Reduced-Complexity Genetic Algorithm Aided and Radial Basis Function Assisted Multiuser Detection for Synchronous CDMA

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Abstract

Radial Basis Function Network aided Multiuser Detection (RBFN-MUD) schemes are capable of detecting the received signal of all users, even if the channel output states are linearly non-separable. However, their complexity may become excessive which renders their real implementation irrealistic, except when the number of users is low. In this contribution a novel reduced-complexity RBFN-MUD is developed, which invokes Genetic Algorithms (GAs) for reducing the number of RBFN-MUD centres. Our computer simulations showed that GAs are capable of considerably reducing the complexity imposed at the cost of a slight performance degradation.

1. INTRODUCTION

Direct Sequence Code Division Multiple Access (DS-CDMA) [1] is the transmission technique employed by all the third generation (3G) systems. However, there are a number of problems associated with transmitting at high bit rates, such as 2*Mb/s*, since the channel-induced dispersion may span several bits, resulting in grave Inter Symbol Interference (ISI) and Inter Chip Interference (ICI). Therefore, in order to jointly mitigate the effects of the ISI, ICI and Multiuser Interference (MUI), the employment of multiuser equalization or detection (MUD) techniques has been proposed in [2] [3].

When communicating over dispersive fading channels, the Channel Impulse Response (CIR) of each user is convolved with the user's spreading code and this signal is superimposed on the composite signal of the $K-1$ interfering users. Hence the superposition of the K dispersed users' phasor constellation may become linearly non-separable even in the absence of noise, which would result in a residual bit error ratio (BER), unless a non-linear MUD is employed, such as the proposed Radial Basis Function (RBF) aided MUD. The RBF-aided MUD has been originally proposed in [4], and was further investigated in [5] [6]. However, until recently its excessive complexity rendered its real-time implementation irrealistic, except when the number of users is low.

The Genetic Algorithm (GA)-based MUD was first proposed by Juntti *et al.* [7] for a synchronous CDMA system communicating over an Additive White Gaussian Noise (AWGN) channel. Yen *et al.* [1, 8] further improved the performance of GA-aided MUD, demonstrating that the performance of the GA-based MUD approaches the single-user performance bound at a significantly lower computational complexity, than that of Verdu's optimum MUD [2].

In this contribution we proposed a GA and RBFN assisted MUD for a synchronous DS-CDMA system transmitting over non-dispersive AWGN and L-path dispersive Rayleigh fading channels using the truncation window approach [2]. According to [1, 6], a RBFN-aided MUD requires 2^K number of RBFN centres, when communicating over a non-dispersive channel using a BPSK modulation scheme, where K is the number of users supported. The required number of RBF centres is set to 2^{LK}, when communicating over a dispersive channel, hence the complexity of RBFN-aided MUD is on the order of $O(2^K)$ and $O(2^{LK})$, respectively. In our proposed GA and RBFN assisted MUD we are capable of reducing the complexity imposed from $O(2^K)$ or $O(2^{LK})$ to $O(P)$, where P is the GA's population size. This is achieved with the aid of finding the RBFN's high-contribution centres and discarding the low-contribution centres, while maintaining a near-single-user performance.

The remainder of this paper is organised as follows. Section 2 briefly describes the philosophy of the RBFN-aided MUD, while Section 3 highlights the concept of the GA-RBFN-aided MUD employed in both non-dispersive and dispersive propagation environments. The GA-RBFN-based MUD's performance is characterised in Section 4, while Section 5 concludes the paper.

