EFFICIENT DESIGN OF ADAPTIVE COMPLEX NARROWBAND IIR FILTERS

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ABSTRACT

In this paper a new adaptive complex digital filter structure is proposed. First, a very low sensitivity second-order complex bandpass (BP) filter section with independent tuning of the central frequency and the bandwidth (BW) is developed (the low sensitivity of the narrow-band realization is ensuring a higher BW-tuning accuracy or better precision in a severe coefficient quantization). Then, a BP/Bandstop (BS) adaptive filter structure is formed around this section, using LMS algorithm to adapt the central frequency. The developed filter circuit is providing a low computational complexity and a very fast convergence, and is convenient for cancellation/enhancement of complex sinusoids or narrowband signals. All theoretical results in the work are verified experimentally.

1. INTRODUCTION

The area of narrowband signal elimination/enhancement has been widely investigated during the last years. A great number of realizations for real narrowband filters has been proposed [1] – [4]. At the same time only a few complex counterparts have been developed and completely studied [5], [6]. In this work a new digital adaptive complex narrowband filter is proposed and investigated.

First, very low magnitude sensitivity narrowband first-order lowpass (LP) filter section is selected. Then, using a general rotation transformation, second-order complex coefficient bandpass (BP) section is obtained and its central frequency is made variable without limitations by tuning the transformation factor θ. The bandwidth (BW) of the section is made tunable by using the spectral LP to LP transformation followed by truncated Taylor series expansion. The new complex filter conducts well in finite wordlength environment and demonstrates very-low coefficient sensitivity.

Having in mind above-mentioned good characteristics of this particular type of complex filter, we use it for the design of an adaptive complex narrowband filter. The behaviour of the filter has been tested with different values of the main filter parameters, namely, step of adaptation and filter bandwidth.

2. COMPLEX DIGITAL FILTER CIRCUIT

In [7], a new method of designing complex coefficient BP and BS filters with independently tunable center frequency and bandwidth is proposed. This method ensures wider range of tuning of the bandwidth, lower stopband sensitivity, reduced complexity and higher freedom of tuning compared to the other well-known methods. We apply this method to obtain a new complex digital adaptive narrow-band second order filter section.

It is well known that if the variable z in a given real digital N-order transfer function $H(z)$ is substituted by

$$z = e^{-j\theta} z \text{ or } z^{-1} = (\cos\theta + j\sin\theta)\, z^{-1}, \qquad (1)$$

the new complex coefficient transfer function $H(e^{-j\theta})$ will be a 2N-order BP/BS filter. The complex circuit realizations always have two inputs and two outputs (both real and imaginary), i.e. it is described by 4 transfer functions:

$$H_{RR}(z) = H_{II}(z) \text{ and } H_{RI}(z) = -H_{IR}(z). \qquad (2)$$

If the starting function H(z) is LP, then all transfer functions (2) are of BP type. If the initial function $H(z)$ is of HP-type, then some of transfer functions (2) are of BS type. The new complex filter may have its central frequency everywhere on the frequency axis (0-π) tuned by changing of θ. The filter's bandwidth will be tuned in some limits using the LP to LP spectral transformations of Constantinides

$$z^{-1} \rightarrow \frac{z^{-1} - \chi}{1 - \chi z^{-1}} = T(z)\,, \qquad (3)$$

followed by truncated Taylor series expansion in order to avoid delay-free loops. But we can avoid the usage of truncated Taylor series if we can tune directly the BW of the initial LP/HP filter by trimming a single multiplier coefficient.

When transformation (1) is applied, it was shown in [8] that all the properties (including sensitivity) of the prototype LP/HP structures will be inherited by the new complex BP/BS filter. Therefore we shall try first to develop or select a very low-sensitivity prototype for a given pole-disposition and then to apply transformation (1) in order to obtain a complex BP section with high accuracy of tuning of the BW.

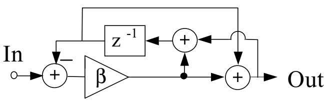
It is well known that mainly narrow-band BP/BS filters are of practical importance. Such filters are obtained starting from very narrow-band LP and wide-band HP prototypes.

As we need a narrow-band BP complex structure, a narrow-band (having poles near $z = +1$) LP structure with very low-sensitivity has to be develop or found.

