RECONSTRUCTION OF IMAGES FROM THEIR OBSERVATION THROUGH BAYER AND HONEYCOMB COLOR FILTER ARRAYS

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ABSTRACT

The general theory of *consistent sampling* by Unser and Aldroubi is applied to reconstruction of color images from their non-ideally observed data through Bayer and honeycomb color filter arrays. An image is assumed to be represented by bivariate box splines over the orthogonal or hexagonal mesh for the case of Bayer or honeycomb arrays, respectively. Then its reconstruction is made by computing the coefficients for its box spline representation so that mathematical re-observation of the spline representation through the array be the same as the observed data.

1. INTRODUCTION

Typical color image sensors used for digital cameras employ an array of color filters, as schematized in Fig. 1, to divide the light intensity at a pixel into its RGB primary color components prior to opto-electronic conversion. Traditional Bayer array [1] has a pixel divided into 2×2 subpixels. One of the four subpixels is devoted to observation of the R component, another observes B, and the other two observe G. A recent arrangement of color filters is called honeycomb or honeycom arrays, where a pixel is composed of a triad of hexagonal subpixels. Each of the three subpixels is assigned to R, G and B.

Through those arrays, each of the primary color components of an image is observed only at a fraction of subpixels. If we want to insist that the image has the resolution as high as the subpixel level, which is the case in the current industry, we have to estimate the missing data of a color component at the subpixels devoted to the other colors. Several techniques [2–4] for estimating the missing data by linear and nonlinear interpolation have been proposed.

Even in the case we are honest to claim only the resolution of the pixel level, the color value of a pixel is exactly represented by the observation at a subpixel only under the strong assumption that an image is a collection of uniformly colored squares or triadic hexagons of the same size as the pixels.

A systematic way of coping with this kind of non-ideal observation devices is the general framework of *consistent sampling* by Unser and Aldroubi [5]. In this framework, a signal is reconstructed as a function from the observed data so that the same data would be obtained if the function is re-injected to the observation device. Then any reconstruction is consistent at least with the data. Besides, if we choose a right signal space to which the function belongs to, we will have a good reconstruction.

A popular signal space assumed for images is generated by bivariate cubic B-splines, which introduces a relatively moderate assumption than the patchwork of uniformly colored polygons of a fixed size. The bivariate box splines (a generalized notion of the bivariate B-splines)[6, 7] over the orthogonal mesh suit Bayer array while the bivariate box splines over the hexagonal mesh suit the honeycomb array.

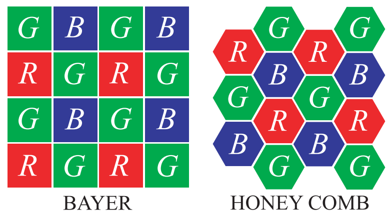

Figure 1: Color filter arrays.

In this paper, images are reconstructed as linear combinations of bivariate box splines over the orthogonal or hexagonal mesh from their observation through Bayer and honeycomb color filter array, respectively, so that the sampling consistency is fulfilled.

2. RECONSTRUCTION WITH BAYER ARRAY

Bivariate cubic B-splines [6] are defined by means of inverse Fourier transform as

$$\varphi_{kl}\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) := \frac{1}{(2\pi)^2}\iint_{\mathbf{R}^2}\left(\frac{1-\mathrm{e}^{-i\omega_x}}{i\omega_x}\right)^4\left(\frac{1-\mathrm{e}^{-i\omega_y}}{i\omega_y}\right)^4 \mathrm{e}^{i(\omega_x(x-k)+\omega_y(y-l))}\,d\omega_x d\omega_y,\quad k,l \in \mathbf{Z}.$$

Assume that primary color components of an image are represented as

$$\begin{aligned} R\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) &= \sum_{k\in\mathbf{Z}}\sum_{l\in\mathbf{Z}} c^R_{kl}\varphi_{kl}\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right), \\ G\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) &= \sum_{k\in\mathbf{Z}}\sum_{l\in\mathbf{Z}} c^G_{kl}\varphi_{kl}\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right), \\ B\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) &= \sum_{k\in\mathbf{Z}}\sum_{l\in\mathbf{Z}} c^B_{kl}\varphi_{kl}\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right). \end{aligned}$$

