A powers-of-two term allocation algorithm for designing FIR filters with CSD coefficients in a min-max sense

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Abstract

In this paper, we consider design problems of linear phase FIR filter with CSD(or SP2) coefficients. When the total number of non zero SP2 terms is given for the design problem, we have to determine the number of non zero SP2 terms allocated for each filter coefficient respectively while keeping the total number. However, it is considered this problem is one of NP-hard problems. Hence, Lim et al.[3] developed a heuristic method for this allocation problem.

In this paper, we propose a new heuristic method for this problem comparing it with traditional heuristic method [3] through several numerical experiments.

1 Introduction

In these decades, several methods have been proposed for the design of FIR filters with SP2(CSD) coefficients `mode2`[1]- [5]. It is well known that the filters whose coefficients are represented as SP2 terms enable implementation of the filters without using multipliers. It has been demonstrated that the advantage can be achieved if different numbers of SP2 terms are allocated to each coefficient value while keeping the given total number of SP2 terms. However, it is difficult to design such filters since it results in an integer programming problem (IP), which is well-known as one of the NP-hard problems [6]. For designing such filters we have to obtain each optimal coefficients based on each number of allocated SP2 terms, but it requires exessive computational time. Hence, it can be effective to use heuristic method for this optimization problem. A well known heuristic method for this allocation problem is given by Lim, Yang, Li and Song [3].

In this paper we present a new heuristic SP2 term allocation method for allocating different number of non-zero SP2 terms to each coefficient subject to a given number of total non-zero SP2 terms for the filter.

In our proposed method, we first solve a usual filter design problem without SP2 coefficients constraints by Linear Programming method (LP). By using these obtained filter coefficients, we heuristically determine an allocation of the number of non-zero SP2 terms.

In order to verify the performance of our proposed allocation method, we obtain optimal filter coefficients for our method and for Lim et al.[3] by using Branch and Bound technique(B&B). Then we compare them through several numerical experiments.

2 Problem Description

Consider a N-tap even symmetric linear phase FIR filter with the nth coefficient value denoted by a_n. For simplisity we suppose N is odd. Ignoring the phase factor, its frequency response $H(\omega)$ can be expressed as

$$H(\omega) = \sum_{n=0}^{K} a_n \cos n\omega. \tag{1}$$

Where $K = (N-1)/2$ when N is odd.
Suppose a desired response $\varphi(\omega)$ is given as follows

$$\varphi(\omega) = \begin{cases} 1, & 0 \le \omega \le \omega_p, \\ 0, & \omega_s \le \omega \le \pi, \end{cases} \tag{2}$$

Let Ω be the approximation band and $\Omega = [0, \omega_p] \cup [\omega_s, \pi]$. Where ω_p is the passband cutoff frequency, and ω_s is the stopband cutoff frequency, respectively. Then, the optimization problem to approximate $H(\omega)$ to $\varphi(\omega)$ in a min-max sense can be written as

$$\min_{a_0,\ldots,a_K} \max_{\omega\in\Omega} |\varphi(\omega) - H(\omega)| \tag{3}$$

3 Allocation Problem of m_k

In the first, we discretize a pass band and a stop band adeqately and solve (3) to obtain the continuous coefficients using the linear programming(LP) method [6]. In this paper we solve the LP using GLPK [7]. Let $\bar{\boldsymbol{a}}$ the obtained continuous filter coefficients vector after solving LP. Then the frequency response $H_{\mathrm{lp}}(\omega)$ can be expressed as

$$H_{\mathrm{lp}}(\omega) = \sum_{n=0}^{K} \bar{a}_n \cos n\omega. \tag{4}$$

Suppose a total number of non-zero SP2 terms is given as M. And m_n represents the number of non-zero SP2 terms for the nth filter coefficient. The SP2 coefficients are represented as d_n for nth coefficient. Then the frequency response $H_{\mathrm{d}}(\omega)$ with d_n is as follows,

$$H_{\mathrm{d}}(\omega) = \sum_{n=0}^{K} d_n \cos n\omega. \tag{5}$$

The value $|d_n|$ ranges between 2^0 and 2^{-U}. Here U is a natural number. For example, if m_n is given, then d_n is represented as follows,

$$d_n = \sum_{i=1}^{m_n} b_i^{(n)} 2^{-q_i^{(n)}}. \tag{6}$$

Where $b_i^{(n)} \in \{-1, 1\}$ and $q_i^{(n)} \le U$ $(1 \le i \le m_n,\ 0 \le n \le K)$.

