NONLINEAR FILTERING APPROACHES FOR INS/GPS INTEGRATION

Audrey Giremus*, Arnaud Doucet†, Anne-Christine Escher* and Jean-Yves Tourneret*

* IRIT/ENSEEIHT/TéSA, 2 rue Charles Camichel, BP 7122, 31071 Toulouse cedex 7, France
† Signal Processing Laboratory, University of Cambridge, Trumpington Street, CB2 1PZ, Cambridge, UK
* ENAC, 7 av Edouard Belin, BP 54005 Toulouse Cedex 4, France

ABSTRACT

Navigation with an integrated INS/GPS approach requires to solve a set of nonlinear equations. In this case, nonlinear filtering techniques such as Particle Filtering methods are expected to perform better than the classical, but suboptimal, Extended Kalman Filter. Besides, the INS/GPS model has a conditionally linear Gaussian structure. A Rao-Blackwellisation procedure can then be applied to reduce the variance of the state estimates. This paper studies different algorithms combining Rao-Blackwellisation and particle filtering for a specific INS/GPS scenario. Simulation results illustrate the performance of these algorithms. The variance of the estimates is also compared to the corresponding posterior Cramer-Rao bound.

1. INTRODUCTION

The global positioning system (GPS) has been extensively used in navigation because of its accuracy and worldwide coverage. GPS augmentations such as satellite-based augmentation systems (SBAS) and ground-based augmentation systems (GBAS) allow to improve the accuracy of the navigation solution. Indeed, ground stations estimate GPS measurement errors that are then transmitted to the GPS receiver thanks to a satellite constellation. However, many tracking channels can be affected simultaneously by interferences, which can result in a loss of the GPS signal. In this case, a self-contained system such as a calibrated inertial navigation system (INS) can ensure the continuity of navigation with a good accuracy.

The principles of the hybridization GPS/INS are recalled in section 1. Section 2 focuses on the nonlinear filtering model which is usually associated to satellite-based systems. Section 3 studies different appropriate estimation strategies for this nonlinear model. These strategies include the extended Kalman filter and particle filtering methods. Simulation results are presented in section 4. Conclusions are reported in the last section.

2. GPS/INS INTEGRATION

GPS is a satellite-based navigation system that provides precise positioning and timing information to any user properly equipped with a receiver. The basic principle is to estimate the user’s position by processing distance measurements from the receiver to GPS satellites of known locations. The ranging information is determined from the propagation delay of a signal transmitted by the satellite and is therefore biased by the receiver clock offset with respect to the satellite time. Consequently, this measurement is called pseudorange (instead of range) to allow for this bias. Moreover, each satellite broadcasts useful information to solve the navigation problem: data relative to the satellite’s current position and different parameters necessary to compensate for some GPS measurement errors (for instance, the satellite clock offset with respect to the GPS time and the atmospheric delays). Assuming these terms are reasonably well corrected, the user position can be obtained as the solution of the so-called pseudorange equations:

\[ \rho_i = r_i + ct_i + u_i, \quad 1 \leq i \leq n_s, \]

where \( \rho_i \) and \( r_i \) denote the pseudorange and the true geometric range from the receiver to the satellite, \( t_i \) is the receiver clock offset from the GPS time, \( c \) is the speed of light \( (3 \times 10^8 \text{ m/s}) \), \( u_i \) are the residual measurement errors and \( n_s \) is the number of visible satellites. Note that a minimum of four equations is required to estimate both the position in free space and the receiver clock offset.

Contrary to satellite-based navigation systems, INS are autonomous onboard navigation systems. They are based on inertial sensors (accelerometers and gyrometers) that measure the rectilinear acceleration and the inertial angular velocity of a vehicle according to the physical laws of motion. The sensor outputs are integrated to maintain an estimate of the vehicle dynamics, provided a precise knowledge of the initial position, velocity and attitude is available. Note that INS are robust to interferences because the inertial sensors do not depend on the reception of an exterior signal. INS consist of an inertial measurement unit containing the cluster of sensors, combined with a computer that determines the navigated trajectory. This paper focuses on strapdown systems, which are characterized by sensors rigidly attached to the host vehicle. The measurements are resolved in the vehicle frame. Consequently, coordinate transformations are required to express the measurements in a frame of interest for the user (usually the locally level frame with the X, Y and Z axis pointing respectively north, east and down).

