KALMAN-BASED ESTIMATION OF MEASURED CHANNELS IN MOBILE MIMO OFDM SYSTEM

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ABSTRACT

In this paper the feasibility of a MIMO OFDM channel tracking algorithm when applied to measured MIMO channels is studied. A time domain channel estimation and tracking based on Kalman filtering is employed while the equalization is performed in frequency domain. Our simulations based on measured MIMO channels show that reliable channel estimation can be performed under realistic conditions. Moreover, some theoretical assumptions are shown to be fulfilled in practice.

1. INTRODUCTION

Radio spectrum is a scarce resource in wireless communication. High spectral efficiency is therefore a major goal of future mobile wireless communications systems. Multiple-input multiple-output (MIMO) systems are promising candidates for such systems since they additionally utilize the spatial domain by doing multiplexing and therefore obtain much higher capacities than conventional single-input single-output (SISO) systems. The only supposition is that the system is operated in a rich scattering environment where a lot of different propagation paths exist between the transmitter and the receiver.

Multicarrier systems, including orthogonal frequency division multiplexing (OFDM) systems, play also an important role in future beyond 3G wireless communication systems. A key benefit of OFDM is its ability to turn a frequency selective channel into a set of parallel narrowband channels, which leads to very simple equalization since the transmission becomes free of Intersymbol Interference (ISI). However, correlated fading at both transmit and receive side and also imperfect channel knowledge at the receiver reduces the theoretical benefits of MIMO and OFDM.

In this paper we investigate the performance of a MIMO OFDM channel tracking algorithm, that was introduced in [5], when applied to real measured channels. Using real channels is an important stage in algorithm design to verify whether the algorithm achieves the same performance as when using channel models. It is also important to verify if the algorithm's theoretical assumptions still hold when using measured data.

Up to now, very little work on MIMO OFDM channel estimation has been done using actual measured channel data. In [4] the performance of a BLAST-OFDM-type of receiver has been investigated when using measured channels. A different Kalman based channel estimator has also been used in [7].

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In this paper we consider a time-domain MIMO channel matrix tracking algorithm stemming from Kalman filter. It takes both the frequency and time selectivity of the channel into account. The channel estimation is followed by equalization in frequency domain. Both zero-forcing and minimum mean square error (MMSE) equalizers are presented. The time-domain approach used in this paper shows highly reliable tracking performance and robustness. It tracks accurately both the amplitudes and phases of the measured MIMO channels. Time-domain method exploits the inherent frequency correlation among the taps. Hence, it is robust to estimation errors that are spread over the complete frequency band.

The rest of the paper is organized as follows. In the next section, we briefly present the MIMO-OFDM system model. In Section 3 we introduce the channel tracking and equalization scheme. In Section 4 the measurement setup and simulation results are discussed. Finally, Section 5 concludes the paper.

2. SYSTEM MODEL

The MIMO-OFDM transmission model used in this paper is presented in Figure 1. A T-transmit / R-receive antenna configuration is considered, with the index t running from 1 to T and r running from 1 to R. The k^{th} modulated OFDM block at transmit antenna t is written as $\tilde{\mathbf{x}}_t(k) = \mathbf{F}_N \mathbf{a}_t(k)$, where $\mathbf{F}_N$ is the $N \times N$ inverse discrete Fourier transform (IDFT) matrix, N being the total number of subcarriers and $\mathbf{a}_t(k)$ is the $N \times 1$ complex symbol vector sent from antenna t.

The received $N \times 1$ signal block, at antenna r, after cyclic prefix insertion, followed by transmission on the wireless channel and cyclic prefix removal is expressed as:

$$\mathbf{r}_r(k) = \sum_{t=1}^{T} \widetilde{\mathbf{H}}_{tr}(k)\tilde{\mathbf{x}}_t(k) + \mathbf{w}_r(k), \qquad r = 1,\ldots,R \tag{1}$$

where $\widetilde{\mathbf{H}}_{tr}(k)$ is the $N \times N$ channel convolution matrix, modeling the wireless environment between the i^{th} transmit and j^{th} receive antennas and it is given by $\widetilde{\mathbf{H}}_{tr}(k) = \mathbf{R}_{CP}\mathbf{H}_{tr}(k)\mathbf{T}_{CP}$. The matrices $\mathbf{R}_{CP}$ and $\mathbf{T}_{CP}$ inserts and removes the cyclic prefix (CP) (see [6] for the structure of these matrices). Due to the insertion and removal operations, $\widetilde{\mathbf{H}}_{tr}$ matrices are circulant, with the (i,l)th entry given by $h_{t,r,(i-l) \mod N}$. The channel taps $\{h_{t,r,l}\}_{l=0,\ldots,L-1}$ are assumed to be invariant over the duration of one ODFM block.