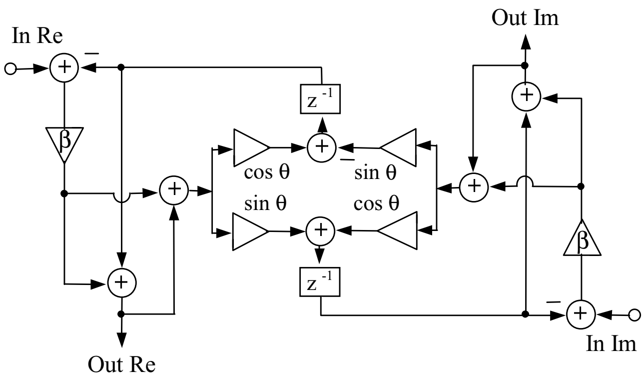
After sensitivities investigation of the most often used first-order sections, it was found that one of the best applicant for LP prototype circuit appeared to be our section LS1 (Fig.1a), proposed in [8]. It has a canonical number of multipliers and delay elements and a unity DC gain. The transfer function of this section is:

$$H_{LP}(z) = \beta \frac{(1+z^{-1})}{1-(1-2\beta)z^{-1}} \qquad (4)$$

Applying the circuit transformation proposed by Watanabe and Nishihara [9] on this LS1 section, a BP second-order complex realization is obtained (Fig.1b). This transformation guarantees also a canonical number of elements for the complex structure.



(a)



(b)

Figure 1: (a) Low-sensitivity first order LP section LS1; (b) Second-order low-sensitivity complex BP structure realization.

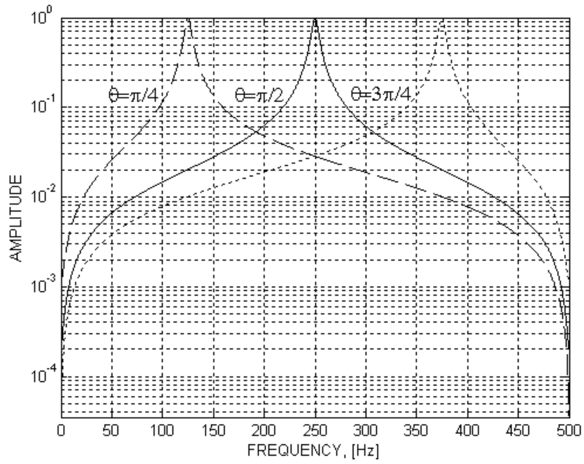
The transfer functions of the low-sensitivity prototype-based complex section (Fig. 1b) are:

$$H_{RR}(z) = H_{II}(z) = \beta \frac{1+2\beta\cos\theta z^{-1} + (2\beta-1)z^{-2}}{1+2(2\beta-1)\cos\theta z^{-1} + (2\beta-1)^2 z^{-2}};$$
$$H_{RI}(z) = -H_{IR}(z) = \beta \frac{2(1-\beta)\sin\theta z^{-1}}{1+2(2\beta-1)\cos\theta z^{-1} + (2\beta-1)^2 z^{-2}}. \qquad (5)$$

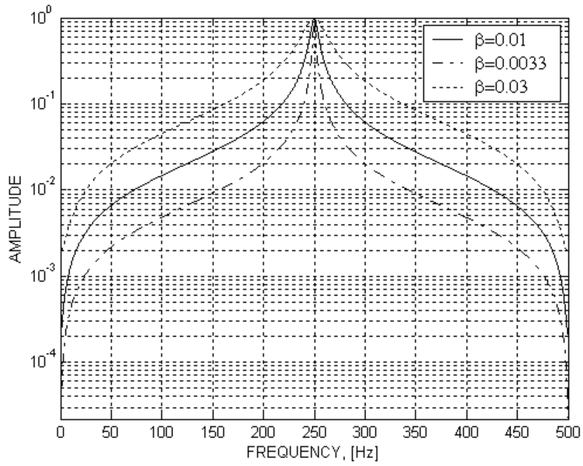
All these transfer functions are of BP type.

In Fig.2a it is shown how the central frequency of the magnitude response of $H_{RR}(z)$ (5) of the narrow-band second-order sections of Fig. 1b is tuned by trimming of θ (for fixed β). The results for the bandwidth tuning by changing β (for fixed θ) are shown in Fig. 2b. It is seen that the bandwidth is tuned without problem over a wide frequency range and the shape of the magnitude is almost not varying in the tuning process.

The behavior of the filter in a limited word-length environment also was investigated. Due to the very low coefficient sensitivity of the initial section, our filter is behaving very well even when the coefficients are severely quantized.



(a)



(b)

Figure 2: Magnitude responses of variable BP complex second-order filter (a) for different values of θ; (b) for different values of β.

3. ADAPTIVE COMPLEX NARROWBAND FILTERING

In Fig. 3 the block-diagram of our narrowband filter section is shown. In the following we consider the input/output relations for corresponding BP/BS filters (Eq.(6)-(13)).

