AN ALGEBRAIC TECHNIQUE FOR THE BLIND SEPARATION OF DS-CDMA SIGNALS

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ABSTRACT

In this paper, we propose a new deterministic technique for the blind separation of DS-CDMA signals received on an antenna array. We start from the observation made by Sidiropoulos *et al.* that the received data exhibit the structure of a Canonical Decomposition in multilinear algebra. We provide a new condition for the uniqueness of this decomposition and we present a new algorithm in which the solution is obtained by means of a simultaneous matrix diagonalization. Next, we consider the special case in which the transmitted signals have constant modulus. In the Analytical Constant Modulus Algorithm by van der Veen and Paulraj the constant modulus constraint leads to an other simultaneous matrix diagonalization. The CDMA structure constraint and the constant modulus constraint can be combined. We derive an alternating least squares algorithm that solves both sets of matrix equations simultaneously.

1. INTRODUCTION

Let us start by introducing a basic algebraic model for CDMA data received by an antenna array. R users transmit information sequences of K symbols spread with a sequence of length J_1. Transmitted signals are received on a network of I antennas. In a first time, we suppose that the channel is noiseless and memoryless.

The kth symbol of the rth information sequence is denoted s_{kr}, the jth chip of the rth spreading sequence c_{jr} and the fading factor between user r and antenna i a_{ir}. Defining y_{ijk} as the output of the ith antenna for chip j and symbol k with $i \in \mathbb{N}_I$, $j \in \mathbb{N}_{J_1}$ and $k \in \mathbb{N}_K$ ($\mathbb{N}_n$ denotes the set of integers between 1 and n), we have:

$$y_{ijk} = \sum_{r=1}^{R} a_{ir}\, c_{jr}\, s_{kr}.$$

This model stays legitimate in case of Inter-Chip Interference (ICI) but no Inter-Symbol Interference (ISI) by adopting a discard prefix or guard chips strategy [4]. One only needs to replace $(c_{jr})_{j\in\mathbb{N}_{J_1}}$ by $(h_{jr})_{j\in\mathbb{N}_J}$, where h_{jr} is the convolution between the spreading sequence associated to rth user and the impulse response of the corresponding channel:

$$y_{ijk} = \sum_{r=1}^{R} a_{ir}\, h_{jr}\, s_{kr}.$$

This equation can be written in a tensor (multi-way array) format as:

$$\mathscr{Y} = \sum_{r=1}^{R} A_r \circ H_r \circ S_r, \tag{1}$$

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in which $\mathscr{Y} \in \mathbb{C}^{I\times J\times K}$, $A_r \in \mathbb{C}^I$, $H_r \in \mathbb{C}^J$ and $S_r \in \mathbb{C}^K$. Eq. (1) is a decomposition of $\mathscr{Y}$ in third-order rank-1 terms. Such a decomposition is called a Parallel Factors Model (PARAFAC) or a Canonical Decomposition (CD) [1, 2, 4]. This multilinear point of view w.r.t. CDMA data was adopted for the first time in [4].

Define $\mathbf{A} = [A_1 \dots A_R]$, $\mathbf{H} = [H_1 \dots H_R]$, $\mathbf{S} = [S_1 \dots S_R]$. Eq. (1) has a number of inherent indeterminacies. First, the order of the rank-1 terms is arbitrary. Secondly, A_r, H_r, S_r may be rescaled ($r \in \mathbb{N}_R$) provided the scaling factors compensate each other.

In [3, 4] it was shown that the CD (1) is unique, apart from the trivial indeterminacies mentioned in the previous paragraph, if

$$k(\mathbf{A}) + k(\mathbf{H}) + k(\mathbf{S}) \geqslant 2(R+1). \tag{2}$$

In this expression, $k(\mathbf{A})$ denotes the "Kruskal-rank" of matrix $\mathbf{A}$ defined as the maximal number such that columns of any submatrix built from k columns of $\mathbf{A}$ are linearly independent.

In Section 2 we will propose a weaker condition. Our proof is constructive. It allows to obtain the canonical components from a simultaneous diagonalization of a set of matrices. Section 3 shows that in this framework it is easy to impose the Constant Modulus (CM) property on the symbol estimates. Section 4 introduces a new Alternating Least Squares (ALS) algorithm for the combined CD / CM problem. Section 5 presents some simulations. Section 6 is the conclusion.