The maximum channel length is assumed to be L, which is also the length of the cyclic prefix, in order to avoid inter-

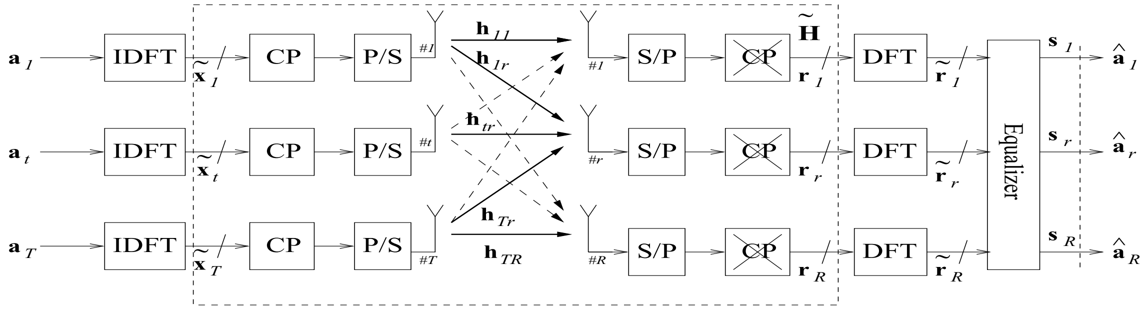

Figure 1: MIMO OFDM transmission.

block interference. The noise term $\mathbf{w}$ is assumed to be circular white Gaussian with covariance matrix $\sigma^2\mathbf{I}$.

Since circulant matrices implement circular convolutions, they are diagonalized by DFT and IDFT operations and thus after having performed the discrete Fourier transform of $\mathbf{r}_t(k)$ we obtain:

$$\tilde{\mathbf{r}}_r(k) = \sum_{t=1}^{T} \mathbf{F}_N^H \widetilde{\mathbf{H}}_{tr}(k)\tilde{\mathbf{x}}_t(k) + \tilde{\mathbf{n}}_t(k), \qquad (2)$$

where $\tilde{\mathbf{n}}_t(k) = \mathbf{F}_N^H \mathbf{w}_t(k)$, $\mathbf{F}_N^H$ is unitary DFT matrix, and the diagonal matrices

$$\begin{aligned}\mathbf{D}_{tr}(k) &= \mathbf{F}_N^H \widetilde{\mathbf{H}}_{tr}(k)\mathbf{F}_N \\ &= diag\left\{\sum_{l=0}^{L-1} h_{t,r,l}(k)\exp\left(-j\frac{2\pi nl}{N}\right)\right\}_{n=0,\dots,N-1} \qquad (3)\end{aligned}$$

contain the frequency response of the channel $\mathbf{h}_{tr}(k)$, evaluated at the subcarrier frequencies. In case $\mathbf{D}_{tr}$ are not diagonal matrices anymore, we face Intercarrier Interference (ICI) and the orthogonality of the OFDM system is lost. In this case both the channel and ICI have to be compensated for [6].

Now, stacking the R equations in (2) in a vector we obtain:

$$\tilde{\mathbf{r}}(k) = \left[\tilde{\mathbf{r}}_1^{\mathrm{T}} \dots \tilde{\mathbf{r}}_R^{\mathrm{T}}\right]^{\mathrm{T}} = \mathscr{D}(k)\mathbf{a}(k) + \tilde{\mathbf{n}}(k), \qquad (4)$$

