf{x}_k(n)$ in the direction of $\mathbf{p}_k$ and the rest of the resulting vector are orthogonal to it. To cancel the scaling of the scalar component, it is multiplied with $-\frac{p_{k,1}}{|p_{k,1}|}$. The HMSWF algorithm, presented in Table 3.2, differs from the conventional MSWF algorithm in the *forward recursion* part.

Table 1: Cross-correlation Based Householder Multistage Wiener Filter (HMSWF)
Algorithm

Initialization: $d_0(n) = d(n)$ and $\mathbf{x}_0(n) = \mathbf{x}(n)$
Forward recursion (For $k = 1, 2, \ldots, D$):

$$\mathbf{p}_k = \frac{E\{d^*_{k-1}(n)\mathbf{x}_{k-1}(n)\}}{\|E\{d^*_{k-1}(n)\mathbf{x}_{k-1}(n)\}\|} \tag{10}$$

$$\mathbf{t}_k(n) = [t_{k,1}(n), \mathbf{t}_L^T(n)]^T = \mathbf{Q}_k\mathbf{x}_{k-1}(n) \tag{11}$$

$$d_k(n) = -\frac{p^*_{k,1}}{|p_{k,1}|}t_{k,1}(n) \tag{12}$$

$$\mathbf{x}_k(n) = \mathbf{t}_L(n) \tag{13}$$

Set: $e_D(n) = d_D(n)$
Backward recursion (For $k = D, D-1, \ldots, 1$):

$$w_k = \frac{E\{d^*_{k-1}(n)e_k(n)\}}{E\{|e_k(n)|^2\}} \tag{14}$$

$$e_{k-1}(n) = d_{k-1}(n) - w_k^* e_k(n) \tag{15}$$

3.3 Generation of the Householder Transformation Matrix

A matrix $\mathbf{Q}_k$ of the form

$$\mathbf{Q}_k = \mathbf{I} - 2\mathbf{v}_k\mathbf{v}_k^H, \|\mathbf{v}_k\| = 1 \tag{16}$$

is called a *Householder transformation matrix* [2]. If a vector is multiplied by $\mathbf{Q}_k$, it is reflected to the other side of the hyperplane orthogonal to the vector $\mathbf{v}_i$. The unit norm vector $\mathbf{v}_i$ is called a *Householder reflector*. It can be easily verified that the Householder matrices are symmetric and unitary. In this section we will describe how to generate the vector $\mathbf{v}_k$ to get a Householder transformation matrix that satisfies Equation (8).

The Householder reflector for HMSWF can explicitly be given as follows:

$$\mathbf{v}_k = \frac{\tilde{\mathbf{v}}_k}{\|\tilde{\mathbf{v}}_k\|} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|p_{k,1}|}}\left[\frac{p_{k,1}}{|p_{k,1}|}\mathbf{e}_1 + \mathbf{p}_k\right] \tag{17}$$

where

$$\tilde{\mathbf{v}}_k = \frac{p_{k,1}}{|p_{k,1}|}\|\mathbf{p}_k\|\mathbf{e}_1 + \mathbf{p}_k = \frac{p_{k,1}}{|p_{k,1}|}\mathbf{e}_1 + \mathbf{p}_k \tag{18}$$

The efficient HMWSF structre is illustrated in the Figure 2. The dashed-line box in the figure shows one stage of the decomposition. The normalized cross-correlation vector $\mathbf{p}_k$ is replaced by $\tilde{\mathbf{p}}_k$, and there are two new coefficients at each stage: a_k and c_k. These coefficients operate on scalar data, so their impact on computational

