WAVELET BASED DE-NOISING FOR CHAOTIC SIGNAL PREDICTION USING THE TRAJECTORY PARALLEL MEASURE

Shigeo Wada and Yuji Koizumi

Graduate School of Engineering, Tokyo Denki University
2-2 Kanda-Nishiki-cho, Chiyoda-ku, Tokyo 101-8457, Japan
phone: +81 3 5280 3314, fax: +81 3 5280 3573, email: wada@cck.dendai.ac.jp

ABSTRACT

In this paper, a new wavelet based de-noising for noisy chaotic signal prediction using the trajectory parallel measure is proposed. As the chaotic signal is similar to noise, the traditional de-noising criterion based on the noise variance is not effective for noisy chaotic signal filtering and prediction. In order to achieve the accurate prediction, the wavelet based prediction with de-noising for attractor using the trajectory parallel measure is applied to noisy chaotic signals. In the de-noising process, the observed signal is judged whether it is chaotic or close to noise using the measure.

To verify the effectiveness of the proposed method, it is demonstrated that noisy chaotic signals are predicted with smaller prediction error compared with the conventional chaotic signal prediction in the simulations.

1. INTRODUCTION

Chaotic behavior can be observed in various fields such as electrical, civil and chemical engineering and brain science and biology [1]. Chaos is a deterministic evolution of a nonlinear system whose orbit sensitively changes in initial conditions. Predicting, identifying and modeling observed chaos or chaotic signal is important problems to control nonlinear systems [2]. As for the prediction problem, it is difficult to estimate chaotic behavior in long-term. However, the short-term prediction of chaos is not comparatively difficult. There are a number of algorithms available for the analysis, synthesis and prediction of chaotic signals [2-9].

In signal and image processing communities, the wavelets become attractive data processing tools (i.e. [10]). The wavelet can be applied to data compression, image processing, spectral analysis and so on. Recently, several wavelet theories are used to the problem of chaotic signal prediction [6-7], [9]. In [6] and [9], approximation method of dynamics of nonlinear system using wavelet expansion is proposed. A nonlinear prediction technique of chaotic signal based on wavelet networks is proposed in [6]. Further, chaotic signal prediction method using wavelet decomposition is developed in [7]. These methods are useful for the prediction of chaotic signals. However, the effect of noise for the observed chaos is not considered in the system. Therefore it is difficult to apply the methods effectively in practical noisy environment. In actual observation process of chaotic signal, the effect of observation noise is essential problem. In addition, the quantization noise in calculation process for prediction by digital computers is also essential [11]. The effect of the noise is important for the accuracy of prediction.

In this paper, an effective wavelet based prediction technique for noisy chaotic signal is investigated. When the conventional de-noising based on noise variance criterion (so-called spectrum subtraction) is used for noisy chaotic signal filtering, the noise such as observation and quantization additive white Gaussian noise are not sufficiently reduced. In order to reduce a large prediction error in the conventional method, a new wavelet based chaotic signal prediction using the trajectory parallel measure is proposed. The wavelet de-noising method for attractor based on the criterion of trajectory parallel measure is used. The observed signal is judged whether it is chaotic or close to noise using the measure in the de-noising process. In the simulations, a noisy chaotic signal is predicted with smaller prediction error compared with the conventional criteria wavelet based chaotic signal prediction.

2. WAVERET BASED PREDICTION OF CHAOTIC SIGNALS

In this section, a wavelet based prediction of chaotic signals is introduced [7]. The multi-resolution (wavelet) representation of a signal $f(t)$ is represented by

$$f(t)=\sum_{k\geq m,i} w_{k,i}\psi_{k,i}(t)+\sum_{i} v_{m,i}\phi_{m,i}(t) \tag{1}$$

where the coefficients for the wavelet basis and the scaling functions are given by $w_{k,i}=\langle f(t),\psi_{k,i}(t)\rangle$ and $v_{m,i}=\langle f(t),\phi_{m,i}(t)\rangle$, respectively. Here $\langle\cdot,\cdot\rangle$ denotes the inner product of signals, and k and i represent the scale and shift parameters, respectively. The signal $f(t)$ can be equivalently represented using the coefficients as

$$\mathbf{f}_{2^J}\equiv\{f(1),f(2),\cdots,f(2^J)\}$$
$$\Leftrightarrow\begin{cases}\mathbf{w}_{2^{J-1}}\equiv\{w_{J-1,1},\cdots,w_{J-1,2^{J-1}}\}\\ \mathbf{w}_{2^{J-1}}\equiv\{w_{J-2,1},\cdots,w_{J-2,2^{J-2}}\}\\ \quad\vdots\\ \mathbf{w}_{2^m}\equiv\{w_{m,1},\cdots,w_{m,2^m}\}\\ \mathbf{v}_{2^m}\equiv\{v_{m,1},\cdots,v_{m,2^m}\}\end{cases} \tag{2}$$

