AN APPROACH TO GLOBAL NOISE CONTROL BY WAVE FIELD SYNTHESIS

A. Kuntz and R. Rabenstein

Telecommunications Laboratory
University of Erlangen-Nuremberg
Cauerstrasse 7, 91058 Erlangen, Germany
E-mail: {kuntz, rabe}@LNT.de

1. INTRODUCTION

Active Noise Control (ANC) has been under investigation for several decades. Topics of recent research were the placement of sensors and actuators, the development of suitable adaptation algorithms, and the real-time implementation of ANC systems. Theoretical advances and applications to several fields of practical interest are described e.g. in [3–5].

ANC has been applied with considerable success in cases where noise reduction is required only in a very small area, e.g. inside a head phone or at the ears of a listener with restricted head movement. On the other hand, achieving satisfactory noise reduction in a larger area where listeners are allowed to move freely (global ANC) is still a very tough problem.

Using current techniques, ANC for large areas is only possible in few special cases under restrictive conditions. An example is ANC of low-frequency duct noise assuming one-dimensional propagation of sound below the cut-on frequency of transversal acoustic modes in the duct. It appears that a major breakthrough is not possible by further refinement of existing technologies only. Instead, we will focus here on the application of a new multichannel spatial sound reproduction method, the so-called wave field synthesis (WFS).

WFS has been successfully applied for spatial rendering of acoustic scenes in large enclosures, where listeners are not restricted in their activities. No head phones or object tracking equipment is required. Full control over the sound field is achieved by carefully designed spatial audio processing for a large number of loudspeaker channels (tens to several hundreds). For a detailed description of WFS techniques for audio reproduction, see [1, 2, 6].

An experiment with the application of WFS to ANC has at first be reported in [8]. In this contribution we present theoretical investigations, simulations and measurements about the utilization for global ANC. Section 2 discusses the traditional approach to global ANC. Section 3 introduces the application of WFS to ANC by theoretical considerations. Experiments for a specific situation are shown in Sec. 4 both by simulations and measurements. Conclusions and directions for further research are presented in Sec. 5.

2. GLOBAL ANC

Most systems for global ANC use a multipoint approach, i.e. they utilize several sensors and actuators distributed within the area of interest, forming a multiple-input multiple-output (MIMO) control system [3, 4]. With this technique, full control over the whole area is in general only given if the sensors and actuators are placed on a sufficiently dense grid, thus spatially sampling the wave field within the area of interest.

The maximum spacing $d_{\text{max}}$ between the transducers is dependent on the maximum frequency $f_{\text{max}}$ of the noise that should be controlled. It must not exceed half of the wavelength of the corresponding acoustic wave as shown in the following equation

$$d_{\text{max}} \leq \frac{c}{2f_{\text{max}}}, \quad (1)$$

where $c$ denotes the speed of sound. For $c = 340 \text{ m/s}$ and $f_{\text{max}} = 1000 \text{ Hz}$ this results in $d_{\text{max}} = 17 \text{ cm}$, which means that approximately 35 transducers are needed to control one square meter of a plane.

Besides the huge number of sensors and actuators required, also theoretical problems arise: As will be shown in section 3, the same area can be controlled using a significantly smaller number of transducers. Therefore, the MIMO system would be highly overdetermined which results in ill-conditioning of the whole system. This causes bad convergence properties of adaptive ANC algorithms and possibly very large actuator driving signals [5].

3. APPLICATION OF WFS/WFA TO ANC

This section gives a short introduction to the basic principles of wave field synthesis (WFS) and wave field analysis (WFA) and its application to active noise control. WFS and WFA are techniques for rendering and recording of spatial audio based on the physical principles of acoustic wave propagation [2]. They both show properties that make them interesting for the application in global ANC systems.
3.1 Theoretical Background

WFS and WFA are based on the Kirchhoff-Helmholtz integral

\[
P(r, \omega) = \frac{1}{4\pi} \int_S P(r_S, \omega) \frac{\partial}{\partial n} \left( \frac{e^{-i|r-r_S|}}{|r-r_S|} \right) \left| \frac{\partial P(r_S, \omega)}{\partial n} \frac{e^{-i|r-r_S|}}{|r-r_S|} \right| dS. \tag{2}
\]

In this equation, \(P(r, \omega)\) denotes the sound pressure at any point \(r\) within a volume of interest surrounded by the surface \(S\). \(r_S\) denotes any point on the surface and \(n\) denotes the surface normal pointing inwards at the point \(r_S\). The angular frequency is denoted by \(\omega\), the wave number by \(\beta\).

