GENERALIZED MULTIPACKET RECEPTION MODEL FOR HETEROGENEOUS NETWORKS

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ABSTRACT

In this paper a generalized Multipacket Reception model for heteregenous systems is presented. In a packet oriented system, heterogenity among terminals or services leads to different instantaneous Packet Error Rates. The main advantage of this model is that it demonstrates that by computing the average Packet Error Rate, it is possible to use classical analysis tools, such as Markov chains, in the design of more efficient MAC protocols. For instance, in systems employing diversity (code, frequency or space), this model allows to incorporate the advantages of multiuser detection techniques and to compute the optimal number of simultaneous transmissions to achieve throughput maximization. The usability of this model is shown by means of an example and simulation results. Furthermore, in the analysis of the results diferences between PHY and MAC optimizations are highlighted and an alternative joint PHY-MAC optimization is proposed.

1. INTRODUCTION

In an OSI layered packet oriented system, the Medium Access Control (MAC) layer is the one in charge of controlling the access to the medium in order to optimize the system performance in terms of packet throughput, delay and jitter and to guarantee some other Quality of Service (QoS) requirements. Recently and as consequence of the great advances in signal processing techniques during the last two decades, MAC designers have started to look at the PHY level and hence, consider reception properties in their designs. The simultaneous reception of multiple packets at the PHY layer is called multipacket capability. Such multipacket capability of the receiver is achieved by means of applying some kind of diversity (code, frequency or space) and using a Multiuser Detector (MUD). In a MUD, thanks to the diversity between users, interference from other users or the so called Multi Access Interference (MAI) can be reduced and then, some information can be recovered from the collision of packets. The use of MUDs for interference cancelling in Digital Subscriber Line (DSL) systems or wireless networks has been widely studied by Cioffi in [1].

Also very recently, the concept of Cross Layer (CL) design have appeared [2] and references therein. The idea behind the CL concept is that in order to achieve optimal performance in one layer of the communication system, it is important that this layer is aware of some parameters or characteristics of the others. Notice that if the design of a MAC incorporates the multipacket reception capability of the receiver, that intrinsically is a CL MAC design.

One of the main tasks of the MAC should be to incorporate the receiver multipacket reception capability and then, no longer avoid collisions as much as possible but control the number of simultaneous transmissions to keep MAI under an optimal threshold. We will refer as multipacket MAC to those MAC protocols designed to take into account the multipacket reception capability of the receiver. Examples of multipacket MAC are [3],[4],[6]. However such designs do not contemplate heterogenous systems.

In a packet oriented system, heterogenity among terminals or services implies for example, different packet lengths, different SNIR , different rate or different traffic model. As a consequence, heterogenity leads to different instantaneous Packet Error Rates (PER) among users and/or services. In this article, we will derive a general multipacket reception model that reduces the problem to the computation of the average PER.

2. PROBLEM STATEMENT

Markov chains are the most widely used tool in the design of MAC protocols. Particularly, Discrete Time Markov Chains (DTMC) are very suitable for the study and evaluation of long term parameters such as throughput or delay in packet oriented time slotted communication systems. Although the design of a DTMC is particular to the MAC under study, each state of a DTMC is always somehow related to the number of packets that will be transmitted. Additionally, the transition probability from one state to another not only depends on traffic parameters such as transmission and *re*transmission probabilities but also on the success of the transmissions or equivalently, on the multipacket reception capability of the receiver.

Based on the idea that the Packet Error Rate ($PER^{(k)}$) depends on the number of simultaneous transmissions or packets that have been simultaneously transmitted (k), the multipacket reception capability of the receiver can be modelled be means of the probability $c_{k,l}$. $c_{k,l}$ is the probability of successfully receive l packets (packets without error) when a total of k packets are transmitted [3]. Then, assuming that the packet error probability is independent between packets,

$$c_{k,l} = \binom{k}{l}(1 - PER^{(k)})^{l}(PER^{(k)})^{k-l} \tag{1}$$

Additionally, the expected number of successfully received packets when k transmissions take place simultaneously is

$$\begin{aligned} C_k &= \sum_{l=0}^{k} l c_{k,l} = \sum_{l=0}^{k} l \binom{k}{l}(1 - PER^{(k)})^{l}(PER^{(k)})^{k-l} \\ &= k(1 - PER_k) \end{aligned} \tag{2}$$