2. RBFN-AIDED MUD

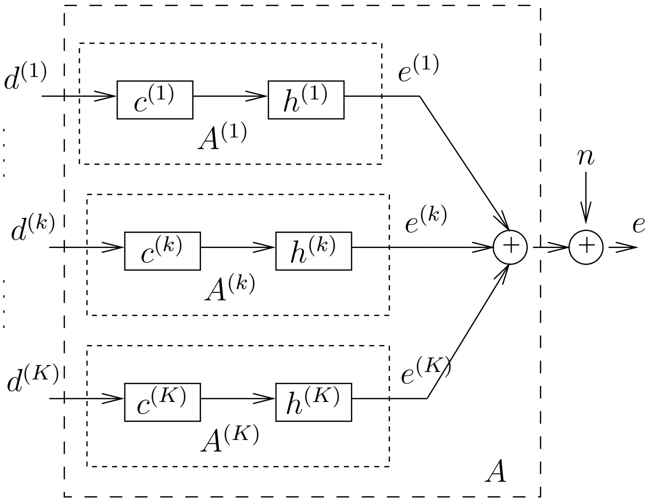

Figure 1: DS-CDMA system model designed for communicating over multipath channels described by the Channel Impulse Response (CIR) of $h^{(i)}$, where $c^{(i)}$ is the ith user's spreading code.

Figure 1 shows the rudimentary model of synchronous CDMA. According to [3], the received signal vector can be written as:

$$\mathbf{e} = \mathbf{A} \cdot \mathbf{d} + \mathbf{n} = \mathbf{v} + \mathbf{n}, \tag{1}$$

where the matrix $\mathbf{A}$ is the so-called DS-CDMA system matrix, where each column is constituted by the convolution of the CIR and the a user's spreading code, as seen in Figure 2 and in more detail on page

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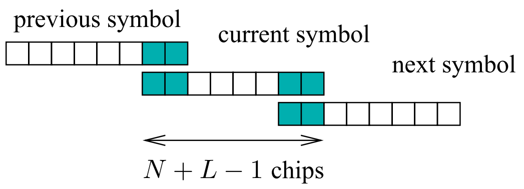

Figure 2: ISI-contaminated DS-spread bits using N-chip spreading in a DS-CDMA system subjected to channel-induced dispersion.

96 of [1]. Furthermore, $\mathbf{e}$ is the composite K-user received signal vector, the vector $\mathbf{d}$ hosts all the users' serially concatenated transmitted data, while the noise vector $\mathbf{n}$ satisfies $E[\mathbf{nn}^T] = I\sigma_n^2$. Finally, the noise variance is $\sigma_n^2 = N_0/2$, where $N_0/2$ is the double-sided noise power spectral density encountered across the signal's bandwidth.

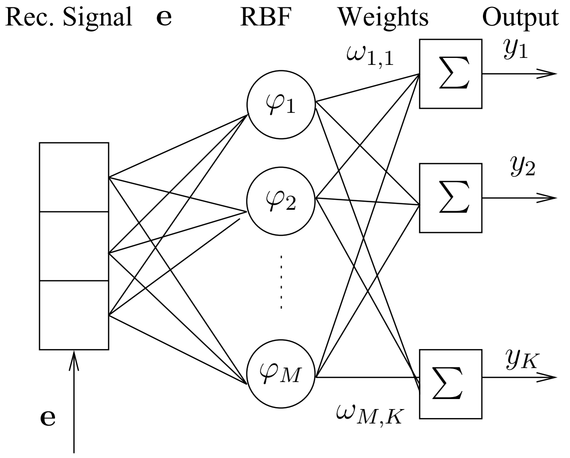

Figure 3: Architecture of the RBF-aided MUD.

The architecture of the RBFN-based MUD is shown in Fig 3, where each of the K outputs can be represented as follows:

$$y_k = \sum_{i=1}^{M} w_{i,k}\varphi_i \cdot (\mathbf{e}) \quad k = 1, 2, ...K, \tag{2}$$

where $w_{i,k}$ denotes the weights [10] of the RBFN-aided MUD, $M = 2^K$ or 2^{LK} is the number of RBF centres, $\varphi_i(\mathbf{x}), i = 1 \ldots M$ is the ith the RBF. Since bit-by-bit MUD is employed, the $N + L - 1$-component vector $\mathbf{e}$ constitutes the MUD's input, while the RBFs $\varphi_i(\mathbf{e})$ are given by [10]:

$$\varphi_i(\mathbf{e}) = exp(-\frac{\parallel \mathbf{e} - \mathbf{v}_i \parallel^2}{2\sigma_i^2}) \quad i = 1...M = 2^K, \tag{3}$$

where $\mathbf{v}_i = [v_1...v_{N+L-1}], i = 1, 2...M$ denotes the vector of noiseless RBF centres [10]. Each of the K outputs of Figure 3 corresponds to the detected signal of one of the K users, where the RBF weights $w_{i,k}$ provide the appropriate weighting for producing a high-confidence K-dimensional received bit vector, and again, $M = 2^K$ or 2^{LK} is the number of the channel output states, which is also equal to the number of RBF centres. All the M RBFN centre outputs are weighted by $w_{i,k}$ and summed up, as seen in Figure 3, resulting in the MUD's output vector $\mathbf{y} = [y_1, y_2, ...y_K]$

When considering a non-dispersive channel, there are $M = 2^K$ possible combinations of the K bits of the K users, which are hosted by the noiseless channel output vector $\mathbf{v}$ corresponding to all possible combinations of the K users' transmitted bits. Each of the $M = 2^K$ possible noiseless channel output vectors can be adopted as an RBFN centre, which are hosted by the vector $\mathbf{v}_i$, $i = 1 \ldots M$:

$$\mathbf{v}_i = \mathbf{A} \cdot \mathbf{d}_i \qquad i = 1, 2, \ldots, M = 2^K, \tag{4}$$

where $\mathbf{v}_i = [v_i^{(1)}, ..., v_i^{(N)}]$, $\mathbf{d}_i = [d_i^{(1)}, ..., d_i^{(K)}]$. Hence $\mathbf{v}_i$ hosts the noiseless channel output corresponding to the transmitted K-bit data vector $\mathbf{d}_i$. However, owing to the channel-induced ISI encountered over a dispersive channel, which was shown in Figure 2 in a stylized format, the effects of the current direct-sequence (DS) spread symbol will spill over to the adjacent symbols, as seen on page 96 of [1]. Hence, we have to take into account both the previous and the next DS-spread symbol, when detecting the current symbol. In other words, the multipath-induced ISI requires us to increase the number of RBF centres quite considerably. For example, in conjunction with BPSK transmission and a 1-bit duration dispersion-induced pre- and post-cursor we have to consider a CIR duration of $L = 3$, which results in $M = 2^{3K}$ RBF centres that maybe obtained by convolving all the 2^K possible number of legitimate combinations of the transmitted data bits with the Channel Impulse Response (CIR). Therefore, we can obtain the vector $\mathbf{v}_i$ and $\mathbf{d}_i$ when communicating over a dispersive channel having an $L = 3$ symbol-duration CIR in the form of: $\mathbf{v}_i = [v_i^{(1)}, ..., v_i^{(3N)}]$, $\mathbf{d}_i = [d_i^{(1)}, ..., d_i^{(3K)}]$.

According to [10], the RBFN weight parameter $w_{i,k}$ may assume a limited set of two values, namely ±1. More explicitly, in our scenario we have:

$$\begin{aligned} w_{i,k} &= +1 \ \text{ if } \ d_i^{(k)} = +1 \qquad (5) \\ w_{i,k} &= -1 \ \text{ if } \ d_i^{(k)} = -1. \qquad (6) \end{aligned}$$

Therefore, the output vector $\mathbf{y}$ of the RBFN-aided MUD seen in Figure 3 can be represented as follows:

$$\mathbf{y} = \mathbf{W} \cdot \varphi, \tag{7}$$

where we have $\varphi = (\varphi_1(e), ..., \varphi_M(e))^T$, and $\mathbf{W}$ is :

$$\mathbf{W} = \begin{bmatrix} w_{1,1} & w_{2,1} & \ldots & w_{M,1} \\ w_{1,2} & w_{2,2} & \ldots & w_{M,2} \\ \vdots & \vdots & \vdots & \vdots \\ w_{1,K} & w_{2,K} & \ldots & w_{M,K} \end{bmatrix}.$$