The sensor outputs are sequentially processed to derive the navigation solution. The accelerometers deliver a non gravitational acceleration called specific force \( f \). This force is first transformed in an appropriate coordinate system, then compensated for the gravity, and finally double-integrated to yield the position. The gyroimeters outputs are used to compute the attitude angles of the vehicle, hence to update the transformation between the vehicle frame and the desired reference frame. The equations relating the sensor outputs to the dynamic states estimated by the INS are referred to as mechanisation equations. They will be denoted as \( \dot{X}_{\text{INS}} = f(X_{\text{INS}}, U_{\text{INS}}) \), where \( X_{\text{INS}} \) is the vector of navigated states and \( U_{\text{INS}} \) is the vector of actual sensor outputs. In contrast, the ideal equations corresponding to the error-free sensor outputs and the actual dynamic states are written \( \dot{X} = f(X, U) \). Due to the integration process, instrumentation and initialization errors result in a linear and parabolic growth of the velocity and position errors. GPS and INS have complementary features. A data fusion approach can take full advantage of this synergism to improve the accuracy and reliability of the navigation solution. On the one hand, interferences do not affect INS sensors, but they may lead to a loss of GPS measurements. On the other hand, the navigation states are computed by a differentiation-based process for the GPS and an integrative-based process for the INS, which entails respectively high frequency and low frequency errors. Allowing for the distinct frequency content of GPS and INS errors, a complementary filter methodology can be applied advantageously: the GPS pseudoranges are used to estimate the inertial errors (rather than the total dynamic states). The system can still rely on the INS in case GPS measurements are lost.
3. THE GPS/INS NONLINEAR FILTERING MODEL

3.1 State model

In the proposed framework, the state vector is composed of the INS error that are defined as the deviation between the actual dynamic quantities and the INS computed values $\delta X = X - X_{\text{INS}}$. The state model describes the INS error dynamic behavior depending on the instrumentation and initialization errors. It is obtained by linearizing the ideal equations around the INS estimates as follows:

$$
\delta X = f(X, U) - f(X_{\text{INS}}, U_{\text{INS}})
$$

where

$$
\begin{align*}
\delta X &= \delta v^n, \delta \rho, b_a, b_g, \delta \lambda, \delta \phi, \delta h, b, d,
\end{align*}
$$

The state vector is usually augmented with systematic sensor errors:

$$
\begin{align*}
\delta X &= \delta v^n, \delta \rho, b_a, b_g, \delta \lambda, \delta \phi, \delta h, b, d, w_n, n_t, w_a, \lambda, \lambda_e
\end{align*}
$$

where

- $v^n$ stands for the velocity relative to the earth centered/earth fixed frame and resolved in the locally level frame,
- $\rho$ is the vector of attitude angles,
- $(\lambda, \phi, h)$ is the geodetic position in latitude, longitude, altitude,
- $b_a$ and $b_g$ represent the accelerometers and gyroimeters biases,
- $b = \sigma_r$ and $d$ are respectively the GPS clock offset and its drift.

A coarse analysis shows that the errors develop through a variety of sources. Horizontal position and velocity experience a pendulum-like motion called Schuler oscillation. In addition, the earth rotation introduces a cross-coupling between the horizontal dynamic components, hence a modulation of the previous oscillations. Earth rotation introduces a cross-coupling between the horizontal and vertical channels, gravity compensation results in an unstable pendulum-like motion called Schuler oscillation. In addition, the linearizing the matrix $A_{\text{INS}}$ equation between the components of $X$ resulted in a block diagonal matrix $A_t$ whose elements are detailed in [5] or in many standard textbooks such as [4, p. 204]. Section 4 explains how this block diagonal structure for $A_t$ can be used to reduce the computational complexity of the estimation algorithm.