Those functions are piecewise cubic polynomials in x and y of which coefficients switch on the grid

$$\Xi := \left\{ \left[\begin{array}{c} x \\ y \end{array}\right] = \left[\begin{array}{c} t \\ l \end{array}\right] \text{ or } \left[\begin{array}{c} k \\ u \end{array}\right] \quad \begin{array}{l} \text{for } t,u \in \mathbf{R} \\ \text{and } k,l \in \mathbf{Z} \end{array} \right\}.$$

The primary color filters of Bayer array are placed in the domains

$$D^R_{mn} := \left\{ \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \middle| \, m \le x < m+\frac{1}{2} \text{ and } n \le y < n+\frac{1}{2} \right\},$$

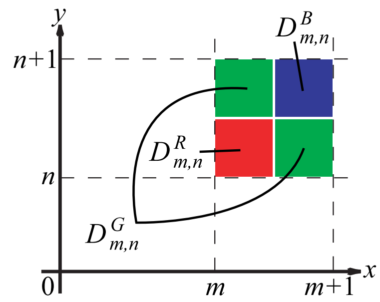

Figure 2: Observation domains in Bayer array.

$$D^G_{mn} := \left\{ \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \middle| m \le x < m + \frac{1}{2} \text{ and } n + \frac{1}{2} \le y < n+1 \right.$$
$$\left. \text{or } m + \frac{1}{2} \le x < m+1 \text{ and } n \le y < n + \frac{1}{2} \right\},$$
$$D^B_{mn} := \left\{ \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \middle| m + \frac{1}{2} \le x < m+1, \ n + \frac{1}{2} \le x < n+1 \right\},$$
$$m, n \in \mathbf{Z}$$

as illustrated in Fig. 2.

Denote by d^R_{mn}, d^G_{mn} and d^B_{mn} the data obtained by observing an image through the color filters in the domain D^R_{mn}, D^G_{mn} and D^B_{mn}, respectively. Then mathematical re-observation of the functions R, G, and B in the same domains must satisfy the linear equations

$$d^R_{mn} = \iint_{D^R_{mn}} R([\begin{smallmatrix} x \\ y \end{smallmatrix}])\, dxdy = \sum_k \sum_l c^R_{kl} f^R_{m-k,n-l},$$
$$d^G_{mn} = \iint_{D^G_{mn}} G([\begin{smallmatrix} x \\ y \end{smallmatrix}])\, dxdy = \sum_k \sum_l c^G_{kl} f^G_{m-k,n-l},$$
$$d^B_{mn} = \iint_{D^B_{mn}} B([\begin{smallmatrix} x \\ y \end{smallmatrix}])\, dxdy = \sum_k \sum_l c^B_{kl} f^B_{m-k,n-l},$$
$$m, n \in \mathbf{Z},$$

where

$$\begin{aligned} f^R_{pq} &:= \tilde{\varphi}([\begin{smallmatrix} p+1/2 \\ q+1/2 \end{smallmatrix}]), \\ f^G_{pq} &:= \tilde{\varphi}([\begin{smallmatrix} p+1/2 \\ q+1 \end{smallmatrix}]) + \tilde{\varphi}([\begin{smallmatrix} p+1 \\ q+1/2 \end{smallmatrix}]), \\ f^B_{pq} &:= \tilde{\varphi}([\begin{smallmatrix} p+1 \\ q+1 \end{smallmatrix}]) \end{aligned}$$

and

$$\tilde{\varphi}([\begin{smallmatrix} x \\ y \end{smallmatrix}]) := \frac{1}{(2\pi)^2} \iint_{\mathbf{R}^2} \left(\frac{1 - \mathrm{e}^{-i\omega_x}}{i\omega_x} \right)^4 \left(\frac{1 - \mathrm{e}^{-i\omega_y}}{i\omega_y} \right)^4$$
$$\left(\frac{1 - \mathrm{e}^{-i\omega_x/2}}{i\omega_x} \right) \left(\frac{1 - \mathrm{e}^{-i\omega_y/2}}{i\omega_y} \right) \mathrm{e}^{i(\omega_x x + \omega_y y)}\, d\omega_x d\omega_y.$$