3. GA AND RBFN ASSISTED MUD

From Figure 3 we can observe that the total number M of channel output states is $M = 2^K$ or 2^{3K} in the *non-dispersive* and *dispersive* scenario, respectively, when no complexity reduction techniques are employed. This complexity may become excessive, if the number of users K is relatively high. In order to circumvent this problem, we invoke GAs for identifying a reduced subset of the high contribution RBFN centres. For a detailed discussion on GAs and on GA-MUD schemes, the interested readers are referred to [1, 8, 9].

Figure 4 shows the structure of the proposed GA and RBFN assisted MUD, where we invoked GAs for selecting a reduced subset of the legitimate RBFN centres, which have the highest contribution to the RBFN-based MUD's output formulated in Equation 2. During the GA-assisted RBFN centre selection process, the specific centres, which have a low contribution to the output of the GA-RBFN MUD formulated in Equation 2 are discarded.

Assuming that the population size of the GA is P, we can reduce the number of RBFN centres required in each generation from $M = 2^K$ or 2^{3K} to P. Then the complexity of the GA and RBF aided

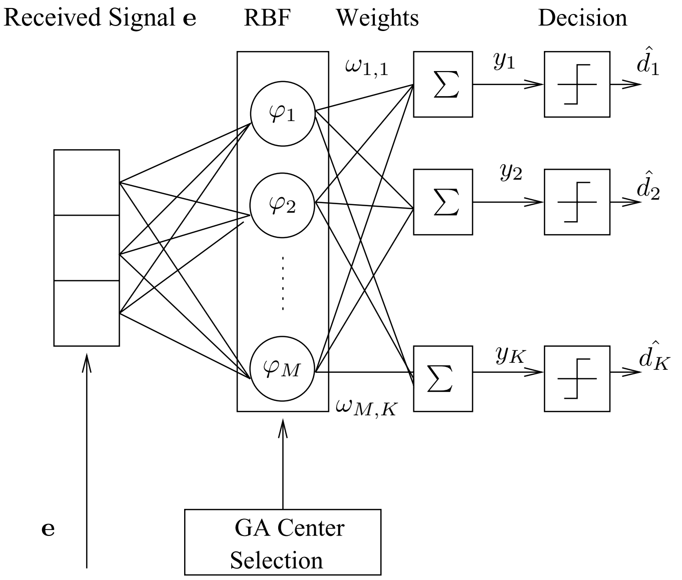

Figure 4: Schematic of the GA-RBF assisted MUD.

MUD is determined by the product of P and the number of generations Y, *i.e.* it is $O(P \cdot Y)$. Our goal is to minimize the MUD's performance degradation, while maintaining an affordable complexity. The GA and RBFN assisted MUD's output vector is expressed as $\mathbf{y} = [y_1, ..., y_K]$, where the kth element of the vector $\mathbf{y}$ is given by:

$$y_k = \sum_{i=1}^{P} w_{i,k} \varphi_i \cdot (\mathbf{e}) \quad k = 1, 2, ...K. \tag{8}$$

Therefore, the output vector $\mathbf{y}$ of the GA and RBFN aided MUD seen in Figure 4 can also be expressed as follows:

$$\mathbf{y} = \mathbf{W} \cdot \varphi, \tag{9}$$

where we have $\varphi = [\varphi_1(e), ..., \varphi_P(e)]^T$ and where $\mathbf{W}$ is :

$$\mathbf{W} = \begin{bmatrix} w_{1,1} & w_{2,1} & \dots & w_{P,1} \\ w_{1,2} & w_{2,2} & \dots & w_{P,2} \\ \vdots & \vdots & \vdots & \vdots \\ w_{1,K} & w_{2,K} & \dots & w_{P,K} \end{bmatrix}.$$

In order to select the most influential RBFN centres having the highest contribution to Equation 8, we employ the RBFN formula of Equation 3 as the GA's objective function. Based on Equation 3, the GA aided selection process will assist us in identifying the specific set of RBFN centres $\mathbf{v}_i = [v_1, v_2, ...v_N]$, $i = 1, ...M$, which are the closest ones to the received signal vector $\mathbf{e}$. Hence these centres have the highest impact on the performance of the GA aided and RBFN assisted MUD. For more information on RBF-assisted receivers, the interested reader is referred to [10].