The wavelet coefficients are efficiently calculated using iterated two-channel analysis filter banks. Using the wavelet and scaling coefficients as input signals, the original signal is completely reconstructed by the corresponding iterated two-channel synthesis filter banks. It is noted that the wavelet coefficients are correspondence to the frequency components at instantaneous time of signals.

The wavelet based chaotic signal prediction is achieved by using the entire wavelet coefficients prediction [7]. The predicted signal is obtained by synthesizing the predicted wavelet coefficients. It is noted that the chaotic topology is preserved in the individual wavelet coefficients [7].

Now the attractor of non-linear dynamics is introduced. It is known that the attractor is an important concept to characterize non-linear dynamics. When the signal is chaotic, the trajectory is restricted in the bounded certain area, so-called attractor. The attractor can be reconstructed in multi-dimensional state space using an observed signal and their delayed signals. In this paper, the signal predictions as well as noise reduction are performed based on the attractor using multi-dimensional state space. Based on the characteristics of the attractor, the local approximation method for reconstructing attractor is executed.

Let the observed chaotic subband signal (the wavelet coefficient) $w_{k,i}, i=1,2,\cdots$ is written by $w(i), i=1,2.\cdots$. The scale parameter *k* is omitted when it is not essential. Then, the *p* step prediction problem based on wavelet decomposition can be formulated.

Let the *m*-dimensional set of delayed signals be

$$\mathbf{w_m}(i) := \{w(i), w(i-\tau), \cdots, w(i-(m-1)\tau)\}. \quad (3)$$

The orbit of $\mathbf{w_m}(i)$ is called reconstruction of attractor. In the first step, the prediction of $w(i+p),\ p>0$ is replace to the prediction of attractor $\mathbf{w_m}(i+p)$. A function *F* that predicts the attractor is introduced.

$$\mathbf{w_m}(i+p) \cong F(\mathbf{w_m}(i)) \quad (4)$$

The local approximation based on the idea that the deviation of attractor in small time is restricted in small range [8] is applied to the prediction of attractor. The *n* numbers of nearest neighbors $\mathbf{w_m}(i'_h)$ of $\mathbf{w_m}(i)$ are selected. Figure 1 shows the local approximation method for reconstruction attractor. If $\mathbf{w_m}(i'_h), h=1,2,\cdots,n$ are neighbors of $\mathbf{w_m}(i)$, then $\mathbf{w_m}(i'_h+p)$ becomes near to $\mathbf{w_m}(i+p)$ in short term range. Then, the element of $\mathbf{w_m}(i+p)$ is approximated by $w(i+p) \cong G(\mathbf{w_m}(i))$. The local approximation function for *G* is selected as a linear approximation shown by

$$\begin{aligned} w(i+p) &\cong G(\mathbf{w_m}(i)) \\ &= a_0 + \sum_{k=1}^{m} a_k x(i-(k-1)\tau) \end{aligned} \quad (5)$$

The coefficients $\{a_0, a_1, \cdots, a_m\}$ are obtained by the following procedures. All Euclidean norms between $\mathbf{w_m}(j)$ in attractor space and $\mathbf{w_m}(i)$ are calculated as

$$r = \left(\sum_{k=0}^{m-1} (w(j-k\tau) - w(i-k\tau))^2 \right)^{1/2}. \quad (6)$$

The values of r calculated by Eq. (6) are sorted in minimum order. The *n* numbers of neighbors $\mathbf{w_m}(i'_h), h=1,2,\cdots,n$ for $\mathbf{w_m}(i)$ are determined. The following error function is defined.

$$\varepsilon = \sum_{h=1}^{n} (w(i'_h+p) - G(\mathbf{w_m}(t'_h)))^2 \quad (7)$$

Then the coefficients $\{a_0, a_1, \cdots, a_m\}$ are obtained as the solution of least square minimization of ε.

Finally, using the resultant coefficients, the predicted signal is obtained based on Eq. (5). As shown in the simulations, when the chaotic signal is observed in noisy environment, the reconstructed attractor is not definite to predict the signal.