Equation (2) shows that full control of the sound field within the area of interest is possible if the sound pressure \(P(r, \omega)\) and its gradient \(\partial P(r_S, \omega)/\partial n\) on the boundary of the area are both known and can be controlled. As a consequence, it is possible to fully control the sound field globally with a distribution of sensors and actuators placed only on the boundary of this area. Therefore, the number of sensors and actuators is greatly reduced when compared to the general multipoint approach.

3.2 Practical Implementation of WFS systems

For audio reproduction, vertical localization is usually assumed to be of much less importance than horizontal localization. Consequently, WFS-systems are designed to synthesize the sound field correctly only in a horizontal plane. For such systems, the two-dimensional continuous distribution of sources as described by the Kirchhoff-Helmholtz integral is degenerated to one-dimensional curved arrays. The maximum interval for spatial sampling is given by (1). For frequencies above \(f_{\text{max}}\) no control over the wave field is possible due to spatial aliasing.

The degeneration from two-dimensional to one-dimensional distributions causes amplitude errors of the synthesized sound field within the reproduction plane. When calculating the driving signals for the array speakers, a gain factor \(g(z)\) has to be determined such that on a chosen reference line the amplitude of the synthesized sound field will be correct. This factor depends on the geometrical arrangement of the virtual source, the loudspeakers and the reference line. For reproduction of a virtual point source with a linear WFS array it is given by [7]

\[
g(z_R) = \sqrt{\frac{z_R - z_l}{z_R - z_p}}. \tag{3}
\]

where \(z_R - z_l\) denotes the distance from the reference line to the loudspeaker array located at \(z = z_l\) and \(z_R - z_p\) denotes the distance from the reference line to the virtual point source at \(z = z_p\) (see Fig. 2).

The amplitude of the synthesized field can thus be calculated as

\[
A_{\text{synth}}(z) = A_{\text{ideal}}(z) \cdot \frac{g(z_R)}{g(z)}, \tag{4}
\]

resulting in a relative amplitude error of

\[
A_{\text{err,rel}}(z) = \frac{A_{\text{err}}(z)}{A_{\text{ideal}}(z)} = 1 - \frac{g(z_R)}{g(z)}. \tag{5}
\]

This amplitude error sets an upper limit to the maximum gain in noise reduction that is achievable for the case when using a linear loudspeaker setup to cancel the field of a primary point source:

\[
G_{\text{max}}(z) = 20\log_{10} \frac{1}{A_{\text{err,rel}}(z)} = 20\log_{10} \frac{1}{1 - g(z_R)/g(z)}. \tag{6}
\]

The upper part of Fig. 1 shows the pressure amplitudes of the sound fields produced by a point source and a virtual point source synthesized by a one-dimensional WFS array. The point source was positioned 1 m behind the loudspeaker array, the reference line was set 0.7 m in front of the WFS speakers. As can be seen, the amplitude of the synthesized field is too high near the WFS array. According to (6), the maximum possible reduction of the noise level can be read out from the lower part of Fig. 1 as the negative of the relative error level.

4. RESULTS

The results presented here are based on simulations and measurements. The scenario under investigation is the cancellation of the sound field of a primary noise source by synthesis of an antiphase sound field by a WFS system. The geometry is shown in Fig. 2.

The primary source is chosen as a point source located on the z-axis at \(z_p = -1\) m, i.e. 1 m behind the WFS array. The WFS array consists of 8 sources on the x-axis with a spacing of \(d = 0.2\) m. The WFS loudspeakers are assumed to act like monopole sources.

The sound fields of the primary and secondary sources are analyzed on a square grid of dimensions 1.43 m \(\times\) 1.43 m which is located 0.07 m in front of the WFS array. The distance between two neighboring analysis points is \(d_a = 0.13\) m, which is small enough to allow for the derivation of global results in this area up to the aliasing frequency of the WFS system.

The noise signal is assumed to be known by the ANC-System, i.e. a perfect reference signal is given. The primary
noise source and the WFS system are both fed with a sinc-function centered at time \( t = 0 \) that is band limited to frequencies below the aliasing frequency of the WFS system, which can be calculated according to (1) as \( f_{\text{max}} = c/2d = 850 \text{ Hz} \).