4. SIMULATION RESULTS

The basic system parameters are listed in Table 1, and the spreading codes employed in our system are m-sequences. In Section 3 we argued that amongst other factors, the GA's convergence accuracy is dramatically influenced by the population size P [1]. As expected, it is seen in Figure 5 that the convergence accuracy and the achievable

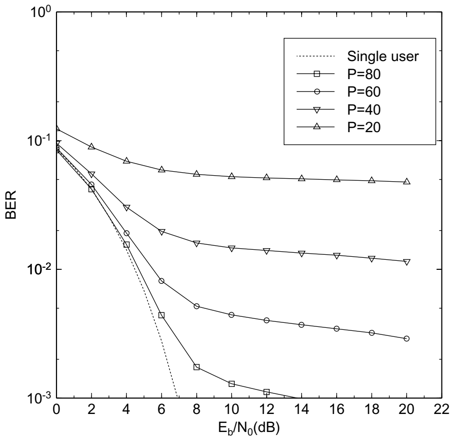

Figure 5: **Effects of the GA's population size**: BER versus E_b/N_0 performance of the GA-RBF assisted MUD, when communicating over **non-dispersive AWGN** channels, while supporting K**=20 users**. The population size was $P = 20, 40, 60, 80$, respectively, and the number of generations was Y=6. The system parameters are summarised in Table 1. **The complexity reduction factor was** $2^{20}/480 \approx 2185$**.**

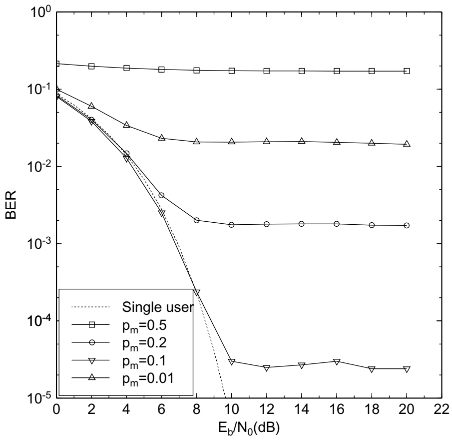

Figure 6: **Effects of the GA's mutation probability**: BER versus E_b/N_0 performance of the GA-RBF assisted MUD when communicating over non-dispersive AWGN channels, while supporting K**=10 users**. The mutation probability was set to p_m = 0.01, 0.1, 0.2 and 0.5, respectively. The other parameters are summarised in Table 1. **The complexity reduction factor was** $2^{10}/6 \cdot 40 \approx 4.3$**.**

Modulation scheme	N=31-chip BPSK/CDMA
GA's selection method	Fitness-proportionate
GA's mutation method	Standard binary mutation
GA's crossover method	Single-point crossover
GA's mutation probability probability p_m	0. 1
GA's crossover probability p_c	0. 5
GA's number of of generations Y	6

Table 1: The configuration of the GA and RBFN assisted MUD employed in our system.

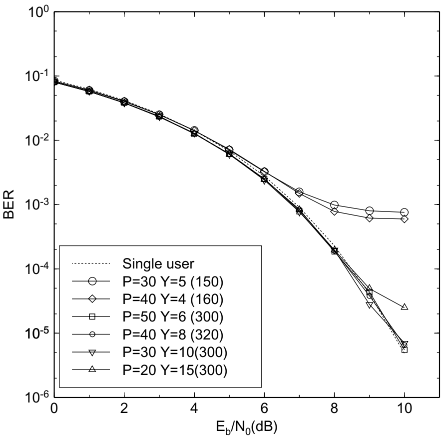

Figure 7: BER versus E_b/N_0 performance of the GA-RBF assisted MUD, when communicating over the non-dispersive AWGN channel, while supporting $K = 10$ **users** employing m-sequences as spreading codes. As seen in the legends, certain configurations have the same or a similar total complexity of $P \cdot Y$, but different P and Y values. **The complexity reduction factor was** $2^{10}/P \cdot Y \approx 3.2 \ldots 6.8$**.**