3.2 Measurement model

The hybridization filter is driven by the GPS pseudoranges. Consequently, the observation equation associated to the $i$th satellite can be defined as:

$$
\begin{align*}
p_i &= \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + \omega_i,
\end{align*}
$$

where $i = 1, \ldots, n_s$ (recall that $n_s$ is the number of visible satellites). The vectors $(x, y, z)^T$ and $(X_i, Y_i, Z_i)^T$ are the positions of the vehicle and the $i$th satellite expressed in the rectangular coordinate system WGS-84 [4]. However, these observations have to be expressed as functions of the state vector components to make the filtering problem tractable. Thus, the position is transformed from the geodetic to the rectangular coordinate system as follows:

$$
\begin{align*}
X_i &= h_r(X_i) + CX_i + w_t,
\end{align*}
$$

where $N = \sqrt{1 - 2\sin^2 \lambda}$. The parameters $a$ and $e$ denote the semi-major axis length and the eccentricity of the earth’s ellipsoid. These expressions have to be substituted in (1) to obtain the highly nonlinear measurement equation:

$$
\begin{align*}
x &= (N + h_{\text{INS}} + \delta h) \cos(\phi_{\text{INS}} + \delta \phi) \\
y &= (N + h_{\text{INS}} + \delta h) \sin(\phi_{\text{INS}} + \delta \phi) \\
z &= (N + h_{\text{INS}} + \delta h) \sin(\lambda_{\text{INS}} + \delta \lambda),
\end{align*}
$$

Accurate models of the instrument biases are required to linearize the state-space model exhibiting nonlinearities as in (2), the popular Extended Kalman Filter (EKF) which approximate the pdf in (1) to obtain the highly nonlinear measurement equation:

$$
\begin{align*}
X_{1:t} &= A_t X_{t-1} + \nu_t, \\
Y_{1:t} &= H_t(X_t - m_{1:t-1}) + h_t(m_{1:t-1}) + C_t X_t + w_t,
\end{align*}
$$

where $H_t = \frac{\partial h_t(x)}{\partial x}|_{x = m_{1:t-1}}$. Consequently, the conditional probability of the state $p(X_t | Y_{1:t})$ can be estimated by a Gaussian pdf whose mean $m_{1:t}$ and covariance $P_{1:t}$ can merely be computed by Kalman recursions.
4.2 Particle Filtering methods

PF methods refer to a class of recursive simulation-based estimation methods that can handle nonlinear possibly non Gaussian state space models. An increase in the computational power has recently drawn much attention on these techniques (see [2] and references therein). PF methods construct an empirical estimate of distributions such as \( p(x_{0:t} | y_{1:t}) \) from weighted random support points (called particles):

\[
p(x_{0:t}(i) | y_{1:t}) \approx \sum_{i=1}^{N} w(x_{0:t}^{(i)}) \delta(x_{0:t} - x_{0:t}^{(i)}).
\]

\( x_{0:t}^{(i)} (i = 1, \ldots, N) \) are the particles and \( w(x_{0:t}^{(i)}) \) the associated weights. The particles are propagated sequentially according to an Importance sampling (IS) method. Indeed, it is usually difficult to draw samples directly from the posterior distribution \( p(x_{0:t} | y_{1:t}) \). IS consists of generating samples from an arbitrary proposal distribution \( q(x_{0:t} | y_{1:t}) \). Weights are then assigned to these particles according to their relevance with respect to the distribution of interest. A simple rule for computing the weights is recalled below:

\[
w(x_{0:t}^{(i)}) \propto \frac{p(x_{0:t}^{(i)} | y_{1:t})}{q(x_{0:t}^{(i)} | y_{1:t})}, \quad i = 1, \ldots, N.
\]

An appropriate importance function allows to preserve the past trajectories \( (x_{0:t+1}^{(i)} = (x_{0:t}^{(i)}, x_{t+1}^{(i)}) \) and evaluate the weights according to the following procedure:

\[
w(x_{0:t+1}^{(i)}) \propto \frac{p(y_{t+1} | x_{0:t+1}^{(i)}) p(x_{0:t}^{(i)} | x_{0:t-1}^{(i-1)})}{q(x_{0:t+1}^{(i)} | x_{0:t-1}^{(i-1)})}.
\]

Unfortunately, the variance of the weights is bound to increase as the number of particles is increased. This problem referred to as degeneracy is classically reduced by a resampling step. In order to concentrate the particles in regions of high probability, a good choice of the importance function is also decisive. This choice is crucial in our GPS/INS application since the small noise variance in the state equation prevents an appropriate exploration. The optimal importance function \( p(x_{t} | x_{0:t-1}, y_{t}) \) was derived in [3]. However, it is not straightforward to sample from this distribution. Some strategies overcoming this difficulty are briefly presented hereafter.