2. CANONICAL DECOMPOSITION

We stack the elements of tensor $\mathscr{Y}$ in a $IJ \times K$ matrix $\mathbf{Y}$:

$$\mathbf{Y} = (\mathbf{A} \odot \mathbf{H})\, \mathbf{S}^T, \tag{3}$$

in which $\odot$ represents the Khatri-Rao or column-wise Kronecker product.

Let a Singular Value Decomposition (SVD) of $\mathbf{Y}$ be given by:

$$\mathbf{Y} = \mathbf{U}\mathbf{D}\mathbf{V}^H. \tag{4}$$

From equations (3) and (4), we have:

$$\begin{cases} \mathbf{A} \odot \mathbf{H} &= \mathbf{U}\mathbf{D}\mathbf{F} \\ \mathbf{S}^T &= \mathbf{F}^{-1}\mathbf{V}^H \end{cases}, \tag{5}$$

where $\mathbf{F}$ is an invertible $R \times R$ matrix.

If matrix $\mathbf{F}$ is known, matrices $\mathbf{S}$, $\mathbf{H}$, and $\mathbf{A}$ can easily be calculated. Obviously, $\mathbf{S} = \mathbf{V}^*\mathbf{F}^{-T}$. Moreover, $\mathbf{A} \odot \mathbf{H} = [A_1 \otimes H_1, A_2 \otimes H_2, \dots, A_R \otimes H_R]$. Let $vec(\mathbf{X})$ denote a vector representation of the $M \times N$ matrix $\mathbf{X} = [X_1, X_2, X_N]$ such as $vec(X) = [X_1^T, X_2^T, \dots, X_N^T]^T$ and $unvec(.)$ denote the inverse operation of $vec(.)$

If we stack each column of $\mathbf{A} \odot \mathbf{H}$ in a $R \times R$ matrix $\mathbf{N_i}$, then

$$\mathbf{N_i} = unvec(A_i \otimes H_i) = H_i A_i^T$$

is theorically a rank-one matrix.
Apart from a scaling factor, H_i is the left singular vector associated with the highest singular value of $\mathbf{N_i}$ and A_i is the conjugate of the right singular vector associated with the highest singular value of $\mathbf{N_i}$, $i \in \mathbb{N}_R$.

The problem is now finding a matrix $\mathbf{F}$ that satisfies equation (5) and evaluating under which conditions this matrix is unique.
Let $\mathbf{E_r}$ be the matrix built by stacking the rth vector of matrix $\tilde{\mathbf{U}} = \mathbf{UD}$ in a $I \times J$ matrix.

$$\begin{aligned}\mathbf{E_r} &= unvec(\tilde{U_r}) \\ &= unvec\left(\left((\mathbf{A} \odot \mathbf{H})\,\mathbf{F}^{-1}\right)_r\right) \\ &= \sum_{k=1}^{R} \left(H_k A_k^T\right)(\mathbf{F}^{-1})_{kr}.\end{aligned}$$

In order to continue, we need the following theorem.
Theorem 1
Consider mapping Φ*:* $(\mathbf{X},\mathbf{Y}) \in \mathbb{C}^{I\times J} \times \mathbb{C}^{I\times J} \longmapsto \Phi(\mathbf{X},\mathbf{Y}) \in \mathbb{C}^{I\times J\times I\times J}$ *defined by:*
$(\Phi(\mathbf{X},\mathbf{Y}))_{ijkl} = x_{ij}y_{kl} + y_{ij}x_{kl} - x_{il}y_{kj} - y_{il}x_{kj}$ *for all* $(i,j,k,l) \in \mathbb{N}_I \times \mathbb{N}_J \times \mathbb{N}_I \times \mathbb{N}_J$.
Given $\mathbf{X} \in \mathbb{C}^{I\times J}$*,* $\Phi(\mathbf{X},\mathbf{X}) = 0$ *if and only if the rank of* $\mathbf{X}$ *is at most one.*
Proof
The case where $\mathbf{X} = 0$ is obvious.
Let $\mathbf{X}$ be a rank one matrix. There exist two vectors $\mathbf{u}$ and $\mathbf{v}$ such that $x_{ij} = u_i v_j$. Then $(\Phi_{xx})_{ijkl} = 2(u_i v_j u_k v_l - u_i v_l u_k v_j) = 0$.
Now, let $\mathbf{X}$ be some matrix verifying $\Phi(\mathbf{X},\mathbf{X}) = 0$. Let the SVD of $\mathbf{X}$ be given by $\mathbf{X} = \mathbf{U\Sigma V}^H$. Then we have:

$$\begin{aligned}x_{ij}x_{kl} - x_{il}x_{kj} &= \sum_{r,s} \sigma_r\sigma_s u_{ir}u_{ks}(v_{jr}v_{ls} - v_{js}v_{lr})^* \\ &= \sum_{r\neq s} \sigma_r\sigma_s u_{ir}u_{ks}(v_{jr}v_{ls} - v_{js}v_{lr})^*.\end{aligned}$$

The tensors with entries $u_{ir}u_{ks}(v_{jr}v_{ls} - v_{js}v_{lr})^*$, $r \neq s$ are linearly independent because matrices $\mathbf{U}$ and $\mathbf{V}$ are unitary. Consequently, $\sigma_r\sigma_s = 0$ if $r \neq s$ and therefore $\mathbf{X}$ and $\mathbf{\Sigma}$ are rank one.■

We can construct a set of R^2 tensors Φ_{rs} defined by

$$\begin{aligned}\Phi_{rs} &= \Phi(\mathbf{E_r},\mathbf{E_s}) \\ &= \Phi\left(\sum_{p=1}^{R} H_p A_p^T(\mathbf{F}^{-1})_{pr},\ \sum_{q=1}^{R} H_q A_q^T(\mathbf{F}^{-1})_{qs}\right).\end{aligned}$$

Due to the bilinearity of Φ, we have:

$$\Phi_{rs} = \sum_{p,q=1}^{R} (\mathbf{F}^{-1})_{pr}(\mathbf{F}^{-1})_{qs}\Phi\left(H_p A_p^T,\ H_q A_q^T\right). \tag{6}$$

Let $\mathbf{B}$ be a $R \times R$ symmetric matrix verifying:

$$\sum_{r,s=1}^{R} \Phi_{rs}\mathbf{B}_{rs} = 0. \tag{7}$$

We can replace Φ_{rs} by expression (6) and we obtain:

$$\sum_{r,s=1}^{R}\sum_{p,q=1}^{R} (\mathbf{F}^{-1})_{pr}(\mathbf{F}^{-1})_{qs}\Phi\left(H_p A_p^T,\ H_q A_q^T\right)\mathbf{B}_{rs} = 0.$$

In accordance with theorem 1, $\Phi\left(H_p A_p^T,\ H_p A_p^T\right) = 0$ for all p in $\mathbb{N}_R$, hence:

$$\sum_{r,s=1}^{R}\sum_{\substack{p,q=1\\ p\neq q}}^{R} (\mathbf{F}^{-1})_{pr}(\mathbf{F}^{-1})_{qs}\mathbf{B}_{rs}\Phi\left(H_p A_p^T,\ H_q A_q^T\right) = 0.$$

Furthermore, due to the symmetry of Φ and $\mathbf{B}$:

$$\sum_{r,s=1}^{R}\sum_{\substack{p,q=1\\ p<q}}^{R} (\mathbf{F}^{-1})_{pr}(\mathbf{F}^{-1})_{qs}\mathbf{B}_{rs}\Phi\left(H_p A_p^T,\ H_q A_q^T\right) = 0. \tag{8}$$