where $\mathbf{a}(k) = [\mathbf{a}_1(k)^{\mathrm{T}} \dots \mathbf{a}_t(k)^{\mathrm{T}} \dots \mathbf{a}_T(k)^{\mathrm{T}}]^{\mathrm{T}}$, $\tilde{\mathbf{n}}(k) = [\tilde{\mathbf{n}}_1(k)^{\mathrm{T}} \dots \tilde{\mathbf{n}}_r(k)^{\mathrm{T}} \dots \tilde{\mathbf{n}}_R(k)^{\mathrm{T}}]^{\mathrm{T}}$ and the $RN \times TN$ matrix $\mathscr{D}(k)$ is:

$$\mathscr{D}(k) = \begin{bmatrix} \mathbf{D}_{11}(k) & \dots & \dots & \dots & \mathbf{D}_{T1}(k) \\ \vdots & & \mathbf{D}_{tr}(k) & & \vdots \\ \mathbf{D}_{1R}(k) & \dots & \dots & \dots & \mathbf{D}_{TR}(k) \end{bmatrix}_{RN\times TN}. \qquad (5)$$

The equalization may be performed in frequency domain as follows:

$$\begin{aligned}\mathbf{u}(k) &= \mathscr{U}(k)\tilde{\mathbf{r}}(k) \qquad (6)\\ &= \mathscr{U}(k)\left[\mathscr{D}(k)\mathbf{a}(k) + \tilde{\mathbf{n}}(k)\right], \qquad (7)\end{aligned}$$

where $\mathscr{U}(k)$ is the $TN \times RN$ equalizer matrix at OFDM block time k. Then, the decisions are carried out on $\mathbf{u}(k)$ in order to obtain the symbol estimate $\hat{\mathbf{a}}(k)$. The zero forcing and MMSE equalizers may be found as follows. The zero forcing (ZF) equalizer is given by: $\mathscr{U}_{ZF}(k) = \mathscr{D}^+(k)$, where $\mathscr{D}^+ = \left(\mathscr{D}^H\mathscr{D}\right)^{-1}\mathscr{D}^H$ is the left pseudoinverse of $\mathscr{D}$. The MMSE equalizer may be found as follows:

$$\mathscr{U}_{MMSE}(k) = \sigma_a^2 \mathscr{D}^H(k)\left[\sigma_a^2 \mathscr{D}(k)\mathscr{D}^H(k) + \sigma^2\mathbf{I}\right]^{-1}, \qquad (8)$$

where $\sigma^2 = \sigma_{\tilde{n}}^2$ is the variance of the noise and σ_a^2 is the average symbol energy.

In order to find the zero forcing equalizer, $\mathscr{D}^H\mathscr{D}$ has to be full rank, i.e. $\mathrm{rank}\{\mathscr{D}^H\mathscr{D}\} = \mathrm{rank}\{\mathscr{D}\} \leq \min(TN, RN)$. Hence $\min(TN, RN) = TN$, i.e. the number of receive antennas must be at least equal to the number of transmit antennas. A closer look to $\mathscr{U}_{ZF}$ and $\mathscr{U}_{MMSE}$ matrices reveals the fact that they are formed by diagonal matrices. By exploiting this special matrix structure, significantly lower complexity can be achieved in the equalization stage since direct matrix inversion in (8) is avoided. This highlights an important property of OFDM transmission: the initial frequency selective channel has been turned into a set of N frequency flat channels. The equalizer is needed here to compensate the flat fading experienced on each subcarrier and also to de-multiplex the transmitted streams $\mathbf{a}_t, t = 1, \dots, T$.

3. TIME-DOMAIN CHANNEL TRACKING

In mobile communications, the channels are time-varying. Hence, the channel needs to be tracked and equalizer coefficients updated periodically. The MIMO-OFDM channel tracking algorithm used in this paper is based on state-space formulation of the transmission model [5].