4.2.1 Local linearization

By linearizing the measurement equation, a Gaussian approximation of the optimal importance function denoted by \( N(m_t, \Sigma_t) \) is obtained [3]. This linearization is performed for each particle and yields the following recursions:

\[
\Sigma_t^{-1} = \Sigma_{0:t}^{-1} + H_t^T \Sigma_{0:t}^{-1} H_t,
\]

\[
m_t = \Sigma_t^{-1} A_t x_{t-1} + H_t^T \Sigma_{0:t}^{-1} (y_t - h_t(A_t x_{t-1} - C_t A_t x_{t-1} - H_t A_t x_{t-1})),
\]

where \( H_t = \frac{dh_t(x)}{dx} |_{x_0 = A_t x_{t-1}}. \)

4.2.2 Auxiliary particle filter

Another approach improving particle exploration is the auxiliary particle filter (APF) [7]. The APF only requires to simulate from the prior density \( p(x_t | x_{t-1}) \). The principle is to propagate the particles that are expected to yield a high value of the likelihood. The propagation of each particle one step ahead enables to compute the predictive density \( p(y_t | \mu_t) \), where \( \mu_t \) is related to \( p(x_t | x_{t-1}) \). The particles \( x_{t-1} (i = 1, \ldots, N) \) that will actually evolve are then obtained by simulating from this predictive density. The standard steps of PF, i.e. Importance sampling and Resampling, are then applied to derive the state estimates.

4.3 Rao-Blackwellization

The performance of the previous PF algorithms can be improved when the state-space model has a block structure. Indeed, Rao-Blackwellization techniques allow to decrease the variance of the state estimates by solving analytically the linear part of conditionally linear Gaussian models ([1]). A convenient partition of the state vector allows to rewrite the INS-GPS system model as follows:

\[
x_{t+1} = \begin{pmatrix} A_t^1 & C_t^2 & 0 \\ C_t^1 & A_t^2 & 0 \\ 0 & 0 & A_t^3 \end{pmatrix} x_t + \begin{pmatrix} B_t^1 \\ B_t^2 \\ D_t \end{pmatrix} w_t,
\]

\[
y_t = h_t(x_{t+1}^2) + C_t^3 x_{t+1}^3 + D_t w_t,
\]

where \( x_{1:t} = (\delta v, \delta p, b_t, b_t)^T, x_{2:t} = (\delta \lambda, \delta \phi, \delta h)^T, x_{3:t} = (b, d)^T \) and \( x_t = ((x_{1:t})^T, (x_{2:t})^T, (x_{3:t})^T)^T \). Note that the measurement equation does not depend on \( x_{1:t}^2 \). Moreover, the observations depend non-linearly on \( x_{2:t} \) and linearly on \( x_{3:t} \). Consequently, the distributions \( p(x_{1:t}^2 | x_{0:t-1}) \) and \( p(x_{3:t}^3 | y_{0:t}, x_{0:t-1}^2) \) are Gaussian. The means and variances of these distributions, denoted \( m_{n,1}^{m,3} \), \( m_{n,3}^{m,3} \), \( P_{n,1}^{m,3} \), \( P_{n,3}^{m,3} \) respectively, can be computed by standard recursions associated to the following Kalman state-space models:

\[
\begin{align*}
X_{1}^{1} &= A_{t}^{1} X_{t-1}^{1} + C_{t}^{2} X_{t-1}^{2} + B_{t}^{1} v_{t}^{1}, \\
X_{2}^{2} &= C_{t}^{2} X_{t-1}^{1} + A_{t}^{2} X_{t-1}^{2} + B_{t}^{2} v_{t}^{2}, \\
X_{3}^{3} &= A_{t}^{3} X_{t-1}^{3} + B_{t}^{3} v_{t}^{3}, \\
Y_{t} &= h_{t}(X_{3}^{3}) + C_{t}^{3} X_{t-1}^{3} + D_{t} w_{t}.
\end{align*}
\]