Let us suppose that the tensors $\left(\Phi\left(H_p A_p^T,\ H_q A_q^T\right)\right)_{p<q}$ are linearly independent. Then equation (8) implies:

$$\sum_{r,s=1}^{R} (\mathbf{F}^{-1})_{pr}(\mathbf{F}^{-1})_{qs}\mathbf{B}_{rs} = \lambda_{pq}\delta_{pq}, \tag{9}$$

in which δ denotes the Kronecker symbol ($\delta_{pq} = 1$ if $p = q$, $\delta_{pq} = 0$ if $p \neq q$).
Equation (9) can be rewritten as:

$$\mathbf{B} = \mathbf{F}\Lambda\mathbf{F}^T, \tag{10}$$

in which Λ is a diagonal matrix whose diagonal elements are λ_{pp}, $p \in \mathbb{N}_R$.
The reverse holds also true: any matrix $\mathbf{B}$ of the form (10) with Λ an arbitrary diagonal matrix satisfies equation (7). Hence the kernel of $\mathbf{P} = [vec(\Phi_{11}), vec(\Phi_{12}), \ldots, vec(\Phi_{RR})]$ yields R matrices.
Finally, the matrix $\mathbf{F}$ can be found from the following simultaneous decomposition:

$$\left\{\begin{aligned}\mathbf{B_1} &= \mathbf{F}\Lambda_\mathbf{1}\mathbf{F}^T \\ \mathbf{B_2} &= \mathbf{F}\Lambda_\mathbf{2}\mathbf{F}^T \\ &\vdots \\ \mathbf{B_R} &= \mathbf{F}\Lambda_\mathbf{R}\mathbf{F}^T\end{aligned}\right. , \tag{11}$$

where $\Lambda_\mathbf{1}, \Lambda_\mathbf{2}, \ldots, \Lambda_\mathbf{R}$ are R diagonal matrices. Algorithms for the computation of this simultaneous decomposition may be found in [1, 4, 8] and the references therein. In the presence of noise, the matrices $\mathbf{B_1}, \mathbf{B_2}, \ldots, \mathbf{B_R}$ are found via the R right singular vectors of $\mathbf{P}$ corresponding to the smallest singular values. These matrices can be weighted in accordance with their expected accuracy (a more accurate estimate corresponding to a smaller singular value).

The number of users R that can be processed in this way is bounded by the condition that all tensors $\left(\Phi\left(H_p A_p^T,\ H_q A_q^T\right)\right)_{p<q}$ are independent (equation (9)). It can be shown that this condition is generically satisfied as long as $R(R-1) \leq (I^2 - I)(J^2 - J)/2$ (proof not included). This means that a number of users can be allowed that depends on the product of I and J, and not on their sum, as suggested by equation (2).

The algorithm can be summarized as follows:

- Stack $\mathscr{Y}$ in a $IJ \times K$ matrix $\mathbf{Y}$.
- Compute the SVD of $\mathbf{Y}$, call $\mathbf{D}$ the diagonal $R \times R$ matrix containing the R highest singular values, $\mathbf{U}$ the matrix of associated left singular vectors and $\mathbf{V}$ the matrix of associated right singular vectors.
- For all $r \in \mathbb{N}_R$, stack rth vector of $\mathbf{UD}$ in a $I \times J$ matrix $\mathbf{E_r}$.
- For all $(r,s) \in \mathbb{N}_R{}^2$, $r < s$, construct the $I \times J \times I \times J$ tensor $\Phi_{rs} = \Phi(\mathbf{E_r},\mathbf{E_s})$ and stack it in a I^2J^2 vector Ψ_{rs}.

- Construct the $I^2J^2 \times R(R-1)/2$ matrix $\mathbf{P} = [\Psi_{12}, \Psi_{13}, \ldots, \Psi_{(R-1)R}]$ and take its R right singular vectors associated with the R lowest singular values.
- Stack each of these vectors in the upper right corner of a matrix $\mathbf{B_r}$ and construct the lower left corner by symmetry.
- Obtain the matrix $\mathbf{F}$ by means of a simultaneous diagonalization of matrices $\mathbf{B_r}$, $r \in \mathbb{N}_R$.
- Estimate $\mathbf{S}$ as $\mathbf{V}^*\mathbf{F}^{-1}$.
- (For all $r \in \mathbb{N}_R$, stack the rth column of matrix $\mathbf{UDF}$ in a $R \times R$ matrix $\mathbf{N_r}$.
- Estimate $\mathbf{H}$ as the matrix which contains the left singular vectors associated with the highest singular value of each matrix $\mathbf{N_r}$ and $\mathbf{A}$ as the conjugate of the matrix which contains the right singular vectors associated with the highest singular value of each matrix $\mathbf{N_r}$.)