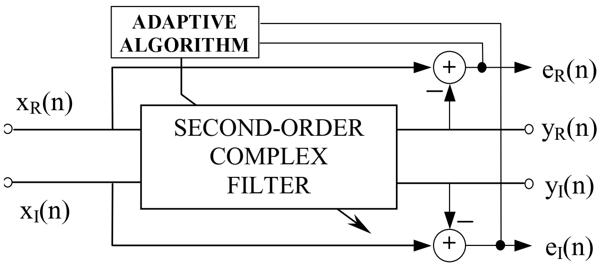


Figure 3: Block-diagram of a BP/BS adaptive complex filter section.

For the BP filter we have the following real output:

$$y_R(n)=y_{R1}(n)+y_{R2}(n)\,, \tag{6}$$

where

$$\begin{aligned}y_{R1}(n)=&-2(2\beta-1)\cos\theta(n)y_{R1}(n-1)-(2\beta-1)^2y_{R1}(n-2)+\\&+2\beta x_R(n)+4\beta^2\cos\theta(n)x_R(n-1)+2\beta(2\beta-1)x_R(n-2)\end{aligned} \tag{7}$$

$$\begin{aligned}y_{R2}(n)=&-2(2\beta-1)\cos\theta(n)y_{R2}(n-1)-(2\beta-1)^2y_{R2}(n-2)-\\&-4\beta(1-\beta)\sin\theta(n)x_I(n-1).\end{aligned} \tag{8}$$

The imaginary output is given by the following equation:

$$y_I(n)=y_{I1}(n)+y_{I2}(n)\,, \tag{9}$$

where

$$\begin{aligned}y_{I1}(n)=&-2(2\beta-1)\cos\theta(n)y_{I1}(n-1)-(2\beta-1)^2y_{I1}(n-2)+\\&+4\beta(1-\beta)\sin\theta(n)x_R(n-1)\end{aligned} \tag{10}$$

and

$$\begin{aligned}y_{I2}(n)=&-2(2\beta-1)\cos\theta(n)y_{I2}(n-1)-(2\beta-1)^2y_{I2}(n-2)+\\&+2\beta x_I(n)+4\beta^2\cos\theta(n)x_I(n-1)+2\beta(2\beta-1)x_I(n-2).\end{aligned} \tag{11}$$

For the bandstop filter we have the real output

$$e\ (n)=x\ (n)-y\ (n)\,, \tag{12}$$

and the imaginary output

$$e_I(n)=x_I(n)-y_I(n)\,. \tag{13}$$

The cost-function is the power of bandstop filter output signal:

$$[e(n)e^*(n)]\,, \tag{14}$$

where

$$e(n)=e\ (n)+je_I(n)\,. \tag{15}$$

We apply a Least Mean Squares (LMS) algorithm to update the filter coefficient responsible for the central frequency as follows:

$$\theta(n+1)=\theta(n)+\mu\mathrm{Re}[e(n)y^{'*}(n)]\,. \tag{16}$$

Where μ is the step size controlling the speed of convergence, (*) denotes complex-conjugate, $y'(n)$ is a derivative of $y(n)=y\ (n)+jy_I(n)$ with respect to the coefficient - subject of adaptation,

$$\begin{aligned}y'_R(n)=&2(2\beta-1)\sin\theta(n)y_{R1}(n-1)-4\beta^2\sin\theta(n)x_R(n-1)+\\&+2(2\beta-1)\sin\theta(n)y_{R2}(n-1)-4\beta(1-\beta)\cos\theta(n)x_I(n-1)\end{aligned} \tag{17}$$

and

$$\begin{aligned}y'_I(n)=&2(2\beta-1)\sin\theta(n)y_{I1}(n-1)+\\&+4\beta(1-\beta)\cos\theta(n)x_R(n-1)+\\&+2(2\beta-1)\sin\theta(n)y_{I2}(n-1)-4\beta^2\sin\theta(n)x_I(n-1).\end{aligned} \tag{18}$$

In order to ensure the stability of the adaptive algorithm we should set the range of the step size μ. We use the results reported in [10]:

$$0<\mu<\frac{\mathrm{K}}{\mathrm{Trace(R)}}\,, \tag{19}$$

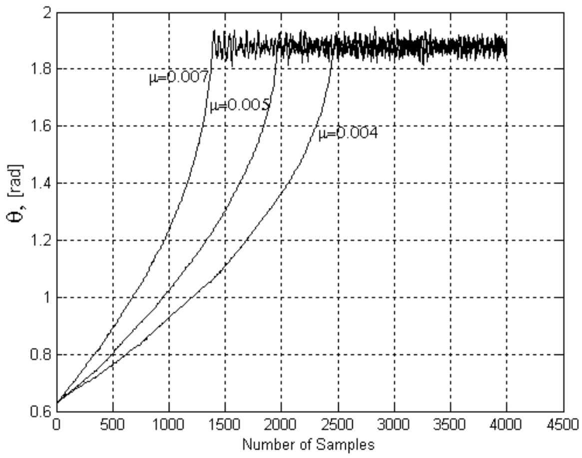
or in a more convenient form:

$$0<\mu<\frac{\mathrm{K}}{\mathrm{L}\,\sigma^2}\,. \tag{20}$$

In our case σ^2 is the power of the signal $y'(n)$, L is the filter order and K is a constant depending on the statistical characteristics of the input signal. In most of the practical situations K is approximately equal to 0.1.