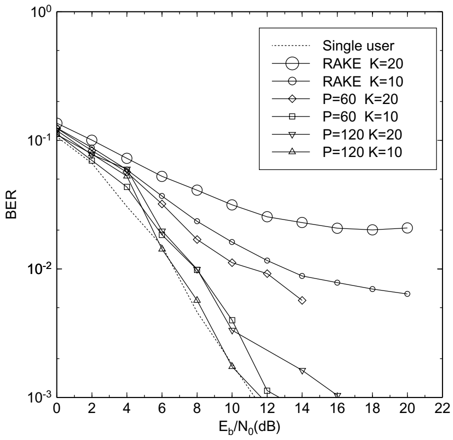

Figure 8: BER versus E_b/N_0 performance of the GA-RBF assisted MUD, when communicating over a three-path Rayleigh-fading channel having a transfer function of $H(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2}$ and different population sizes P, while supporting $K = 10$ **and** $K = 20$ users. The other system parameters are listed in Table 1. **The complexity reduction factor was** $2^{K=20}/6 \cdot 120 \approx 1456$**.**

BER of the detector improved as the population size was increased, and vice versa.

According to Figure 6 we may observe that from the set of mutation probabilities considered, $p_m = 0.1$ offers the best performance, although the conventionally recommended value of p_m typically found in the literature [9] is confined to the range of $0.001 - 0.01$. The associated effects of the mutation probabilitiy are interpreted as follows. If the mutation probability is too low, the diversity of individuals in the population will be insufficient for exploring the entire 2^K-element transmitted data space. This phenomenon hence might lead to premature GA convergence, finding a local rather than global optimum owing to the associated lack of population diversity. By contrast, if the mutation probability is excessive, it may lead to a poor final K-bit solution due to the birth of an excessive number of low-fitness individuals. Therefore, there is a trade-off between achieving sufficient GA population diversity and a high convergence rate. Again, in our specific MUD problem, the choice of $p_m = 0.1$ provided the best trade-off.

In Figure 7 we can observe that the achievable BER performance is similar for the different GA configurations having the same computational complexity related to $O(P.Y)$, regardless of the specific choice of P and Y. The GA and RBFN assisted MUD exhibits a reduced complexity owing to the reduction of the number of RBFN centres from 2^K to P.

Figure 8 portrays the performance of the GA aided and RBFN assisted MUD, when communicating over a three-path Rayleigh fading channel. We can observe in the figure that the GA and RBF assisted MUD is capable of jointly suppressing the effects of the MUI, ISI and ICI, attaining a considerably better performance, than the classic Rake receiver [1]. As seen in Figure 8, the GA-RBF assisted MUD is capable of approaching the single-user performance, provided that its complexity is sufficiently high for exploring the 2^{3K}-element, K-user search space. Yet, its complexity is substantially lower than that of Verdu's optimum MUD [2], reducing the complexity imposed from $O(2^{3K})$ to $O(P.Y)$.

5. CONCLUSIONS

In this contribution we investigated a range of GA-RBFN assisted MUDs in terms of their computational complexity and achievable BER performance, when communicating over both non-dispersive AWGN channels and dispersive Rayleigh-fading channels. We demonstrated that the GA-RBFN assisted MUD exhibits a significantly lower complexity, than the traditional RBFN-aided MUD, although this complexity reduction is achieved at the cost of a slight performance degradation. The proposed technique is capable of approaching the optimum performance of the full-complexity RBFN-based MUD at the cost of increasing the population size, also approaching the single-user performance.

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