3. CONSTANT MODULUS CONSTRAINT

If the transmitted information sequences are CM, then this constraint can easily be combined with equation (11).
According to equation (5), matrix $\mathbf{V}$ containing the right singular vectors of $\mathbf{Y}$ satisfies:

$$\mathbf{V}^H = \mathbf{F}\mathbf{S}^T. \tag{12}$$

This is the classical expression of an $(R \times R)$ instantaneous mixture of CM source signals. In [8] it is shown that the demixing matrix may be found from the simultaneous matrix decomposition

$$\left\{ \begin{array}{lcl} \mathbf{M_1} & = & \mathbf{F}^{-H}\Omega_\mathbf{1}\mathbf{F}^{-1} \\ \mathbf{M_2} & = & \mathbf{F}^{-H}\Omega_\mathbf{2}\mathbf{F}^{-1} \\ \vdots & & \\ \mathbf{M_R} & = & \mathbf{F}^{-H}\Omega_\mathbf{R}\mathbf{F}^{-1} \end{array} \right. , \tag{13}$$

where matrices $(\mathbf{\Omega_i})_{i\in\mathbb{N}_R}$ are diagonal and where $(\mathbf{M_i})_{i\in\mathbb{N}_R}$ are obtained from $\mathbf{V}$. For the computation of the matrices $(\mathbf{M_i})_{i\in\mathbb{N}_R}$, we refer to [8]. Because this system is very similar to the system obtained from the CDMA structure constraint (11), they can be solved jointly.

4. AN ALTERNATING LEAST SQUARES ALGORITHM

In this section, we present a new ALS algorithm for the simultaneous diagonalization of systems (11) and (13). This algorithm is a generalization of the algorithm proposed in [4]. An ALS algorithm consists of an iteration over conditional least-squares updates of unknown factors.
Writing $\tilde{\mathbf{F}} = \mathbf{F}^T$, we have:

$$\left\{ \begin{array}{lcl} \mathbf{B_1} & = & \mathbf{F}\Lambda_\mathbf{1}\tilde{\mathbf{F}} \\ \mathbf{B_2} & = & \mathbf{F}\Lambda_\mathbf{2}\tilde{\mathbf{F}} \\ & \vdots & \\ \mathbf{B_R} & = & \mathbf{F}\Lambda_\mathbf{R}\tilde{\mathbf{F}} \\ \mathbf{M_1} & = & \tilde{\mathbf{F}}^{-*}\Omega_\mathbf{1}\mathbf{F}^{-1} \\ \mathbf{M_2} & = & \tilde{\mathbf{F}}^{-*}\Omega_\mathbf{2}\mathbf{F}^{-1} \\ & \vdots & \\ \mathbf{M_R} & = & \tilde{\mathbf{F}}^{-*}\Omega_\mathbf{R}\mathbf{F}^{-1} \end{array} \right. . \tag{14}$$

An iteration step consists of the subsequent minimization of the cost function $\sum_{i=1}^{R}\left(\left\|\mathbf{B_i} - \mathbf{F}\Lambda_\mathbf{i}\tilde{\mathbf{F}}\right\|^2 + \left\|\mathbf{M_i} - \tilde{\mathbf{F}}^{*-1}\Omega_\mathbf{i}\mathbf{F}^{-1}\right\|^2\right)$ with respect to $\Lambda_\mathbf{i}$ and $\Omega_\mathbf{i}$, then with respect to $\mathbf{F}$, and finally with respect to $\tilde{\mathbf{F}}$. In order to initialize the algorithm, we can take $\mathbf{F}_{init}$ equal to the eigenmatrix of $\mathbf{B_1}\mathbf{B_2}^{-1}$ and $\tilde{\mathbf{F}}_{init}$ equal to the transpose of $\mathbf{F}_{init}$.
An iteration step can be implemented as follows.

