IMPULSE NOISE CORRECTION IN AN IMAGE TRANSMISSION SYSTEM BY MEANS OF AN OVERSAMPLED FILTER BANK CODE

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ABSTRACT

Quantized frame expansions based on block transforms and oversampled filter banks (OFB) have been considered recently as joint source-channel codes for erasure and error resilient signal transmission over noisy channels. This paper examines the problem of syndrome decoding and especially of error localization and correction in quantized OFB signal expansions. The error localization problem is treated as an M-ary hypothesis testing problem. The tests are derived from the joint probability density function of the syndromes under various hypothesis of impulse noise positions and in a number of consecutive windows of the received samples (to account for the encoding memory of the convolutional code). The error amplitudes are then estimated from the syndrome equations by solving them in the least square sense. The message signal is reconstructed from the corrected received signal by a pseudoinverse receiver. The algorithm is applied to joint source and channel coding (JSCC) of images based on oversampled wavelet filter banks.

1. INTRODUCTION

In the joint source and channel coding based on oversampled transform codes the error control coding and the signal decomposition are integrated in a single block by using an oversampled filter bank (OFB). The error protection in this approach is introduced before the quantization allowing suppression of quantization noise effects. So far the research in this area has been concentrated on the oversampled transform codes (OTC) which are the OFB codes with polyphase filters orders equal to zero. The OTC can therefore be viewed as real number block codes while the OFB codes with higher order polyphase matrices can be associated to real number convolutional codes.

Decoding of real number block codes has been considered in [1, 2, 3, 4]. Here we are considering the decoding of the real number convolutional codes in presence of impulse noise errors and the background noise. This problem has been treated in [5] in the context of fault tolerant systems and channel coding for communications, and recently in [6] in the context of joint source and channel coding (JSCC). In [5] the detection of the increased noise statistics due to impulse errors is motivated by an M-ary hypothesis testing theory and employs likelihood ratios of quadratic forms [5, 1]. In [6] the error localization is also treated as an M-ary hypothesis testing problem and differs from [5] in the way the likelihood values for hypothesis testing are calculated.

The approach presented here is inspired from [5]. We use a minimum total probability of error test [7], that is, we compute the a posteriori probability of each hypothesis and choose the largest. The a posteriori probabilities are derived from the joint pdf of the syndrome vectors under various hypothesis of impulse noise positions and in a number of consecutive windows of the received samples (to account for the encoding memory of the convolutional code). The error amplitudes are then estimated by solving the syndrome equations in the least square sense. The message is reconstructed from the corrected received sequence by a pseudoinverse receiver. The algorithm is applied to joint source and channel coding (JSCC) of images based on oversampled wavelet filter banks.

2. GENERAL FRAMEWORK AND PROBLEM STATEMENT

The block diagram of an analysis and synthesis bank of with N filters and decimation factor K is shown in Fig.1. In the analysis filter bank, an input signal $x[n]$

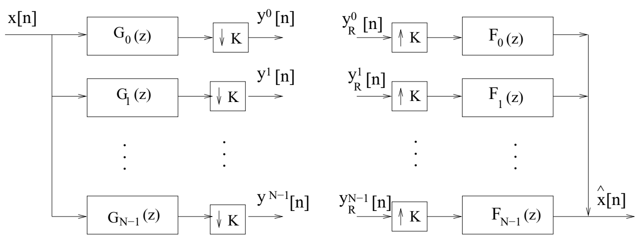

Figure 1:

is split into N signals $y^k[n]$, $k = 0, \ldots, N-1$. The sequence $y^k[n]$ is obtained by downsampling the output of the filter k with a factor K, where $K \leq N$. The sequences $y^k[n]$ are then quantized and transmitted over the channel which introduces impulse noise errors. Due to quantization and channel errors he received signal $y^k_R[n]$ differs from $y^k[n]$ and can be written as

$$y^i_R[n] = y^i[n] + e^i[n] + n^i[n] \quad (1)$$

where $n^i[n]$ is the quantization noise modeled as a Gaussian random variable with a zero mean and variance σ_q^2 and $e^i[n]$ is an impulse channel error modeled as a Bernoulli Gaussian process $e[n] = a[n]b[n]$ where $a[n]$ is a sequence of ones and zeros with probability $P(a[n] = 1) = p$ and $b[n]$ is a Gaussian random variable with zero mean and variance σ_i^2. It is assumed that the variance of the impulse noise is significantly larger than that of a quantization noise $\sigma_i^2 >> \sigma_q^2$. The aim is to use the redundancy introduced by oversampling for correction of the channel errors and/or suppression of the quantization noise.