The received $RN \times 1$ signal block can be written as:

$$\mathbf{r}(k) = \tilde{\mathbf{X}}(k)\mathbf{h}(k) + \mathbf{w}(k), \qquad (9)$$

where the channel vectors corresponding to each MIMO branch are stacked into the vector $\mathbf{h}(k) = \left[\mathbf{h}_{11}^{\mathrm{T}}(k) \dots \mathbf{h}_{T1}^{\mathrm{T}}(k) \dots \mathbf{h}_{1r}^{\mathrm{T}}(k) \dots \mathbf{h}_{Tr}^{\mathrm{T}}(k) \dots \mathbf{h}_{1R}^{\mathrm{T}}(k) \dots \mathbf{h}_{TR}^{\mathrm{T}}(k)\right]^{\mathrm{T}}$. Each $\mathbf{h}_{tr}$ vector contains L channel taps. The data matrix of size $RP \times TRL$ is the following $\tilde{\mathbf{X}}(k) =$

$$\begin{bmatrix} \tilde{\mathbf{X}}_{11}(k) & \cdots & \tilde{\mathbf{X}}_{T1}(k) & \cdots & & \cdots & \mathbf{0}_{P\times L} \\ \vdots & & & \cdots & & & \vdots \\ \mathbf{0}_{P\times L} & \cdots & & \cdots & \tilde{\mathbf{X}}_{1R}(k) & \cdots & \tilde{\mathbf{X}}_{TR}(k) \end{bmatrix}_{RP\times TRL}, \qquad (10)$$

where $P = N + L$ is the total OFDM symbol length. The time evolution of the channel can be described as:

$$\mathbf{h}(k) = \mathbf{A}\mathbf{h}(k-1) + \mathbf{v}(k), \qquad (11)$$

where $\mathbf{A}$ is the state transition matrix and $\mathbf{v}$ is the state noise. The spectral radius of the state transition matrix has to be less than one in order to ensure the stability of the system. This matrix can be also estimated from the received data, see e.g., [2]. Equations (9) and (11) form the state-space model describing our transmission system. Considering that the model is linear and the noise is Gaussian, Kalman filtering (KF) [1] can be applied to estimate the state vector $\mathbf{h}$. Note that when applying KF, the observation and state covariance matrices have to be known. In our simulation studies in Section 4, these quantities have been chosen manually. However, they can be reliably estimated [2].

The overall structure of the receiver is illustrated in Figure 2. The algorithm works as follows. For OFDM symbol at time k:

1. Decode the received vector $\mathbf{r}(k)$ and obtain the symbol estimate $\hat{\mathbf{a}}(k)$, using $\hat{\mathbf{h}}(k-1|k-1)$, i.e. the filtered estimate of the channel at symbol time $k-1$.
2. Re-modulate $\hat{\mathbf{a}}(k)$: $\hat{\tilde{\mathbf{x}}}(k) = \mathbf{F}_N \hat{\mathbf{a}}(k)$.
3. Build the estimate $\hat{\tilde{\mathbf{X}}}(k)$ using $\hat{\tilde{\mathbf{x}}}(k)$.
4. Run the Kalman filter to finally obtain $\hat{\mathbf{h}}(k|k)$.

At this point one may also refine the estimates by re-decoding the symbol $\hat{\mathbf{a}}(k)$ using the filtered estimate $\hat{\mathbf{h}}(k|k)$.

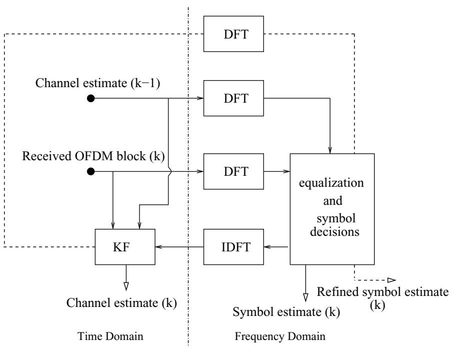

Figure 2: Time-domain channel estimation and tracking, frequency domain equalization.

Except for the first OFDM symbol, the algorithm works in a decision directed mode. The major computational cost lies in the calculation of the matrix inversion in the Kalman gain expression. By applying the matrix inversion lemma, the number of operations can be reduced to $O(LN^2)$ when tracking is done in time-domain. The complexity is of order $O(N^3)$ if one is applying KF in the frequency domain. In practice $L \ll N$, hence significantly lower complexity is achieved.

4. MEASUREMENT SETUP AND SIMULATIONS

4.1 Measurement setup

The channel measurements used in this paper took place at the Institute of Communications and Radio-Frequency Engineering, Vienna University of Technology. They were performed together with Elektrobit, using the Elektrobit PropSound channel sounder. We measured the impulse response between each transmit and receive antenna at a center frequency of 2.4GHz using a total measurement bandwidth of 200MHz, which gives a delay resolution of 5ns. At transmit side an 8-element circular array of monopoles with 7 elements on the circle and one in the center and at receive side a 16-element 4x4 rectangular patch array were used. Both arrays had an inter-element spacing of 0.5λ at 2.55GHz. During the measurement the receiver has been moved (with pedestrian speed), leading to a time-varying MIMO channel.