The above equations can be solved for the coefficients c^R_{kl}, c^G_{kl} and c^B_{kl} by the inverse digital filters derived by means of Poisson sum formula. Then the functions R, G and B with those coefficients give the reconstructed image.

Figure 3 show the test images having the resolution of 500×500 subpixels. Reconstructed images from their observation through Bayer filter array are shown in Fig. 4.

Figure 3: Test images.

Those obtained by the cubic spline interpolation technique are shown in Fig. 5 for the purpose of comparison. Texture of the hat, the lips, and the calyxes are a little better reproduced in Fig. 4 than Fig. 5.

3. RECONSTRUCTION WITH HONEYCOMB ARRAY

By the directional vectors $a_1 = \left[\begin{smallmatrix} 1/2 \\ -\sqrt{3}/2 \end{smallmatrix}\right]$, $a_2 = \left[\begin{smallmatrix} 1/2 \\ \sqrt{3}/2 \end{smallmatrix}\right]$, and $a_3 = a_1 + a_2 = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$, we can construct a hexagonal mesh consisting of equilateral triangles

$$\Delta := \left\{ \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] = t a_1 + k a_2 + l a_3 \text{ or } k a_1 + t a_2 + l a_3 \right.$$
$$\left. \text{or } k a_1 + l a_2 + t a_3 \quad \begin{matrix} \text{for } t \in [0,1) \\ \text{and } k, l \in \mathbf{Z} \end{matrix} \right\}.$$

Bivariate box splines over this mesh [7] defined by

$$\psi_{kl}([\begin{smallmatrix} x \\ y \end{smallmatrix}]) := \frac{1}{(2\pi)^2} \iint_{\mathbf{R}^2} \left(\frac{1 - \mathrm{e}^{-i[\omega_x \ \omega_y] a_1}}{i[\omega_x \ \omega_y] a_1} \right)^2$$

Figure 4: Reconstructed images by consistent sampling from observation through Bayer filter array.

Figure 5: Reconstructed images by bi-cubic spline interpolation technique.

$$\left(\frac{1-\mathrm{e}^{-i[\omega_x\ \omega_y]a_2}}{i[\omega_x\ \omega_y]a_2}\right)^2 \left(\frac{1-\mathrm{e}^{-i[\omega_x\ \omega_y]a_3}}{i[\omega_x\ \omega_y]a_3}\right)^2$$
$$\mathrm{e}^{i[\omega_x\ \omega_y]\left(\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]-[a_1\ a_2]\left[\begin{smallmatrix}k\\l\end{smallmatrix}\right]\right)}|a_1\ a_2|\,d\omega_x d\omega_y,\quad k,l\in\mathbf{Z},$$

are used in this experiment, which generalize the bivariate B-splines to have extra grid lines added in a new direction and all the grid lines slanted to make unilateral hexagons. Assume that primary color components of an image are represented as

$$\begin{aligned} R(\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]) &= \sum_{k\in\mathbf{Z}}\sum_{l\in\mathbf{Z}} c^R_{kl}\psi_{kl}(\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]),\\ G(\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]) &= \sum_{k\in\mathbf{Z}}\sum_{l\in\mathbf{Z}} c^G_{kl}\psi_{kl}(\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]),\\ B(\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]) &= \sum_{k\in\mathbf{Z}}\sum_{l\in\mathbf{Z}} c^B_{kl}\psi_{kl}(\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]).\end{aligned}$$