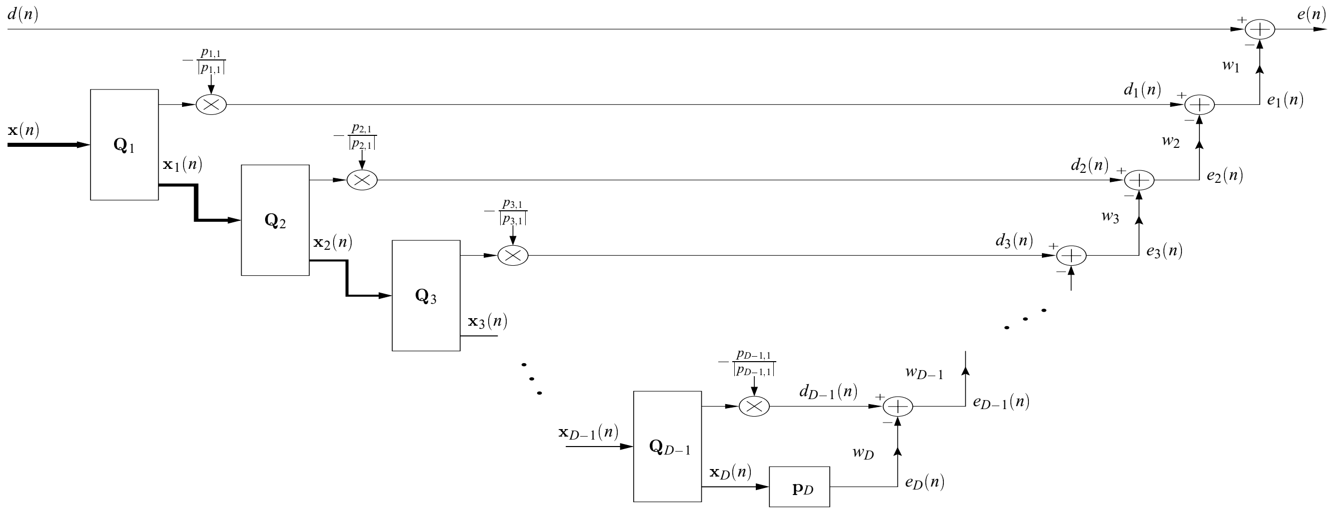

Figure 1: Cross-correlation Based Householder Multistage Wiener Filter (HMSWF)

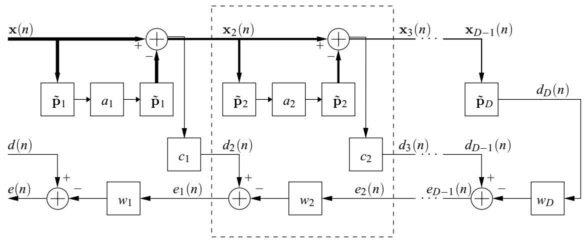

Figure 2: An efficient structure of HMSWF

complexity is very small. The coefficients and the modified cross-correlation vector are calculated as follows:

$$a_k = \frac{1}{1+|p_{k,1}|} \quad (19)$$

$$c_k = -\frac{p_{k,1}}{|p_{k,1}|} \quad (20)$$

$$\tilde{\mathbf{p}}_k = \frac{p_{k,1}}{|p_{k,1}|}\mathbf{e}_1 + \mathbf{p}_k \quad (21)$$

3.4 Computational Complexity Issues

There are two important aspects concerning the computational complexity related to the use of different blocking matrices in the MSWF: (1) the complexity in *constructing* the blocking matrix, and; (2) the complexity of the *blocking operation* or in other words the multiplication $\mathbf{B}_k\mathbf{x}_k(n)$.

The computational complexity for the generation of the blocking matrix may affect its practical use due to the explicit dependency on the cross-correlation vector $\mathbf{p}_k$. In practice $\mathbf{p}_k$ is estimated using batch processing of a block of N consecutive input vectors. In batch processing, the estimate will be constant for that particular block of data and, therefore, $\mathbf{B}_k$ remains constant during the processing. As a consequence, a computationally costly generation of $\mathbf{B}_k$, e.g., using singular-value decomposition (SVD), may only be acceptable if N is much larger than M (dimension of $\mathbf{x}(n)$). The blocking matrix referred to as the correlation subtractive structure (CSS) [6] is probably the most widely used in the literature due to its simple form $(\mathbf{B}_k = \mathbf{I} - \mathbf{p}_k\mathbf{p}_k^H)$. The CSS blocking matrix is a nonunitary matrix of rank $M-1$ and its use can lead to potential problems in a finite precision environent [2]. As for the HMSWF proposed here, given $\mathbf{p}_k$, the only additional computations required to form the reflectors at stage k are due to $|p_{k,1}(k)|$ and $1/|p_{k,1}(k)|$.

The computational complexity of the blocking operation, i.e., the multiplication $\mathbf{B}_k\mathbf{x}_k(n)$, depends on the structure of the blocking matrix. If $\mathbf{B}_k$ is generated through SVD or other unitary decomposition, the computational complexity of the blocking operation may render its use impractical. For the HMSWF decomposition (i.e., blocking and generation of desired signal), the number of multiplications and additions at stage k is approximately $2(M-k+1)$.