When $m_0, \ldots, m_K$ are given, the convex optimization problem to determine the filter coefficients $\boldsymbol{d}$ is abstractly as follows. Here $\boldsymbol{d} = (d_0, \ldots, d_K)^T$.

$$\begin{array}{ll} \min & f(\boldsymbol{d}) \\ \text{sub. to} & \text{the number of SP2 terms for } d_n \\ & \quad \text{is at most } m_n \ \ (n = 0, \ldots, K). \end{array} \tag{7}$$

In our case $f(\boldsymbol{d}) = \max \left|\varphi(\omega) - \sum_{n=0}^{K} d_n \cos n\omega\right|$, $(\omega \in \Omega)$.

3.1 Our proposed method

Our method is proposed focusing on the convexity of the design problem. Let $\boldsymbol{e}_n$ be a unit vector whose n th element is 1 and the rest are 0. Let $\lceil \bar{a}_n \rceil$ be the least SP2 upper bound and $\lfloor \bar{a}_n \rfloor$ be the largest SP2 lower bound for each continuous coefficient $\bar{a}_n$. In each iteration we select an index n by following one of four rules in step 3, and we add 1 to m_n.

The new value of $\lceil \bar{a}_n \rceil and \lfloor \bar{a}_n \rfloor$ approximates $\bar{a}_n$ better. We propose this algorithm since it is expected to obtain SP2 solutions so as not to increase error function f much since f is a convex function.

Step 1: For each n, if $|\bar{a}_n| < \varepsilon$, $m_n = 0$; otherwise $m_n = 1$.

Step 2: Evaluate the following equations for each of n such that $m_n \ge 1$

$$f_n^{\mathrm{U}} = f(\bar{\boldsymbol{a}} + (\lceil \bar{a}_n \rceil - \bar{a}_n)\boldsymbol{e}_n), \tag{8}$$

$$f_n^{\mathrm{L}} = f(\bar{\boldsymbol{a}} + (\bar{a}_n - \lfloor \bar{a}_n \rfloor)\boldsymbol{e}_n). \tag{9}$$

Step 3: Calculate f_n by using one of the following four rules, and $f_{n^*} = \max_n \{f_n\}$.

`mode1:` (max-max rule) $f_n = \max\{f_n^{\mathrm{U}}, f_n^{\mathrm{L}}\}$.
`mode2:` (max-min rule) $f_n = \min\{f_n^{\mathrm{U}}, f_n^{\mathrm{L}}\}$.
`mode3:` (average rule) $f_n = f_n^{\mathrm{U}} + f_n^{\mathrm{L}}$.
`mode4:` $f_n = \lceil \bar{a}_n \rceil - \lfloor \bar{a}_n \rfloor$.

Step 4: Let $m_{n^*} = m_{n^*} + 1$ and $M = M - 1$.
Step 5: If $M = 0$, stop; otherwise, go to Step 2.

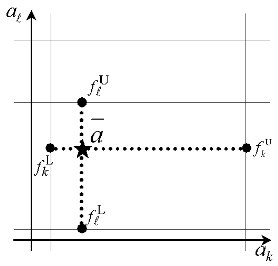

Figure 1: Our allocation algorithm

In general, the computational cost is not large to calculate $f(\boldsymbol{d})$. A well-known heurisic to determine m_n from $\bar{a}_n$ is given by Lim et al. [3]. Hence, we compare our proposed method with this Lim et al.'s method in the follows.

3.2 Branch and Bound technique

The algorithm to solve for (7) by using B & B is represented as follows. Each subproblem is expressed as

$$\begin{array}{ll} \min & f(\boldsymbol{d}) \\ \text{sub. to} & \ell_n \le d_n \le u_n \ \ (n = 0, \ldots, K), \\ & \text{the number of SP2 terms for } d_n \\ & \quad \text{is at most } m_n \ \ (n = 0, \ldots, K). \end{array} \tag{10}$$

$\ell_n = 0, u_n = 1$ $(n = 0, \ldots, K)$ in the problem (10) corresponds to (7) (original problem). The following relaxation problem whose constraints on SP2 are eliminated is solved.

$$\begin{array}{ll} \min & f(\boldsymbol{d}) \\ \text{sub. to} & \ell_n \le d_n \le u_n \ \ (n = 0, \ldots, K). \end{array} \tag{11}$$

Since the relaxation problem(11) is a convex programming problem with lower and upper bound constraints, it can be solved easily. (11) is easily transformed into an LP-problem by discretizing Ω. Let $\overline{\boldsymbol{d}}$ be an optimal solution of (11). Then we calculate $\lceil \overline{d}_n \rceil$ and $\lfloor \overline{d}_n \rfloor$ for $\overline{d}_n$,and operate as follows.