4.2 Simulation results

In the simulations, we consider a 2x2 MIMO system, therefore we only use the first two transmit and receive antennas from the available data. From the measured impulse responses, we extracted 8 consecutive taps, where the first tap was chosen such that it covers the first significant delay tap in the impulse response. In order to create a channel suitable for our simulations, we interpolated between the snapshots that were measured every 37ms such that the coherence time of the channel equals the OFDM symbol duration. Considering that the OFDM system has $N = 128$ subcarriers and a cyclic prefix of $100ns$, the total OFDM symbol duration becomes $740ns$.

The employed symbol modulation is QPSK. The MMSE equalizer (8) was chosen, as it does not suffer from noise enhancement as the zero forcing approach does. Frequency offsets between transmitters and receivers are assumed to be compensated for [6]. Channels are considered to remain stationary during the OFDM block time.

Since the equalization stage operates in the frequency domain, accuracy in estimating frequency responses of the channels at the subcarrier frequencies needs to be investigated. Figure 3 and 4 respectively show amplitude and phase responses, for the true and estimated channel, at a given OFDM block time. Since time-domain estimation performs well, channel transfer functions are consequently also modeled accurately, in both amplitude and phase. Re-training symbols are sent periodically (every 100 blocks, in our simulations), in order to avoid loosing the track, which could occur if all MIMO subchannels fall into a deep fade simultaneously. This would be a problem if the MIMO channels are highly correlated and rank of the channel matrix is low.

Tracking in time turns out to be robust to estimation errors because the frequency correlation of the taps can be efficiently exploited. Furthermore, estimation errors are spread over the whole transmission spectrum, and not concentrated on a given set of subcarriers.

The receiver's performance criterion is the bit error rate as a function of noise variance, presented in Figure 5. A lower bound for the performance of the tracking algorithm is given by using the ideal channel state information (CSI), i.e. perfectly known channel at the receiver side. As shown by simulation curves, tracking in time-domain provides us with results close to the ones obtained with known channel.

4.3 Discussion

The state transition matrix $\mathbf{A}$ is a crucial parameter for Kalman Filter convergence. In our experiments we have used a matrix close to identity matrix, with the diagonal elements less then one. This implied that the channels are varying independently in time, and from the simulation results we can draw the conclusion that we made a good assumption. This might be also explained by the fact that in the indoor scenario we considered, rich scattering at both transmit and receive

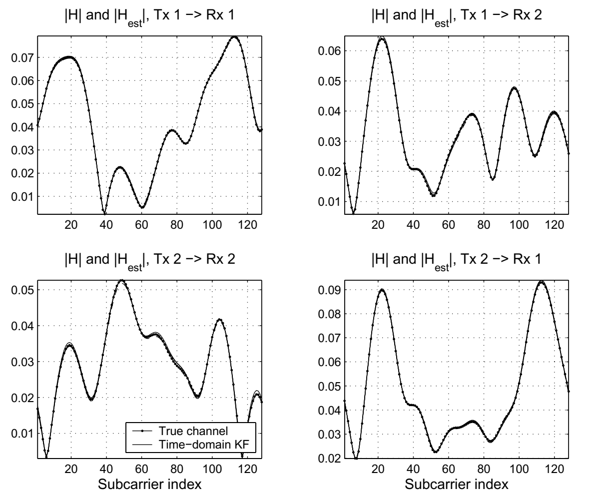

Figure 3: Amplitude responses (SNR = 15 dB, measured channel).