The posterior distribution of the state \( x_{2:t}^3 \) (non linearly related to the observations) is estimated by one of the PF methods described before:

\[
p(X_{2}^{3} | y_{1:t}, x_{0:t-1}^{2}) \approx \sum_{i=1}^{N} w_{i}^{(i)} \delta(x_{2}^{3} - x_{2}^{3,i}).
\]

Note that the Kalman filters associated to each particle \( x_{2:t}^{3,i} \) provide both the importance distribution \( p(x_{2}^{3} | x_{0:t-1}^{2}) \) and the likelihood \( p(y_{t} | x_{2}^{3,i}) \). The pdfs \( p(x_{2}^{3} | y_{t}, x_{0:t-1}^{2}) \) \((k=1,3)\) are finally approximated by a mixture of Gaussian distributions:

\[
p(X_{2}^{3} | y_{t}, x_{0:t-1}^{2}) \approx \sum_{i=1}^{N} w_{i}^{(i)} N(x_{2}^{3,i}, m_{n,3}^{m,3,k}, P_{n,3}^{m,3,k}).
\]

4.4 The posterior Cramer Rao Bound (PCRB)

The PCRB provides a lower bound on the mean square errors (MSEs) of the state estimates. It can be viewed as a reference to which the state MSEs of suboptimal algorithms can be compared. The PCRB is often referred to as the Bayesian version of the Cramer-Rao Bound. The recursive formula allowing to compute the PCRB are detailed in [1]. As the altitude is supposed to be known, this paper focuses on the PCRB for the horizontal INS position error denoted by HPCRB. We recall here that the horizontal position error HE can be defined as a function of the latitude \( \delta \lambda \) and longitude \( \delta \phi \) inertial errors by \( HE = (R \delta \lambda)^2 + (R \delta \phi \cos \lambda)^2 \), where \( R \) stands for the earth radius [6].

Figure 1 illustrates the intuitive dependence of the HPCRB on the number \( n_s \) of visible satellites. Indeed, the higher \( n_s \), the lower the PCRB. Further investigations show that the number of visible satellites is not the only important feature. In navigation, a well-known parameter referred to as Horizontal Dilution Of Precision (HDOP) quantifies the effect of the GPS satellites configuration on the horizontal position estimation error. The HDOP represents the
The lower the HDOP, the lower the HPCRB, as expected.

The different estimation strategies detailed in the previous sections are applied to an INS/GPS scenario. First, the vehicle trajectory is generated according to the position-velocity-acceleration model (the acceleration is modeled as a Gauss-markov process). The INS estimates of the vehicle dynamics are then computed for low cost inertial sensors with an accelerometer bias of $\sigma_a = 10$ meters and the number of visible satellites ensures a good observability (i.e. $n_s \geq 4$).

The results obtained with the EKF and the PF (with 5000 particles) are compared for a simulation duration of 500s. For each method, the horizontal positioning root mean square error HRMSE is computed from 100 Monte carlo runs. The RMSEs are also compared to the corresponding posterior bound $\sqrt{\text{HPCRB}}$. Figure 3 shows the results obtained with the EKF and the local linearization (the APF approach has not been plotted since it performs very similarly to the APF). After 50s, the HRMSEs of both state estimates converge to a constant which is close to $\sqrt{\text{HPCRB}}$ ($\approx 2.3$ meters). Figure 3 shows that the EKF and the PF perform quite similarly in nominal situations where the error dynamics are reasonable (i.e. when the local linearization is appropriate).

### 6. CONCLUSIONS

This paper has studied alternatives to the classical EKF to solve the nonlinear INS/GPS filtering problem. PF methods do not require any approximation on the state-space model, contrary to the EKF. A Rao-Blackwellization procedure was applied to decrease the variance of the estimates. However, simulation results conducted in nominal situations (with a good observability) show that quite similar performance is achieved with the different filtering strategies. A comparison in critical situations (such as loss of observability or presence of multipath) is currently under investigation.

### REFERENCES