In the following bold letters denote vectors, $()^T$ and $\tilde{()}$ denote transpose and paraconjugate operation, respectively. The analysis filter bank's outputs and the corresponding received samples for time instant n are represented as:

$$\mathbf{y}[n] = \begin{bmatrix} y^0[n] & \dots & y^{N-1}[n] \end{bmatrix}^T$$
$$\mathbf{y}_R[n] = \begin{bmatrix} y_R^0[n] & \dots & y_R^{N-1}[n] \end{bmatrix}^T, n = 0, \dots L$$

where $\mathbf{y}[n] = \mathbf{0}$ and $\mathbf{y}_R[n] = \mathbf{0}$ for $n < 0$ and $n > L$.

3. OVERSAMPLED FILTER BANK CODE

Frame expansions based on oversampled filter banks can be associated to convolutional codes on the real or complex fields with corresponding generator and parity check matrices. The encoding operation performed by an oversampled filter bank with N channels and downsampling factors K can be compactly described in the polyphase domain as

$$\mathbf{Y}(z) = \mathbf{E}(z)\mathbf{X}(z), \tag{2}$$

where $\mathbf{X}(z)$ and $\mathbf{Y}(z)$ are the polyphase representations of the input and the output signals for the analysis filter bank and $\mathbf{E}(z)$ is an $[N \times K]$ analysis polyphase matrix with elements $E_{i,j}(z) = \sum_{k=0}^{L_E} g_i(Kk+j)z^{-k}$ where g_i is the ith filter's impulse response and L_E is the smallest integer exceeding L_G/K, where L_G denotes the largest filter length. The polyphase analysis matrix $\mathbf{E}(z)$ is referred to as a generator matrix of an OFB code.

Similarly, the parity check matrix is defined as

$$\begin{array}{l} \mathbf{P}(z)\mathbf{E}(z) = \mathbf{0} \\ \mathbf{P}(z) = \sum_{i=0}^{L_P} \mathbf{P}_{L_P - i} z^{-i} \end{array} \tag{3}$$

The error correction strategy which makes use of this property of the parity check matrix is referred to as syndrome decoding. The syndromes $\mathbf{S}(z)$ are hence obtained by filtering the received signal $\mathbf{Y}_R(z)$ with the parity check filters as

$$\mathbf{S}(z) = \mathbf{P}(z)\mathbf{Y}_R(z) \tag{4}$$

From Eqs. (2), (3) and (4) we can observe that filtering sequence $\mathbf{Y}(z)$ with a party check filters yields zero syndromes. On the other hand, if the transmitted signal is corrupted by noise we have

$$\mathbf{S}(z) = \mathbf{P}(z)\mathbf{e}(z) + \mathbf{P}(z)\mathbf{n}(z) \tag{5}$$

where $\mathbf{e}(z)$ and $\mathbf{n}(z)$ are the polyphase representations of the impulse and the quantization noise. Therefore syndromes can be used to detect and correct impulse noise errors.

4. DESCRIPTION OF THE DECODING ALGORITHM

As the convolutional code codeword is very long the syndrome decoding algorithm operates on the segments of the codeword in a sequential manner. The errors are estimated for the first window of the received signal, their influence is removed and hence the decoding process for the next window is the same as that for the first window.