Considering that the maximum number of simultaneous transmissions is M ($1 \leq k \leq M$), the maximum throughput that a MPR channel can offer, independent of MAC protocols, is

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$$\eta = \max_{k=1..M} C_k \tag{3}$$

In other words, this is the *average* of the channel capacity in packets/slot defined at the MAC layer. Furthermore, we could compute the value of k that maximizes (3). In the event that more than one value of k maximizes (3), the minimum argument is chosen for power saving considerations. Formally,

$$m_0 = \min\{\arg \max_{k=1..M} C_k\} \tag{4}$$

According to (4) and for throughput maximization, the multipacket MAC should try to achieve m_0 simultaneous transmissions. This is easy to achieve under the assumption that each time the MAC gives access for the transmission of a packet, such transmission takes place. However, when this is not always true, i.e. considerations on data traffic are taken, the analysis of the system is more complex and usually DTMC must be used.

Clearly, in (1) it is assumed that the $PER^{(k)}$ is the same for all the transmissions in the system with independence of the subset of k transmissions that is considered. This in particular, is the approach taken in many multipacket MAC protocols such as [3], [4], [6] and others. However, there are many systems where formulating the equal $PER^{(k)}$ assumption is not so evident. For instance, systems that use an iterative MUD, Code Division Multiple Access (CDMA) systems with different cross-correlation between any pair of codes, in some DSL systems or Space Division Multiple Access (SDMA) systems where usually the $PER^{(k)}$ not only depends on the number of transmissions k but also on the channel realization, or in heteregenous systems with different QoS requirements, channel coding or packet lengths. Apparently, equations (1), (2) and (3) should not be valid in such systems. However, starting from a more general formulation, in the following chapters we will derive a mathematical framework that allows to use equations (1), (2) and (3) in any heterogeneous system employing code, frequency or space diversity. Furthermore, in section 5 we give an example for a SDMA system with a Zero Forcing receiver and in section 6 some simulation results are presented and a joint PHY-MAC optimization is proposed.

3. GENERALIZED MULTI PACKET RECEPTION MODEL

Let us consider a multiaccess Multiple Input Multiple Output (MIMO) communication link. Many terminals with multiple antennas whish to communicate with an other terminal, namely Receiver Terminal (RT). The multiaccess MIMO link considered is equivalent to the uplink of a centralized system but could also be considered as one of the many possible communication links in an ad-hoc network. Independent of the number of terminals, a total of M antennas can transmit packets to the RT. By means of N antennas, the RT has multipacket reception capability. Time axis is divided into slots, each antenna can transmit up to one packet per slot and transmissions of different antennas are synchronized. Notice that for any system employing code or frequency diversity, M would be the number of codes or frequencies and N would be set to 1.

The link between the m^{th} transmitting antenna and the RT during the slot t is parametrized by $\gamma_m^{(t)}$. We assume that the values of $\gamma_m^{(t)}$ for $m = 1..M$ and $t \in N$ are independent and indentically distributed with probability distribution function $F(\gamma)$. On the event there are k simultaneous transmissions or equivalently, that k different antennas transmit a packet, we define a vector $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, ..., \gamma_k)$ that contains the link state of these transmissions during such slot. Additionally, we can also define a vector $\boldsymbol{\Theta}_k = (\theta_k^{(1)}, \theta_k^{(2)}, ..., \theta_k^{(k)})$ that represents the packet success at the end of the slot, i.e., $\theta_k^{(i)}$ is a binary value that indicates with a 1 that the packet from transmission i has been successfully received and with a 0 on the contrary. Clearly, at the end of a slot with outcome $\boldsymbol{\Theta}_k$, the number of successfully received packets is