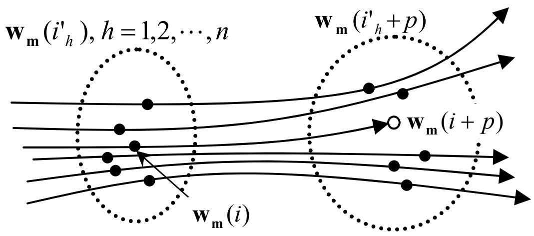


Figure 1: Local approximation for reconstruction attractor.

3. TRAJECTORY PARALLEL MEASURE FOR WAVELET DE-NOISING

It is known that the geometrical structure of attractor is preserved for neighbourhood space composed of neighbouring points in the short term range. The variance of tangential directions of attractor can be used as a measure whether the signal is chaotic or not [12]. The unit tangent vector $\mathbf{u}(i)$ for $\mathbf{w_m}(i)$ and the unit tangent vectors $\mathbf{u}(i'_h)$ for $\mathbf{w_m}(i'_h)$ are computed as tangent line for the circular arc of hyper elliptic. Using the tangent vectors, the trajectory parallel measure is calculated by

$$p(i) = \frac{1}{4n} \sum_{h=1}^{n} \| \mathbf{u}(i) - \mathbf{u}(i'_h) \|^2. \quad (8)$$

Then, the *k* numbers of arbitrary points are selected to calculate the trajectory parallel measures. The following average is computed to obtain the statistical measure for the entire attractor.

$$P = \frac{1}{k} \sum_{t=1}^{k} p(t) \quad (9)$$

When the measure *P* is nearly equal to zero, the signal is estimated as chaotic. While it nearly equals to 0.5, the signal has stochastic property. The signal is far from chaotic [12].

Next, an observed chaotic signal $y(t)$ is assumed to be represented by $y(t) = f(t) + n(t)$ where $f(t)$ represents a chaotic signal and $n(t)$ represents an additive white Gaussian background noise. The wavelet coefficients of the observed signal is represented by $w_{k,i} = d_{k,i} + e_{k,i}$ where $d_{k,i}$

and $e_{k,i}$ represent wavelet coefficients of $f(t)$ and $n(t)$, respectively.

The de-noising process for the wavelet coefficients is represented by

$$\hat{w}_{k,i} = \mathrm{sgn}(w_{k,i})(|w_{k,i}| - th^k) \tag{10}$$

where

$$th^k = \alpha |w_{k,i}| \quad 0 < \alpha \le 1 . \tag{11}$$

In this paper, the parameter α is determined based on the trajectory parallel measure shown in Eq. (9). It is noted that the traditional selection of the threshold is base on the variance of the additive noise.

4. SIMULATIONS

In this section, computer simulation results are shown to verify the effectiveness of the proposed prediction method.

4.1 Observed chaotic signals

In the computer simulations, the x component of the following Rossler differential equation is used as original chaotic signals.

$$\frac{dx}{dt} = -y - z, \quad \frac{dy}{dt} = x + ay, \quad \frac{dz}{dt} = b + z(x - c)$$

where $a = 0.2$, $b = 0.4$ and $c = 5.7$ are selected. The Fourier spectrum for the original chaotic signal with the additive white Gaussian noise is shown in Fig. 2. The chaotic signal is buried by the white Gaussian noise.

To predict the noisy chaotic signals in various SNR conditions, Daubesechies' wavelet with level 3 is used for the observed signal of length 2^{13}. The parameters for calculating attractors and the trajectory parallel measure are selected by $m = 3$, $\tau = 1$, $n = 5$ and $k = 0.01 \times 2^{13}$. The parameter for local approximation is selected by $n = 20$.

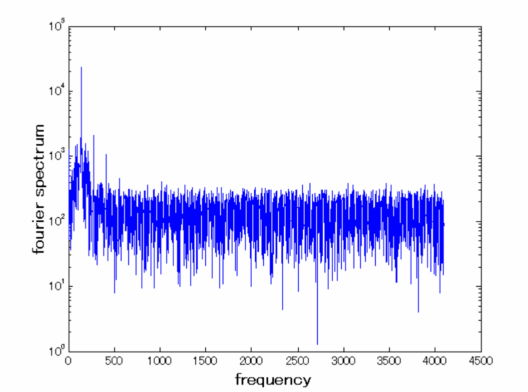


Figure 2: Fourier spectrum for noisy observed chaotic signal of x-component of Rossler equation (SNR=10 dB).