The syndrome equations for one window can be written as

$$\mathbf{S}^j = \mathbf{P}\mathbf{y}_R^j \tag{6}$$

where

$$\begin{array}{l} \mathbf{S}^j = \begin{bmatrix} \mathbf{s}^T[j] & \mathbf{s}^T[j+1] & \dots & \mathbf{s}^T[j+M-1] \end{bmatrix}^T \\ \mathbf{s}[j] = \begin{bmatrix} s_1[j] & s_2[j] & \dots & s_{N-K}[j] \end{bmatrix}^T \\ \mathbf{y}_R^j = \begin{bmatrix} \mathbf{y}_R^T[j-L_P] & \dots & \mathbf{y}_R^T[j+M-1] \end{bmatrix}^T \end{array} \tag{7}$$

and $\mathbf{P}$ is a $[(N-K)M \times N(M+L_P)]$ matrix given by

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_0 & \mathbf{P}_1 & \dots & \mathbf{P}_{L_P} & \mathbf{0} & \dots & \dots \\ \mathbf{0} & \mathbf{P}_0 & \mathbf{P}_1 & \dots & \mathbf{P}_{L_P} & \mathbf{0} & \dots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{P}_0 & \mathbf{P}_1 & \dots & \mathbf{P}_{L_P} \end{bmatrix} \tag{8}$$

The decoding algorithm consists of the following steps: impulse error localization and impulse error amplitude estimation step. After amplitude estimation step the error estimates are subtracted from the received signal, new syndrome values are calculated and the error correction procedure moves to the syndrome and received data window $\mathbf{S}^{j+M}$ and $\mathbf{y}_R^{j+M}$, respectively.

4.1 Error Localization

The approach presented here is based on the M-ary hypothesis testing with the Bayes Criterion. In particular we use the minimum total probability of error test [7] which selects the hypothesis with the maximum a posteriori probability.

We assume that impulse errors are sparse and that we can have at most one error within a few consecutive windows of the received data. Each possible position of an impulse error within a window of the received data is considered as a separate hypothesis. The null hypothesis means that there no impulse errors. We note that in the first window $[\mathbf{y}_R^T(-L_P) \dots \mathbf{y}_R^T[-1]]$ is a zero vector. As we assume that there is no error propagation the first $L_P N$ samples in the proceeding data windows are corrupted only by quantization noise. The effective window size for impulse error localization and correction is therefore MN samples. That is, we consider $MN+1$ hypothesis: null hypothesis H_0 and hypothesis H_i, $i = 1, \dots MN$ which says that there is an impulse error in position $NL_P + i$ within the window of the received data $\mathbf{y}_R^j$.

The joint probability density function (p.d.f) of the syndromes under hypothesis H_i is a multivariate Gaussian distribution given by

$$\begin{array}{r} p(s_1, \dots, s_D \mid H_i) = \frac{1}{(2\pi)^{D/2}\det(\mathbf{M}_i)^{1/2}} \times \\ \exp([-\frac{1}{2}(\mathbf{S}^j - \bar{\mathbf{S}}^j)^T \mathbf{M}_i^{-1}(\mathbf{S}^j - \bar{\mathbf{S}}^j)]) \end{array} \tag{9}$$

where in order to simplify the notation the $D = (N - K)M$ elements in $\mathbf{S}^j$ are denoted by $s_1, \ldots s_D$, $\mathbf{M}_i$ is the syndrome covariance matrix under hypothesis i and $\bar{\mathbf{S}}^j$ is a vector of syndromes mean values under hypothesis H_i. The $\mathbf{M}_i$ and $\bar{\mathbf{S}}^j$ are given by

$$\begin{aligned}\mathbf{M}_0 = \quad & E\{\mathbf{S}^j \mathbf{S}^{j^T} \mid H_0\} = \mathbf{P}\text{diag}\{\sigma_q^2, \ldots, \sigma_q^2\}\mathbf{P}^T \\ \mathbf{M}_i = \quad & E\{\mathbf{S}^j \mathbf{S}^{j^T} \mid H_i\} = \mathbf{P}\Big(\text{diag}\{\sigma_q^2, \ldots, \sigma_q^2\} \\ & +\text{diag}\{0, \ldots, 0, \sigma_i^2, 0, \ldots 0\}\Big)\mathbf{P}^T \\ \bar{\mathbf{S}}^j = & E\{\mathbf{S}^j \mid H_i\} = E\{\mathbf{P}\mathbf{y}_R^j \mid H_i\} = \mathbf{0}\end{aligned}$$