4. SIMULATION RESULTS

We test our filter for elimination/enhancement of narrow-band complex signals. Input signal is a mixture of white noise and complex (analytic) sinusoidal signal.



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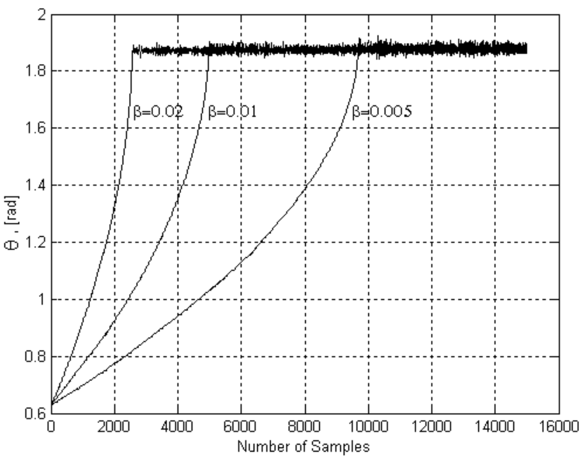

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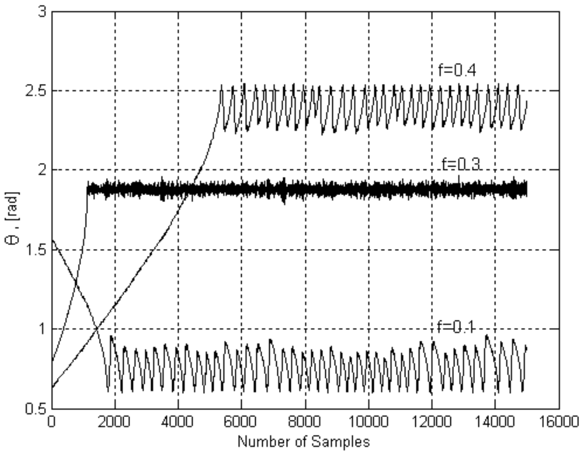

(c)

Figure 4: Trajectories of filter coefficient θ (a) for different step size μ; (b) for different bandwidth β; (c) for different frequency *f*.

In Figure 4a the learning curves for different values of step size μ are shown. It can be observed that the larger is the step size the higher speed of adaptation could be achieved. In Figure 4b the results for different filter bandwidth are presented. It is obvious that by narrowing the filter bandwidth we make the process of convergence slower. Finally, in Figure 4c we show the behaviour of our filter for a wide range of sinusoidal frequencies. In all the cases our filter converges to the proper frequency value.

In the next experiment we investigate the dependence on β and its debiasing effect. Parameter β defines the filter bandwidth and for $\beta \approx 0.004$ the resulting filter is a good approximation of an ideal notch filter. For the purpose of this study, the input signal consists of a complex sinusoid at a frequency of 0.25 and white noise. Step size μ is set to 0.01. We have conducted 30 independent trials with a data-length of 256. The results for θ , which true value is 0.5 π, are summarized in Table 1. It can be seen that the estimates improve continuously with β approaching 0.004.

Table 1: Simulation results for one complex sinusoid in white noise as a function of β and for a data length of 256.

β	θ[rad]	
	mean	standard deviation
0.050	0.5603 π	0.0086 π
0.045	0.5533 π	0.0075 π
0.040	0.5471 π	0.0065 π
0.035	0.5411 π	0.0057 π
0.030	0.5351 π	0.0051 π
0.025	0.5278 π	0.0042 π
0.020	0.5207 π	0.0031 π
0.015	0.5139 π	0.0026 π
0.010	0.5074 π	0.0018 π
0.005	0.5004 π	0.0011 π

5. CONCLUSIONS

A very low sensitivity real LP filter section was transformed in this work to a complex BP section permitting a very precise tuning of the BW in much wider frequency range compared to other known sections (based on truncated Taylor series). The transformation factor θ is used to tune (also adaptively, by applying an LMS algorithm) the central frequency of the complex BP filter so obtained. The convergence of the algorithm for the developed adaptive complex filter circuit is investigated experimentally and the efficiency of the adaptation is clearly demonstrated.

The main advantages of the proposed adaptive structure are in its low computational complexity, fast convergence (less than 100 iterations) and the convenience for implementation with CORDIC processors. The very low sensitivity of the initial LP section ensures a high tuning accuracy even with severely quantized multiplier coefficients.

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