$$\rho(\boldsymbol{\Theta}_k) = \sum_{i=1}^{k} \theta_k^{(i)} \tag{5}$$

Following notations in [5], we can define the function $\Phi^{(k)}(\boldsymbol{\gamma}; \boldsymbol{\Theta}_k)$ as the probability of $\boldsymbol{\Theta}_k$ on the condition that there are k simultaneous transmissions whose link states are $\boldsymbol{\gamma}$. Then, the probability that l packets are successfully received in one slot when there are k simultaneous transmissions and $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, ..., \gamma_k)$ is the sum of probabilities of all the outcomes $\boldsymbol{\Theta}_k$ with $\rho(\boldsymbol{\Theta}_k) = l$, that is

$$\Gamma_l^{(k)}(\gamma_1, \gamma_2, ..., \gamma_k) = \sum_{\rho(\boldsymbol{\Theta}_k)=l} \Phi^{(k)}(\gamma_1, \gamma_2, ..., \gamma_k; \boldsymbol{\Theta}_k) \tag{6}$$

A more general formulation for equations (1) and (2) is

$$\begin{aligned} & c_{k,l}(F(\boldsymbol{\gamma})) = \\ & = \int \Gamma_l^{(k)}(\gamma_1, \gamma_2, ..., \gamma_k)\; p(\gamma_1)\; ...\; p(\gamma_k)\; d\gamma_1\; ...\; d\gamma_k \end{aligned} \tag{7}$$

$$\begin{aligned} & C_k(F(\boldsymbol{\gamma})) = \\ & = \sum_{l=0}^{k} l \int \Gamma_l^{(k)}(\gamma_1, \gamma_2, ..., \gamma_k)\; p(\gamma_1)\; ...\; p(\gamma_k)\; d\gamma_1\; ...\; d\gamma_k \end{aligned} \tag{8}$$

Where $p(\gamma_i) = dF(\gamma_i)/d\gamma_i$. Notice though, that since (7) and (8) present an average over the link statistics, the result obtained when computing (3) corresponds to the *average* in a wide sense (over transmissions and time) of the throughput in a MPR channel.

In the event that k simultaneous transmissions take place, we define $\boldsymbol{\gamma} = (PER_1, PER_2, ..., PER_k)$, where $\gamma_i = PER_i$ corresponds to the ith transmission instantaneous PER. Additionally, $PER_1, PER_2, ..., PER_k$ are i.i.d random variables and, denoting $\overline{PER_i^{(k)}}$ as the average of PER_i, $\overline{PER_1^{(k)}} = \overline{PER_2^{(k)}} = .. = \overline{PER_k^{(k)}} = \overline{PER^{(k)}}$. Using probability methods, it can be easily seen that $\Gamma_l^{(k)}(PER_1, PER_2, ..., PER_k)$ is a linear function with respect to each of its variables PER_i and consequently, it can be shown that $c_{k,l}(F(\gamma))$ becomes

$$c_{k,l}(F(\gamma)) = \binom{k}{l}(1 - \overline{PER^{(k)}})^l (\overline{PER^{(k)}})^{k-l} \tag{9}$$

Moreover, the expected number of successfully received packets when k simultaneous transmissions occur is

$$C_k(F(\boldsymbol{\gamma})) = k(1 - \overline{PER^{(k)}}) = k\overline{PSR^{(k)}} \tag{10}$$

In (10), $\overline{PSR^{(k)}} = (1 - \overline{PER^{(k)}})$ is the average Packet Success Rate (PSR) conditioned that k simultaneous transmissions take place. Hence, with the knowledge of $\overline{PER^{(k)}}$ for $1 \leq k \leq M$ we can then use equations (1) and (2). Therefore the problem stated in section 2 has been reduced to the computation of the $\overline{PER^{(k)}}$ or equivalently, the $\overline{PSR^{(k)}}$. Notice that when γ_i is not a random variable and is the same for all the transmissions, (9) and (10) match with equations (1) and (2).