The a posteriori probabilities of each hypothesis can be calculated as

$$p(H_i \mid s_1, \ldots, s_D) = \frac{p(ls_1, \ldots, s_D \mid i) p_a(H_i)}{p(s_1, \ldots, s_D)} \tag{10}$$

where $p_a(H_i)$ is the a priori probability hypothesis H_i. These are calculated as $p_a(H_0) = (1 - p)^{MN}$ and $p_a(H_j) \approx (1 - p)^{MN-1} p$, $j = 1, ..., MN$.

4.2 Error Tracking

Due to the memory of the convolutional code the errors can be tracked by considering syndrome segments $\mathbf{S}^{j+1} \ldots \mathbf{S}^{j+L_P}$ under the same set of hypothesis as in the window corresponding to $\mathbf{S}^j$. That is, the hypothesis testing should indicate the same error location in respect to window $\mathbf{S}^j$ for each of these syndrome segments. We therefore introduce a parameter T which specifies how many times the error location has to be confirmed in order to be considered as a true error location. The tracking of errors is necessary if the structure of matrix $\mathbf{P}$ in Eq. (8) is such that not all error positions can be detected by considering only syndrome segment $\mathbf{S}^j$.

4.3 Amplitude Estimation

Once located the errors' amplitudes are calculated by solving the syndrome equations in Eq. (5) in the least square sense. Since impulse errors are sparse one can consider additional syndrome equations in order to have better estimate of the error amplitudes. Considering additional syndrome equations is necessary when the matrix $\mathbf{P}$ in Eq. (8) is such that the system of syndrome equations in Eq. (5) is underdetermined for some error positions. For the amplitude estimation we consider a set of equations corresponding to a following syndrome segment $\Big[\mathbf{S}^{j^T} \quad \mathbf{s}^T[j+M] \quad \ldots \quad \mathbf{s}^T[j+M+E-1]\Big]^T$ where E is a parameter which determines the number of additional syndrome equations.

5. MESSAGE RECONSTRUCTION

It has been shown in [8] that if the output of an OFB is corrupted by a quantization error which can be modeled by an additive white Gaussian noise and if the noise sequences in different channels are pairwise uncorrelated the pseudoinverse is the best linear reconstruction operator in the mean square sense. Assuming that after impulse error correction the received sequence is corrupted only by the quantization noise, the message is reconstructed by applying the pseudoinverse receiver.

The polyphase matrix of the synthesis filter bank corresponding to the pseudoinverse receiver is obtained as

$$\mathbf{R}(z) = \Big[\tilde{\mathbf{E}}(z)\mathbf{E}(z)\Big]^{-1} \tilde{\mathbf{E}}(z) \tag{11}$$

6. PERFORMANCE RESULTS

As current signal compression systems already use critically sampled filter banks for signal decomposition, the simplest way to introduce redundancy at this point in the system is to use a subsampling factor which is smaller than the number of channels.

Here we consider an application of the presented decoding algorithm to an image coding system with subband signal decomposition by a $N = 2$ channel biorthogonal 9/7 wavelet filter bank. The 16 subband image decomposition is obtained by performing vertical and horizontal filtering two times. The redundant signal representation is obtained by removing the downsamplers in the last horizontal filtering stage. Therefore each subband is protected by an $(N, K) = (2, 1)$ oversampled filter bank code with the generator and parity check matrix given by

$$\mathbf{E}(z) = [H_0(z) \quad H_1(z)]^T \quad \mathbf{P}(z) = [H_1(z) \quad -H_0(z)]$$

where $H_0(z)$ and $H_1(z)$ are the z transforms of the two channel wavelet filter bank impulse responses. The subband signals are quantized with a scalar quantizer with a dead zone and quantization step sizes corresponding to 6 bits for the lowest (first) subband, 3 bits for subbands 2-3, 2 bits for subbands 4-6, and 1 bit for the last 10 subbands. This quantization scheme yields entropy of 0.66 bits per sample. The channel introduces impulse noise errors with $p = 0.001$ and a variance which is hundred times greater than that of the highest quantization noise variance. The parameter M which determines the window size is set to $M = L_P + 1 = 9$ and the parameter E which determines the number of additional syndrome equations in the amplitude estimation procedure is $E = 3$.