Table 3.4 shows the computational complexity (in terms of number of multiplications[1]) of a D-stage decomposition (the analysis part) for the Householder MSWF, the MSWF with a CSS (MSWF-CSS) [6], and the MSWF with a unitary blocking matrix of no special structure (MSWF-SVD). Table 3.4 also shows the additional complexity due to the estimation of $\mathbf{p}_k$. It can be seen that the HMSWF is of lower complexity than the MSWF-CSS for both the decomposition and estimation of $\mathbf{p}_k$. The reason is that dimension of the input-signal $\mathbf{x}(n)$ is not reduced at each stage of the MSWF-CSS, as is the case for the HMSWF and MSWF-SVD.

Table 2: Computational complexity of three MSWF structures, M is the dimension of $\mathbf{x}(n)$, N is the sample support, and D is the rank.

STRUCTURE	# MULT
MWF-SVD	$[MD(M-D)+D(D+1)(D-1)/3]N$
MWF-CSS [6]	$2MDN$
HMWF	$[2MD-D^2]N$
Estimate $\mathbf{p}_k \in \mathbb{C}^{(M-k+1)\times 1}$	$[2MD-D^2]N$
Estimate $\mathbf{p}_k \in \mathbb{C}^{M\times 1}$	$2MDN$

4. SIMULATION RESULTS

In the following, simple anti-jamming examples using a space-time array as a front-end to the actual GPS receiver are presented. A 7-element linear array is used with $\lambda/2$ spacing. Each element has a tapped delay line of 7 taps associated with it and critical sampling in time domain is employed.

The desired navigation signals are C/A codes 22dB below the noise level so there is no danger of canceling the signal of interest. Total of $N = 250$ snapshots are observed by the array. The receiver is receiving navigation data from 2 satellites. Five narrowband jammers with DoAs $[-70, -45, -15, 15, 45]$ degrees are present. Their normalized frequencies (wrt. L1 bandwidth)

[1]The number of additions is roughly the same

are $[0.300, 0.375, 0.525, 0.625, 0.700]$ where 0.5 corresponds to carrier frequency. The jammer to noise ratios (JNR) are [40, 30, 40, 35, 35]dB, respectively. In addition, there are 5 wideband jammers that are all 30dB above the noise level (JNR 30dB). The DoAs of the wideband jammers are $[-60, -30, 0, 30, 60]$. The Householder MSWF is used to estimate the space-time array weights so that the jammers are canceled. The weights are computed using the minimum variance distortionless response beamformer (MVDR) principle that may be formulated as generalized sidelobe canceler (GSC) in space-time domain. The MVDR beamformer was steered towards one of the GPS satellites using a single constraint.

The performance is plotted as a function of the number of stages (rank) in the nested structure. Theoretical bounds for the performance are given by the sample matrix inversion (SMI) method. HMSWF and MSWF had equal performance in the simulation. It appears that in the simulation scenarios used in this study, typically rank 20-25 solution achieves the theoretical bound, see Figure 4. This corresponds to roughly 25% lower computational complexity for the HMSWF as compared to the widely used structure using CSS blocking matrices.

The beampattern produced by the space-time MVDR beamformer obtained via HMSWF processing is plotted in Figure 4, where the array is steered towards the GPS satellite at -10 degrees. Deep nulls associated with wideband jammers can be seen as canyons parallel to the frequency axis. The nulls associated with narrowband jammers appear as deep and narrow pits in the plot. All the jammers are reliably nulled.

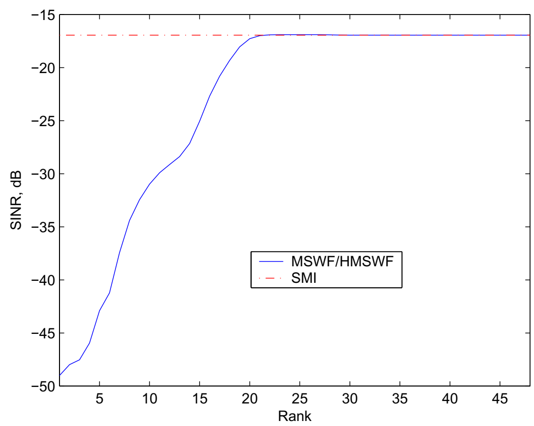

Figure 3: SINR after anti-jamming processing of the signal from satellite 1 as a function of rank of conventional and Householder multistage nested Wiener filters.