4. THE $\overline{PSR}$ UNDER A CL PERSPECTIVE

In any multiuser channel similar to the one presented in previous section, the instantaneous Signal-to-Noise-and-Interference Ratio (SNIR) generally depends on the MAI which in turn depends on the receiver implementation, the instantaneous channel realization and the number of interferers $(k-1)$. The $SNIR^{(k)}$ for user i has the form

$$SNIR_i^{(k)} = \frac{P}{\sigma_w^2 \alpha_i (k-1)} \quad (11)$$

In (11), P is the transmitted power, σ_w^2 is Gaussian noise power and $\alpha_i(k-1)$ is the ith transmission factor associated to the MAI interference when there are $k-1$ interferers. The parameter α_i is a random variable whose statistics depend on the channel statistics through the receiver implementation. However, assuming that all transmissions suffer from the same channel statistics, i.e., α_i for $1 \le i \le k$ are i.i.d. variables, the user index i can be omitted. Applying the approximation presented in [7], considering both, the use of hard-decision decoding of perfect linear binary block codes for a packet of length P_l and α as a random variable with p.d.f. $p(\alpha)$, the average Packet Success Rate is

$$\overline{PSR}^{(k)} = \int_0^{+\infty} PSR^{(k)}(\alpha)\ p(\alpha)\ d\alpha = \quad (12)$$

$$= \int_0^{+\infty} \sum_{i=0}^{t} (\binom{P_l}{i} (C_1 \exp(-C_2 \frac{P}{\sigma^2 \alpha (k-1)}))^i$$

$$(1 - C_1 \exp(-C_2 \frac{P}{\sigma^2 \alpha (k-1)}))^{P_l - i})\ p(\alpha)\ d\alpha$$

In (12), $t = \left\lfloor \frac{1}{2}(d_{\min} - 1) \right\rfloor$ is the number of correctable errors being $d_{\min}$ the minimum distance of the code and $\lfloor x \rfloor$ the largest integer contained in x. Similar approaches can be obtained when convolutional codes are used. Besides, as mentioned above, the p.d.f. of α depends on both, the channel statistics through the receiver implementation and the number of simultaneous transmissions k. Then, in order to compute (12) and consequently (9) and (10), the p.d.f. of α must be known at the MAC level. This, intrinsically leads to the concept of CL between MAC and PHY levels.

5. $\overline{PSR}$ IN A RAYLEIGH CHANNEL

For a communication link as the one described in previous sections with $N \ge M$, assume that k ($k \le M$) simultaneous transmissions take place. The following is a model for a synchronous multiaccess antenna-array communication link

$$\mathbf{y} = \mathbf{Hs} + \mathbf{w} \quad (13)$$

In (13), $\mathbf{s} = (s_1, s_2, .., s_k)^T$ is a vector where s_i is the the transmitted symbol of the ith antenna, $\mathbf{H} = (\mathbf{h}_1^T, \mathbf{h}_2^T, ..., \mathbf{h}_k^T)$ is $N \times k$ channel matrix where the scalar $h_{j,i}$ represents the fading of the ith transmitting antenna at the jth receiver antenna. The entries are complex Gaussian distributed. The vector $\mathbf{w}$ is a complex-valued, background Gaussian noise with zero mean and variance σ_w^2. If the MUD employed is a zero forcing linear receiver then, as it is shown in [8], the user instantaneous $SNIR_i^{(k)}$ is

$$SNIR_i^{(k)} = \frac{P}{\sigma_w^2 [(\mathbf{H}^H \mathbf{H})^{-1}]_{ii}} \quad (14)$$

Although the dependence on k is not explicit in (14), notice that the value of $[(\mathbf{H}^H \mathbf{H})^{-1}]_{ii}$ directly depends on the columns in $\mathbf{H}$, i.e., on k. Let us redefine α as $\alpha = \frac{2}{[(\mathbf{H}^H \mathbf{H})^{-1}]_{ii}}$. Then, α is a weightened Chi-Square distributed variable with $2(N-k+1)$ degrees of freedom. Hence, (12) becomes

$$\overline{PSR}^{(k)} = \sum_{i=0}^{t} \binom{P_l}{i} \frac{1}{2^{n/2}\Gamma(\frac{n}{2})} (\int_0^{+\infty} (C_1 \exp(-C_2 \frac{P\alpha}{2\sigma^2}))^i$$

$$(1 - C_1 \exp(-C_2 \frac{P\alpha}{2\sigma^2}))^{P_l - i}\ \alpha^{\frac{n}{2}-1} \exp(-\frac{\alpha}{2})\ d\alpha) \quad (15)$$

In (15), $n = 2(N-k+1)$ and $\Gamma(\frac{n}{2})$ is the Gamma function.