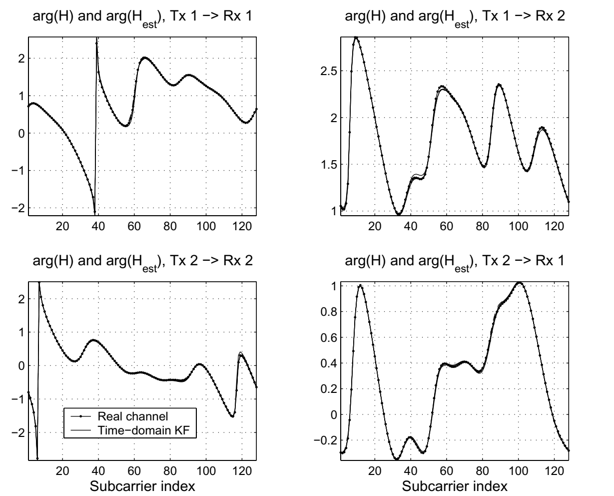

Figure 4: Phase responses (SNR = 15 dB, measured chanel).

side took place. However, if the channels exhibit correlation among each other the state transition matrix has to be a full matrix. It is of high interest to estimate this matrix [2] in order to get a close to optimum performance of the Kalman Filter.

5. CONCLUSIONS

In this paper, channel estimation and tracking experiments for mobile MIMO-OFDM systems have been carried out for MIMO measured channels. The performance of the method was demonstrated showing that the theoretical assumptions match with real channels very well. Future work includes studying the estimation of channel parameters from the measured data, to ensure a good performance of the channel estimation algorithm in any type of scenario.

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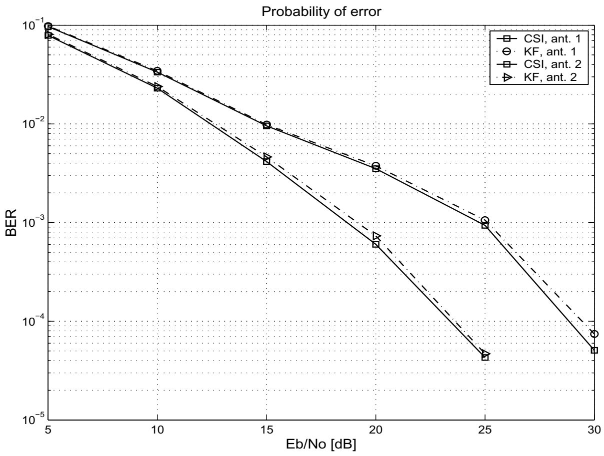

Figure 5: Bit error rate performance (over 5000 OFDM blocks).

measurements taken at the Institute of Communications and Radio-Frequency Engineering and to D. Schafhuber for the interesting discussions and suggestions. Furthermore the authors would like to thank Hüseyin Özcelik for helping a lot with the measurements and also Elektrobit for doing the measurements with us.

REFERENCES

[1] C.K. Chui, G. Chen, "Kalman Filtering with Real-Time Applications", Springer, 1999.

[2] M. Enescu, V. Koivunen, "On the estimation of state transition matrix and noise statistics in state-space models", Vehicular Technology Conference, 2002. Proceedings. VTC 2002-Fall. 2002 IEEE 56th , Volume: 4 , 2002 Page(s): 2192 -2196.

[3] Molisch, A.F., "Wideband wireless digital communications", Prentice Hall, 2000.

[4] R. J. Piechocki, P. N. Fletcher, A. R. Nix, C. N. Canagarajah, J. P. McGeehan, J.P., "Performance evaluation of BLAST-OFDM enhanced Hiperlan/2 using simulated and measured channel data ", Electronics Letters , Vol. 37, No. 18, August 2001, pp.:1137 - 1139.

[5] T. Roman, M. Enescu, V. Koivunen, "Time-domain method for tracking dispersive channels in MIMO OFDM systems", IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Vol. 4, pp. 393 - 396, Hong Kong, April 2003.

[6] T. Roman, M. Enescu, V. Koivunen, "Joint time-domain tracking of channel and frequency offset for MIMO OFDM systems", 14th IEEE international Symposium on Personal Indoor and Mobile Radio Communications, Beijing, China, 2003.

[7] D. Schafhuber, G. Matz, F. Hlawatsch, "Kalman tracking of time-varying channels in wireless MIMO-OFDM systems," in Proc. of Asilomar-03, Pacific Grove (CA), Nov. 2003.

[8] Z. Wang, G.B. Giannakis, "Wireless multicarrier communications", IEEE Signal Processing Magazine, Vol. 17 Issue: 3 , May 2000, Page(s): 29-48.