4.2 Prediction results

The noisy chaotic signal of 16 step ahead is predicted. Figure 3 shows the relationship between SNR and trajectory parallel measure values. The blue line with triangles and the red line with circles represent the trajectory parallel measure values in noisy and de-noised environment, respectively. The green line with squares represents the value by the conventional spectrum subtraction. As the observed additive noise is appropriately removed by the wavelet based de-noising using the trajectory parallel measure compared with the traditional method. It is noted that the value of P becomes under 0.03 where the signal is judged chaotic.

Further, Fig. 4 shows the corresponding relationship between SNR and the prediction error (normalized square error). The blue line, the green line and the red line represent the errors for without de-noising, with the traditional method and with the proposed de-noising, respectively. The conventional method shows large error and the prediction is failed, while the proposed method is effective even in the SNR is small.

Furthermore, Fig. 5 shows attractors of chaotic signals. Figure 5 (a) shows the original reconstructed attractor in noiseless environment. Figure 5 (b) shows the reconstructed attractor in noisy environment. The trajectory becomes random by the noise influence. Figure 5 (c) shows the reconstructed attractor with de-noising. The attractor is effectively restored by the proposed de-noising process. Figure 5 (d) shows the predicted 250 points of attractor ($\times$) and the reconstructed actual attractor ($\circ$). It is confirmed that the prediction is successfully performed with smaller errors. It is noted that the precise prediction is also effectively simulated when the prediction step is under 16.

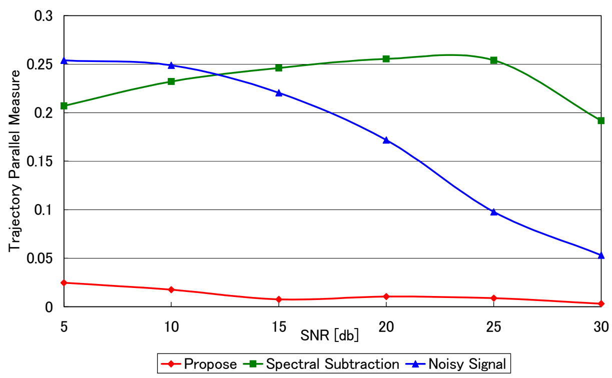


Figure 3: The relationship between SNR and trajectory parallel measure values.

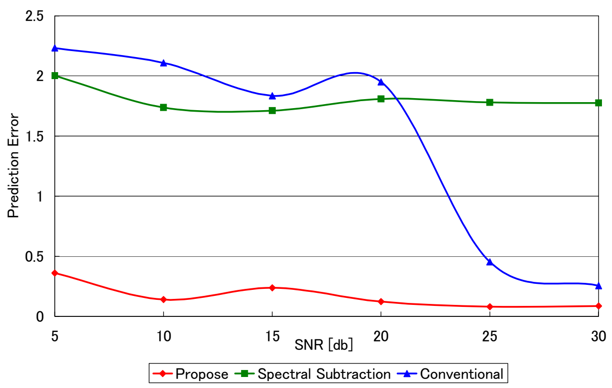
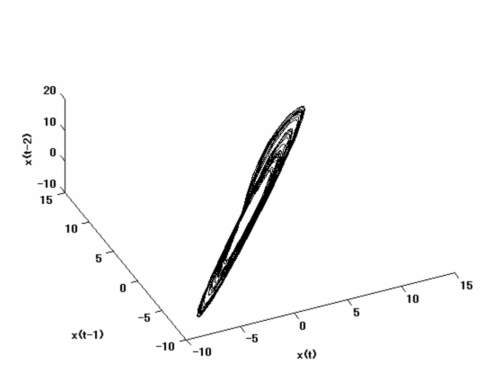
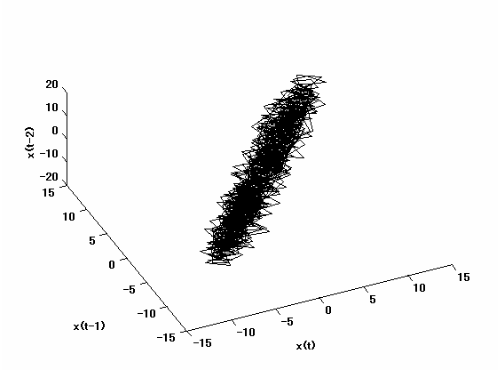


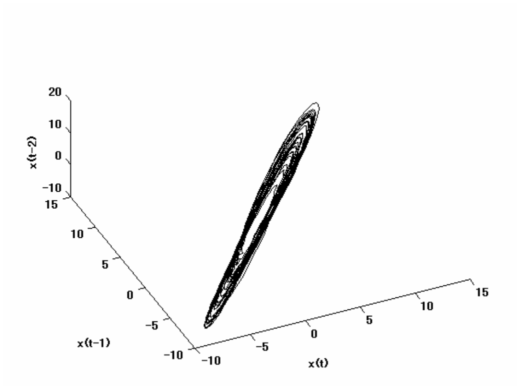
Figure 4: The relationship between SNR and prediction errors.