The peak signal to noise ratio (PSNR) for the system with no impulse noise errors is 31.9545 dB while the PSNR for the system with impulse noise errors and reconstruction by a pseudoinverse receiver is 30.8291 dB.

Table 1 shows PSNR, probability of detection P_d and a probability of the false alarm P_f for the various values of the parameter T defined in 4.2. We can observe that

PSNR	P_d	P_f	T
31.6323 dB	0.7025	4.6319×10^{-04}	3
31.7344 dB	0.7085	2.0356×10^{-04}	6
31.4659 dB	0.4677	6.6355×10^{-05}	8

Table 1: Performance of the syndrome decoding algorithm for various values of the parameter T

for $T = 3$ and $T = 6$ the PSNR and probability of detection are approximately the same. Increasing the parameter T to 8 reduces the probability of false alarm, however, the probability of the detection is reduced as well which has as a consequence worse PSNR results.

Table 2 shows the performance of the algorithm described in [5] and thresholds calculated based on the mean of the likelihood values under the various hypothesis as suggested in [1]. Clearly the localization procedure based on the a posteriori probabilities of the hypothesis yields better results. However, the algorithm in [5] can be seen as more robust as it does not require the knowledge of the a priori probabilities.

PSNR	P_d	P_f	T
31.3077 dB	0.4731	2.0000×10^{-03}	3
31.3970 dB	0.4337	3.0341×10^{-04}	6
31.1955 dB	0.2406	1.6158×10^{-05}	8

Table 2: Performance of the algorithm described in [5] for various values of the parameter T

Figures 2 and 3 show the reconstructed image without and with syndrom decoding.

Figure 2: Reconstructed image, no syndrome decoding

7. CONCLUSIONS

It has been shown that an oversampled filter bank can be viewed as a joint source and channel code as it can be used for both subband signal decomposition and impulse noise correction. In particular, we have examined the performance of the syndrome decoding algorithm where the error localization in presence of the background noise is treated as an Mary hypothesis testing problem. The error positions are treated as separate hypothesis. Localization procedure selects the hypothesis with a maximum a posteriori probability.

Figure 3: Reconstructed image after syndrome decoding T=6

REFERENCES

[1] G. Robert Redinbo, *Decoding Real Block Codes: Activity Detection, Wiener Estimation*, IEEE Transactions on Information Theory, vol. 46, no.2, pp. 609-623, March 2000.

[2] J.K. Wolf, *Redundancy, the Discrete Fourier Transform, and Impulse Noise Cancellation*, IEEE Trans. on Communications, Vol. 31, No. 3,pp. 458-461, March 1983.

[3] J. Wu and J. Shiu, *Discrete Cosine Transform in Error Control Coding*, IEEE Trans. on Communications, Vol 43, No.5, pp. 1857-1861, May 1995.

[4] G. Rath and Christine Guillemot, *Characterization of a Class of Error Correcting Frames and Their Application to Image Transmission*, PCS 2003.

[5] G. Robert Redinbo, *Decoding Real-Number Convolutional Codes: Change Detection, Kalman Estimation*, IEEE Transactions on Information Theory, vol 43. no. 6, pp. 1864-1876, November 1997.

[6] J.C. Chiang, M. Kieffer and P. Duhamel, *Oversampled Filter Banks Seen as Channel Codes: Impulse Noise Correction*, IEEE Intl. Conf. on Acoustic and Signal Processing, ICASSP'03, April 2003.

[7] Harry L. Van Trees, *Detection, Estimation, and Modulation Theory*, John Wiley Sons, 1968.

[8] J. Kovacevic, P.L. Dragotti and V. Goyal, *Filter Bank Frame Expansions With Erasures*, IEEE Trans. on Information Theory, Vol. 48, No. 6, pp. 1439-1450, June 2002.