The overall performance of the proposed HMSWF is practically similar to the conventional MSWF. However, computational complexity is lower and numerical stability is better, which is important for example in practical implementations where fixed-point arithmetics are used.

5. CONCLUSION

In this paper we proposed a computationally efficient and numerically stable method for anti-jamming navigation receivers. The receiver algorithm is based on Householder transformations and multistage Wiener filtering. The method has lower computational complexity than conventional MSWF in terms of computing the blocking matrix as well as the blocking operation itself. The method produced blocking matrices by design. The proposed anti-jamming processor cancels effectively large number of different type of jammers. A typical example showed a 25% reduction in computations using the proposed method as compared with a widely used standard approach without compromising the performance.

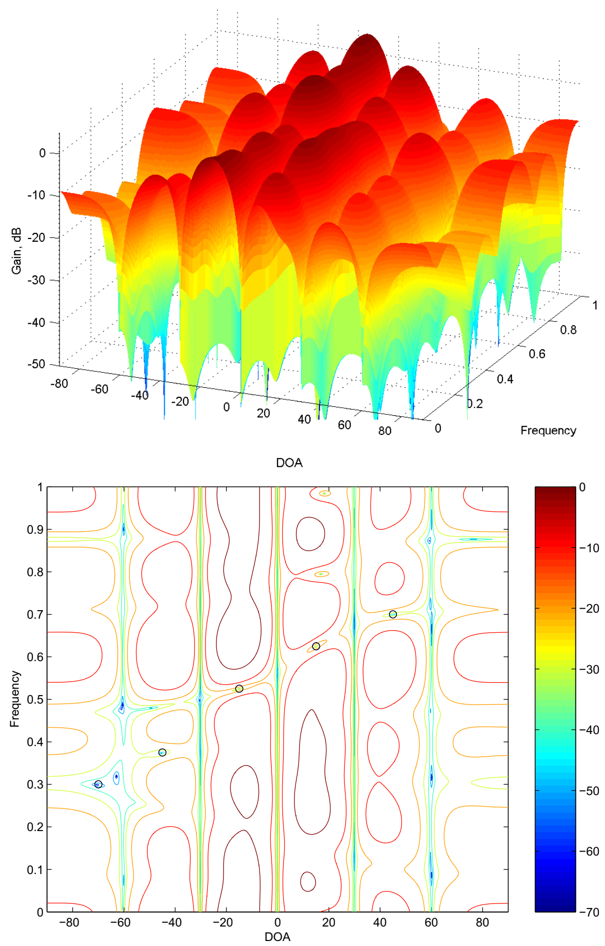

Figure 4: Beampattern of the ST-array obtained via Householder MSWF processing. Deep nulls associated with wideband jammers can be seen as canyons parallel to the frequency axis. The nulls associated with narrowband jammers appear as deep pits in the plot. The locations of the narrowband jammers in terms of angle of arrival and frequency are marked by small circles in the lower plot.

REFERENCES

[1] Kaplan, E., (editor) (1996) *Understanding GPS Principles and Applications*, Artech House Publishers, 1996.

[2] M. L. R. de Campos, S. Werner, J. A. Apolinrio Jr., "Constrained adaptation algorithms employing Householder transformation," *IEEE Trans. on Signal Processing*, vol. 50, pp. 2187-2195, Sept. 2002.

[3] Misra, P., Enge, P., *Global Positioning System–Signals, Measurements and Performance*, Ganga-Jamuna Press, 2001.

[4] J.S. Goldstein, I.S. Reed, and L.L. Scharf, "A multistage representation of the Wiener filter based on orthogonal projections," *IEEE Trans. Inform. Theory*, vol. 44, pp. 2943-2959, Nov. 1998.

[5] W.L. Myrick, J.S. Goldstein, and M.D. Zoltowski, "Low complexity anti-jam space-time processing for GPS," *Proc. ICASSP'2001*, Salt Lake City, USA.

[6] D. C. Ricks, J. S. Goldstein, "*Efficient structures for implementing adaptive algorithms,*" in Proc. Antenna Applications Symp., Allerton Park, Monticello, IL, 2000.