The position of the primary source is also provided to the WFS system such that the field of a virtual secondary point source located at the same position can be synthesized.

### 4.1 Simulations

All the following simulations are done for three dimensional sound propagation assuming free field propagation, i.e. assuming a reflection-free environment.

Figure 3 shows the pressure field \( p(x,z) \) produced by the primary noise source at time instance \( t = 5.2 \text{ ms} \). The error field, i.e. the difference between the pressure field of the primary source and the field of a virtual source synthesized by the loudspeaker array is shown in Fig. 4.

The reference line of WFS system was placed at \( z_R = 0.7 \text{ m} \). It can be seen that on the reference line cancellation of the unwanted noise occurs. The relatively high errors near the corners of the WFS array are caused by the truncation of the WFS array at these points.

Fig. 5 shows the energies of the error signals relative to the energies of the primary noise signals. This figure shows the same behavior like the theoretically derived Fig. 1. The maximum gain values (which can be found on the reference line \( z_R = 0.7 \text{ m} \)) range from 18.7 dB to 22.9 dB.

### 4.2 Measurements

We performed measurements according to the setup described by Fig. 2 to verify the theoretical and the simulation results. The measurements were carried out in a room of size \( 6m \times 6m \times 3m \), and a reverberation time \( T_{60} \approx 200 \text{ ms} \) with one linear 8-speaker array of our WFS system. The WFS array loudspeakers and the the primary noise source were of the same loudspeaker type and arranged at the same height in the room.

12 omnidirectional microphones were mounted on a linear array with a microphone spacing of \( d_a = 0.13 \text{ m} \) that was positioned in parallel to the x-axis. We measured the impulse responses from every loudspeaker (including the primary noise source) to every microphone of the array. These measurements were done for 12 different \( z \)-locations of the microphone array, so that the whole area of interest was sampled at a spacing of 0.13 m.
At first we were primarily interested in validating the basic mechanisms of the WFS-ANC approach. Therefore the results shown in Fig. 6 are based on the early parts of the impulse responses. Using only the first 25 ms of the impulse responses, most of the energy of the investigated signals arises from the direct sound, not from the reflections. The obtained results are closer to free field propagation due to the truncation of the impulse responses.

In principle, this result shows the same characteristics as the simulation results from Fig. 1 and Fig. 5: (1.) Maximum gain in noise reduction can be obtained on the reference line (10.0 dB to 15.9 dB), (2.) the noise reduction level slowly decreases when moving further away from the WFS array down to a mean value of 8.6 dB, (3.) there is a zone of noise amplification very close to the WFS speakers (mean gain: −3.6 dB).

It is expected that the noise reduction level will decrease in reverberant environments for our scenario. As we only use a linear array on one side of the area of interest, we cannot cancel reflections from other directions. We can show this effect by using the full measurement-length (170 ms) of the impulse responses. The resulting relative error signal energies are shown in Fig. 7. Again, the same behavior as before can be seen, although at lower levels of noise reduction (maximum gain on reference line: 4.0 dB, mean gain at $z = 1.43$ m: −0.7 dB, mean gain near loudspeakers: −3.9 dB).

5. CONCLUSION AND OUTLOOK
An approach to global noise control has been investigated which avoids the large number of acoustical sensors required in current multi-point-methods. Theoretical considerations predict a considerable reduction of noise emitted by a point-source. For a typical scenario, these predictions where shown to hold both by sound field simulations and by measurements on a dense grid of sensors. The proposed noise reduction arrangement is particularly attractive in enclosures, where a loudspeaker array for spatial audio reproduction is already installed. Based on these results, a number of directions for further research can be identified:

Besides a single point source, also other spatial noise distributions have to be controlled, e.g. multiple point sources, plane waves, etc. Furthermore, the position of noise sources above or below the listening plane require attention.

For reverberation times as in our measurements or higher, a loudspeaker array at only one side of the listening room is not sufficient. Additional line arrays have to be placed at all reflecting walls.

The assumption that the exact noise signal is known to the system at any time is too idealistic in many cases. Instead adaptive multichannel algorithms for noise reduction have to be applied. Here, wave domain adaptive filtering based on the wave field analysis techniques described above are promising candidates.

REFERENCES