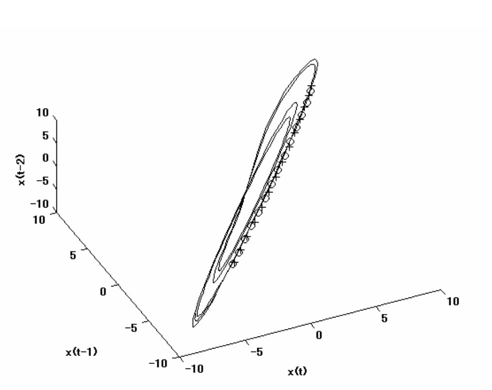
(a) Reconstructed attractor without noise.



(b) Reconstructed attractor in noisy environment.



(c) Reconstructed attractor with de-noising.



(d) Predicted 250 points of attractor (×) and the reconstructed actual attractor (○).

Figure 5: Attractors of chaotic signals.

5. CONCLUSIONS

In this paper, a wavelet based noisy chaotic signal prediction method using the trajectory parallel measure was investigated. The traditional de-noising criterion based on the noise variance is not sufficient for noisy chaotic signal prediction. A large prediction error occurred in applying to actual observed signals. In order to reduce the influence of observed noise (additive white Gaussian noise) or quantization noise, a wavelet based prediction with de-noising for attractor using the trajectory parallel measure was proposed. Simulation results were given to demonstrate the effectiveness of the proposed method. It was shown that noisy chaotic signals were effectively predicted with smaller prediction error compared with the traditional so-called spectrum subtraction method.

REFERENCES

[1] K. Aihara ed, *Chaotic time series analysis of basis and application*, Sangyo Tosyo Publishing Co., 2000.

[2] E. Ott, T. Sauer and J. A. Yorke, *Coping with chaos*, John Wiley & Sons, 1994.

[3] J. D. Farmer and J. J. Sidorowich, “Predicting chaotic time series,” *Phys. Rev. Lett.*, vol.59, no.8, pp.845-848, Aug. 1987.

[4] M. Casdagli, “Nonlinear prediction of chaotic time series,” *Physica*, D35, no.35, pp.335-356, May 1989.

[5] T. Sauer, “Time series prediction using delay coordinate embedding,” in *Time series prediction: Forecasting the future and understanding the past*, ed. A. S. Weigend, N. A. Gershenfeld, pp.175-193, Addison-Wesley, 1994.

[6] L. Cao, Y. Hong, H. Fang and G. He, “Predicting chaotic time series with wavelet networks,” *Physica*. D85, pp.225-238, 1995.

[7] N. Masuda and K. Aihara, “The prediction of chaotic time series with wavelet coefficients,” *IEICE Trans.*, Vol.J82-A, No.11, pp.1710-1718, Nov. 1999.

[8] K. Itoh, “An improved method for predicting chaotic time series,” *IEICE Trans.*, Vol.J80-A, No.6, pp.986-993, June 1997.

[9] W. L. B. Constantine and P. G. Reinhall, “Wavelet-based in-band denoising technique for chaotic sequences,” *Int. J. Bifurcation and Chaos*, vol.11, no.2, pp.483-495, 2001.

[10] D. L. Donoho, “De-noising by soft-thresholding,” *IEEE Trans. Information Theory*, Vol.41, No.3, pp.613-627, May 1995.

[11] T. Suzuki, T. Ikeguchi and M. Suzuki, “On evaluation of noise levels for quantized observed data,” *Technical Report of IEICE.*, NLP2001-116, pp.107-114, 2001.

[12] Y. Fujimoto, T. Iokibe and T. Tanimura, “Trajectory parallel measure method for measuring the degree of deterministic property in the observed time series data”, *Journal of Japan Society for Fuzzy Theory and Systems*, Vol.9, No.4, pp.580-588, 1997.