1. *Updating the estimate of* $\Lambda_\mathbf{i}$ *and* $\Omega_\mathbf{i}$
We call $diag(\Lambda_\mathbf{i})$ the vector that contains the diagonal values of $\Lambda_\mathbf{i}$. Equation $\mathbf{B_i} = \mathbf{F}\Lambda_\mathbf{i}\tilde{\mathbf{F}}$ can be rewritten as:

$$vec(\mathbf{B_i}) = \left(\tilde{\mathbf{F}}^T \odot \mathbf{F}\right) diag(\Lambda_\mathbf{i}).$$

For all $i \in N_R$, $\Lambda_\mathbf{i}$ follows from this linear set of equations.

Likewise, the equation $\Omega_\mathbf{i} = \tilde{\mathbf{F}}^*\mathbf{M_i}\mathbf{F}$ can be rewritten as:

$$diag(\Omega_\mathbf{i}) = \left(\mathbf{F}^T \odot \tilde{\mathbf{F}}^*\right) vec(\mathbf{M_i}).$$

$\Omega_\mathbf{i}$, $\forall i \in N_R$, follows immediately.

2. *Updating the estimate of* $\mathbf{F}$
Define $\delta_1 = [\Lambda_\mathbf{1}\tilde{\mathbf{F}}, \Lambda_\mathbf{2}\tilde{\mathbf{F}}, \ldots, \Lambda_\mathbf{R}\tilde{\mathbf{F}}]$ and $\delta_2 = [\mathbf{B_1}, \mathbf{B_2}, \ldots, \mathbf{B_R}]$.
According to (14), $\delta_2 = \mathbf{F}\delta_1 = \mathbf{I_R}\mathbf{F}\delta_1$, hence:

$$vec(\delta_2) = \left(\delta_1^T \otimes \mathbf{I_R}\right) vec(\mathbf{F}).$$

We also define $\gamma_1 = [(\tilde{\mathbf{F}}^*\mathbf{M_1})^T, (\tilde{\mathbf{F}}^*\mathbf{M_2})^T, \ldots, (\tilde{\mathbf{F}}^*\mathbf{M_R})^T]^T$ and $\gamma_2 = [\Omega_\mathbf{1}^T, \Omega_\mathbf{2}^T, \ldots, \Omega_\mathbf{R}^T]$.
According to (14), $\gamma_2 = \gamma_1\mathbf{F} = \gamma_1\mathbf{F}\mathbf{I_R}$, hence:

$$vec(\gamma_2) = (\mathbf{I_R} \otimes \gamma_1)\, vec(\mathbf{F}).$$

We obtain:

$$\left[\begin{array}{c} \delta_1^T \otimes \mathbf{I_R} \\ \mathbf{I_R} \otimes \gamma_1 \end{array}\right] vec(\mathbf{F}) = \left[\begin{array}{c} vec(\delta_2) \\ vec(\gamma_2) \end{array}\right]. \tag{15}$$

$\mathbf{F}$ follows from this overdetermined set of equations.

3. *Updating the estimate of* $\tilde{\mathbf{F}}$
First, we define $\delta_3 = \left[(\mathbf{F}\Lambda_\mathbf{1})^T, (\mathbf{F}\Lambda_\mathbf{2})^T, \ldots, (\mathbf{F}\Lambda_\mathbf{R})^T\right]^T$ and $\delta_4 = \left[\mathbf{B_1}^T, \mathbf{B_2}^T, \ldots, \mathbf{B_R}^T\right]$.
According to (14), $\delta_4 = \delta_3\tilde{\mathbf{F}} = \delta_3\tilde{\mathbf{F}}\mathbf{I_R}$, hence:

$$vec(\delta_4) = (\mathbf{I_R} \otimes \delta_3)\,.vec(\tilde{\mathbf{F}}).$$

We also define $\gamma_3 = [\mathbf{M_1}\mathbf{F},\ \mathbf{M_2}\mathbf{F},\ \ldots,\ \mathbf{M_R}\mathbf{F}]$ and $\gamma_4 = [\Omega_\mathbf{1},\ \Omega_\mathbf{2},\ \ldots,\ \Omega_\mathbf{R}]$.
According to (14), $\gamma_4{}^* = \tilde{\mathbf{F}}\gamma_3{}^* = \mathbf{I_R}\tilde{\mathbf{F}}\gamma_3{}^*$, and we get:

$$vec(\gamma_4{}^*) = \left(\gamma_3^H \otimes \mathbf{I_R}\right) vec(\tilde{\mathbf{F}}).$$

We obtain:

$$\left[\begin{array}{c} \mathbf{I_R} \otimes \delta_3 \\ \gamma_3^H \otimes \mathbf{I_R} \end{array}\right] vec(\tilde{\mathbf{F}}) = \left[\begin{array}{c} vec(\delta_4) \\ vec(\gamma_4{}^*) \end{array}\right]. \tag{16}$$

$\tilde{\mathbf{F}}$ follows from this overdetermined set of equations.