(i) If $\lceil \overline{d}_n \rceil = \lfloor \overline{d}_n \rfloor$ $(n = 0, \ldots, K)$, $\overline{\boldsymbol{d}}$ is an optimal solution of the subproblem. Hence, if the objective function value $f(\overline{\boldsymbol{d}})$ is smaller than the value of the incumbent best solution, we let $f(\overline{\boldsymbol{d}})$ be a new incumbent value.

(ii) If n exists such that $\lceil \overline{d}_n \rceil$ is bigger than $\lfloor \overline{d}_n \rfloor$, we select a n and generate a subproblem in which u_n is changed to $\lfloor \overline{d}_n \rfloor$ and a subproblem in which ℓ_n is changed to $\lceil \overline{d}_n \rceil$. We continue these operations until we finish searching all of subproblems.

We adopted a depth first search rule as a rule of selection of unsearched subproblems. There are some selection rules of n in (ii). For example we can consider

$\max_n\{\lceil \overline{d}_n \rceil - \lfloor \overline{d}_n \rfloor\}$, $\max_n\{u_n - \ell_n\}$, and their combination. However we adopt the rule $\max_n\{\lceil \overline{d}_n \rceil - \lfloor \overline{d}_n \rfloor\}$ since it is found that this rule is superior to others as a result of preliminary experiments.

4 Computational Experiments

We executed some computational experiments to certify the performance of the proposed filter design method. Where $N = 9, \ldots, 105$, $\omega_p = 0.15$, $\omega_s = 0.25$, $U = 16$. ω_p is passband cutoff frequency and ω_s is stopband cutoff frequency.

4.1 Comparison of Optimal Value

We assumed $M = 2 \times K + 1$, $\sum_{n=0}^{K} m_n = M$. We demonstrate the comparison of the optimal value for (7). Where we adopted `mode2`, since `mode2` is the best solution among four `modes`.

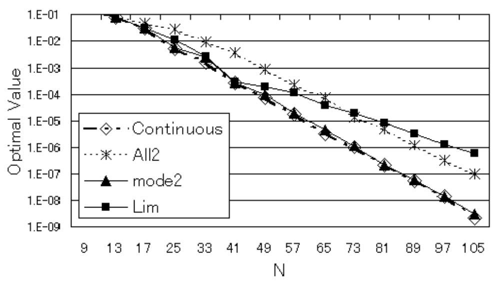

Figure 2: Optimal Value ($N = 9...105$)

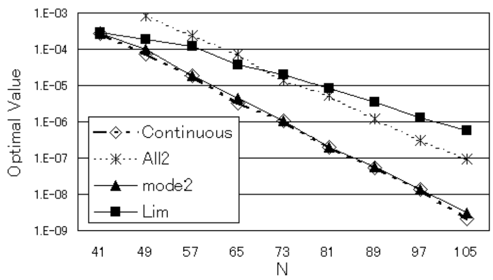

Figure 3: optimal Value ($N = 41 \ldots 105$)

The figure 2 shows the continuous solution of (3) ($N = 9, \ldots, 105$), and the optimal solution of (7) by `all2`, `Lim` and `mode2`. The figure 3 is an extension of figure 2. "All2" means an optimal value for an allocation of two non-zero SP2 terms for each d_n, i.e.,$(m_0, \ldots, m_K) = (2, \ldots, 2)$ and "Lim" means a solution by method of Lim et al. [3].

The figure 2 and figure 3 show that the optimal value of `mode2` is better than All2 and Lim when N is more than 41 and show that the optimal value of `mode2` is the closest to that of continuous. It was shown that our allocation method is effective for minimizing max-error through this numerical experiments. However we notice here that a magnitude response of FIR filters with SP2 coefficients is not complete equi-ripple in spite of the optimal value being good.

4.2 Comparison of m_n Allocation Methods

The table1 and the table2 show comparisons of optimal value of All, `mode2`, Lim and All2 when $N = 11$ and $N = 13$. A description "All" means the smallest optimal value among the optimal values for all allocation patterns of m_n.

It is demonstrated that the optimal value of our proposed method (`mode2`) is the smallest among all2, Lim and `mode2`. And it is also shown that the optimal value of `mode2` is very close to that of "All". From these results, our proposed method was successful in allocating m_n.