The primary color filters of the honeycomb array are placed in the domains

$$\begin{aligned} H^R_{mn} &:= \left\{\left[\begin{matrix}x\\y\end{matrix}\right]=ta_1+ua_2+va_3\ \middle|\ m\le t<m+\frac{1}{3},\right.\\ &\qquad \left. n\le u<n+\frac{1}{3},\ 0\le v<\frac{1}{3}\right\},\\ H^G_{mn} &:= \left\{\left[\begin{matrix}x\\y\end{matrix}\right]=ta_1+ua_2+va_3\ \middle|\ m+\frac{1}{3}\le t<m+\frac{2}{3},\right.\\ &\qquad \left. n+\frac{2}{3}\le u<n+1,\ 0\le v<\frac{1}{3}\right\},\\ H^B_{mn} &:= \left\{\left[\begin{matrix}x\\y\end{matrix}\right]=ta_1+ua_2+va_3\ \middle|\ m+\frac{2}{3}\le t<m+1,\right.\\ &\qquad \left. n+\frac{1}{3}\le u<n+\frac{2}{3},\ 0\le v<\frac{1}{3}\right\},\\ &\qquad m,n\in\mathbf{Z}\end{aligned}$$

as illustrated in Fig. 6.

Denote by d^R_{mn}, d^G_{mn} and d^B_{mn} the data obtained by observing an image through the color filters in the domains H^R_{mn}, H^G_{mn} and H^B_{mn}, respectively. Then mathematical reobservation of the functions R, G, and B in the same domains must satisfy the linear equations

$$d^R_{mn}=\iint_{H^R_{mn}} R(\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right])\,dxdy=\sum_k\sum_l c^R_{kl}g^R_{m-k,n-l},$$

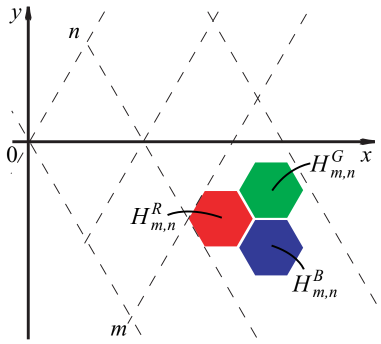

Figure 6: Observation domains in the honeycomb array.

$$d^G_{mn} = \iint_{H^G_{mn}} G\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) dxdy = \sum_k \sum_l c^G_{kl} g^G_{m-k,n-l},$$

$$d^B_{mn} = \iint_{H^B_{mn}} B\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) dxdy = \sum_k \sum_l c^B_{kl} g^B_{m-k,n-l},$$

$$m,n \in \mathbf{Z},$$

where

$$\begin{aligned} g^R_{pq} &:= \iint_{H^R_{pq}} \psi_{00}\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) dxdy, \\ g^G_{pq} &:= \iint_{H^G_{pq}} \psi_{00}\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) dxdy, \\ g^B_{pq} &:= \iint_{H^B_{pq}} \psi_{00}\left(\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]\right) dxdy. \end{aligned}$$

The above equations can be solved for the coefficients c^R_{kl}, c^G_{kl} and c^B_{kl} by the inverse digital filters. Then the functions R, G and B with those coefficients give the reconstructed image as shown in Fig. 7. They look almost the same as those in Fig. 5 obtained from observation through Bayer color filters. Since a hexagonal subpixel is $2/\sqrt{3}$ times larger than Bayer's in area, the honeycomb array may be more efficient in observing images.

4. CONCLUSIONS

The framework of consistent sampling was applied to reconstruction of images from their observation through Bayer and honeycomb color filter arrays under the assumption that the images are represented by linear combinations of bivariate cubic B-splines and box splines, respectively. Reconstruction by consistent sampling from the data observed through Bayer array was certainly better than those estimated by the classical bi-cubic spline interpolation technique. Reconstruction from observation through the honeycomb array was almost the same as that through Bayer's in spite that the subpixel size is a little larger than Bayer's.

Figure 7: Reconstructed images by consistent sampling from observation through the honeycomb filter array.

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