We decide that the algorithm has converged when the Frobenius norm of the difference between the estimation at iteration k and the the estimation at iteration $k+1$ is less than a certain tolerance ε.

5. SIMULATION RESULTS

Figures 1 and 2 depict Symbol Error Rate (SER) versus Signal Noise Ratio (SNR) for our ALS algorithm combining CD and CM constraints.
Figure 1 corresponds to the case of $R = 6$ users, $I = 4$ antennas, $K = 100$ symbols and a spreading factor $J = 5$. Symbols are QPSK modulated. Results have been averaged over 30 simulations.
Figure 2 corresponds to the case where I=3, J=4, K=100 and R=6. Note that the Kruskal-bound (2) (yielding a maximum of 5 users) is surpassed; nevertheless our algorithm still works well.

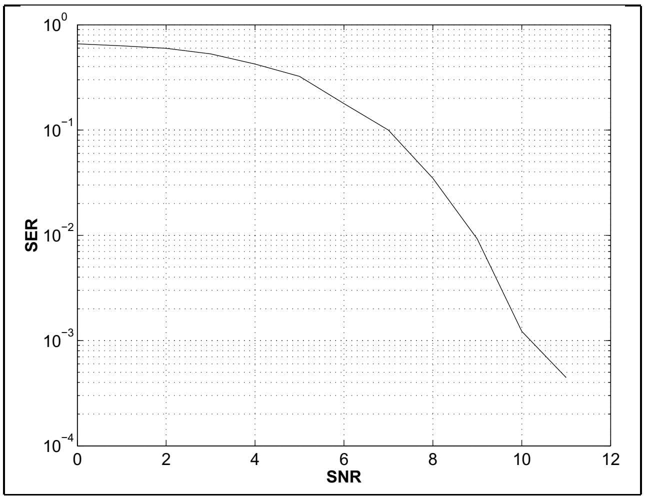

Figure 1: SER versus SNR for I=4, J=5, K=100, R=6

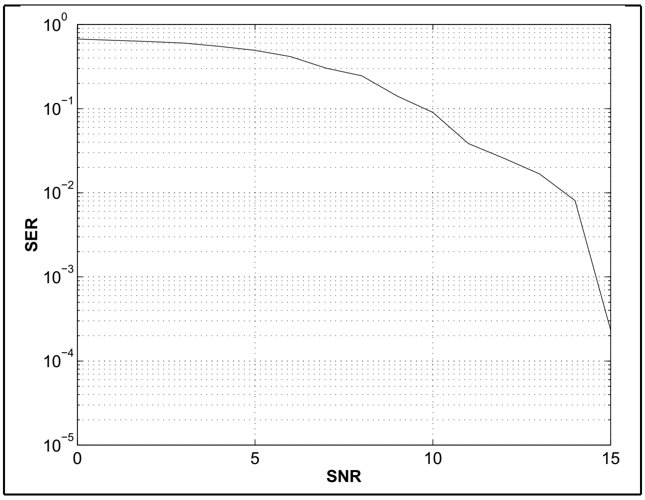

Figure 2: SER versus SNR for I=3, J=4, K=100, R=6

6. CONCLUSION

In order to separate DS-CDMA signals impinging on an antenna array, one can resort to a CD in multilinear algebra. In this paper we showed that the Krustal-bound on the number of users can be surpassed. We provided a new sufficient condition for the uniqueness of the decomposition. We presented a new algorithm based on a simultaneous matrix diagonalization. Furthermore, we showed that CD and CM properties can be combined by deriving an ALS algorithm. Principles exposed in this paper are also useful for other telecommunication applications in which the CD plays a role [5, 6, 7].

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