Table 1: Optimal Value(N=11)

All	0.07520084902536860000
`mode2`	0.07796325860073640000
Lim	0.08448930648197490000
All2	0.08448930648197490000

Table 2: Optimal Value(N=13)

All	0.03701944731815920000
`mode2`	0.03942173986241200000
Lim	0.05720465158750860000
All2	0.06418419326499370000

4.3 Comparison of Magnitude response

In figure 4 and figure 5, the magnitude responses of `mode2`, Lim et al. and Continuous coefficients for $N = 63$ are shown. In this figure a description "Cont" means continuous. It is observed that the filter of `mode2` can obtain better performance than the filter of Lim et al. in pass band and stop band. We verified whether the relation between M and N influences on the magnitude response. We adopted `mode2`.

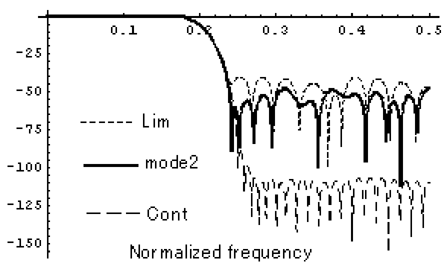

Figure 4: Magnitude response $N = 63$

A figure 6 and a figure 7 show magnitude responses of low pass filter for $N = 71 (M = 73(2K + 3))$ and $N = 47(M = 73(3K + 4))$. In case that $N = 71$ ($M = 73$), two non-zero SP2 terms can be allocated to each filter coefficient and in case that $N = 47$ ($M = 73$), three non-zero SP2 terms can be allocated to each filter coefficient.

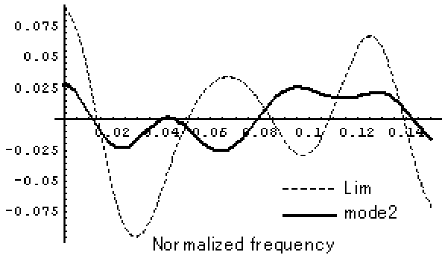

Figure 5: Magnitude response $N = 63$

These figures indicate that the magnitude response of $N = 47$ can obtain better performance than that of $N = 71$ both in the passband and in the stopband.

According to these results, it was shown that the number of SP2 terms m_n for each coefficient has an influence on a magnitude response.

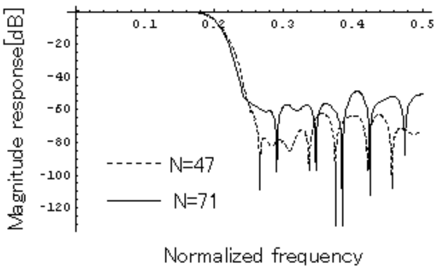

Figure 6: Magnitude response $N = 47$ and $N = 71(M = 73)$

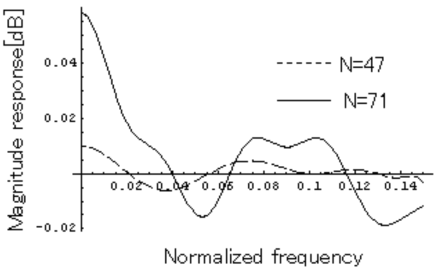

Figure 7: Magnitude response $N = 47$ and $N = 71(M = 73)$

4.4 Computational Time

A figure 8 shows a computational time of our proposed algorithm($N = 9 \ldots 121$). This time is a total time to obtain a solution. It means that it includes time to solve LP and to determine an allocation of SP2 terms and to obtain an optimal solutions by Brnch and Bound technique.

This figure indicates that it takes less than 10 seconds for $N = 97$ and it takes about 21 seconds for $N = 121$

According to this experiments, our algorithm gave us much better performance in spite of it using Branch and Bound technique.

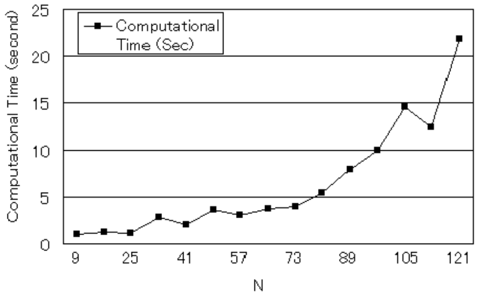

Figure 8: Computational Time $N = 9 \ldots 121$

5 Conclusion

In this paper, we proposed a new heuristic SP2 allocation method for the design of FIR filters with SP2 coefficients. We executed numerical experiments to verify the effectiveness for improving the performande of the filter and also verified the number of SP2 terms m_k for each coefficients has an influence on a magnitude response.

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