6. SIMULATION RESULTS

For all the simulations, we consider that the transmitted symbols are QPSK modulated. Then, following [7], constants C_1 and C_2 become $C_1 = 0.2$ and $C_2 = \frac{7}{2^{3.8}+1}$. Additionally, a packet length of $P_l = 320$ *bits*, which corresponds to the minimum length of a frame in the IEEE 802.11 standard, is considered. Notice that synchronization preamble has not been taken into account because synchronization issues are assumed to perform correctly. We have also considered two situations, no channel code is used, i.e., $t = 0$ and a perfect linear block code is used with $t = 10$, i.e., about the 3% of the bits can be corrected. The average Signal-to-Noise Ratio ($\overline{SNR} = \frac{P}{\sigma^2}$) is set to $7dB$. The number of receiver antennas N is set to 10 and the number of simultaneous transmissions k is in the range $[1, M]$, with $M = 10$.

In figure 1, the throughput as defined in (3) of the communication link presented in section 5 is evaluated as a function of the number of simultaneous transmissions k. We see, that depending on the error correction capacity of the code the number of simultaneous transmissions that maximizes throughput changes, obtaining a maximum throughput equal to 5.9139 *packets/slot* for $k = 7$ transmissions when $t = 0$ and a maximum throughput equal to 7.6306 *packets/slot* for $k = 8$ transmissions when $t = 10$. The difference between the capacity with $t = 0$ and $t = 10$ when $k \geqslant 4$ is because more the number of errors that can be corrected, more the MAI that can be tolerated. On the other hand, we see that when $k < 3$, the level of noise (background + interferences) is so low that no codification is needed and hence, both throughputs coincide. We also have observed that simulated results, which are obtained by means of Montecarlo simulations, totally match with theoretical curves. Recall, that the number of simultaneous transmissions that maximize throughput corresponds to the number of simultaneous transmissions that the MAC should allow to take place in one slot. The set of k simultaneous transmissions has been chosen randomly but could be chosen depending on other QoS issues such as access priorities.

Additionally, we have considered an instantaneous scheduling system that uses an instantaneous Signal-to-Noise-Interference Ratio threshold ($SNIR_{th}$) as a parameter of QoS. The Receiver Station consists of a Zero Forcing detector where access is given only to transmissions whose instantaneous $SNIR$ is over the $SNIR_{th}$. Hence, from an scheduling point of view, the number of transmissions in the channel is not constant. In figure 2 we compare the performance of both systems. The straight lines, correspond to the throughput that is achieved when the optimal number of simultaneous transmissions is computed by averaging over the channel statistics as in section 5. Such values of the throughput coincide with the maximums in figure 1. On the other hand, we see that for the other system, throughput depends on the $SNIR_{th}$. When using a $SNIR$ threshold as a criterion to decide whether accept a transmission or not, we are selecting those transmissions with better probability of success. However, if the $SNIR_{th}$ is too high, although the instantaneous PSR is very high, the number of simultaneous transmissions is too low and then, throughput decreases. If on the contrary, the $SNIR_{th}$ is too low, the number of transmissions that simultaneously take place is too high and the excess of MAI decreases throughput. We observe that when $t = 0$, a maximum of 7.4799 $packets/slot$ is achieved at $SNIR_{th} = 10\ dB$. In that situation, the average number of simultaneous transmissions in one slot is 7.7767 $transmissions$. For the case of $t = 10$, the maximum throughput is equal to 8.7953 $packets/slot$ when $SNIR_{th} = 8\ dB$ and the average number of simultaneous transmissions is 8.845 $transmissions$.

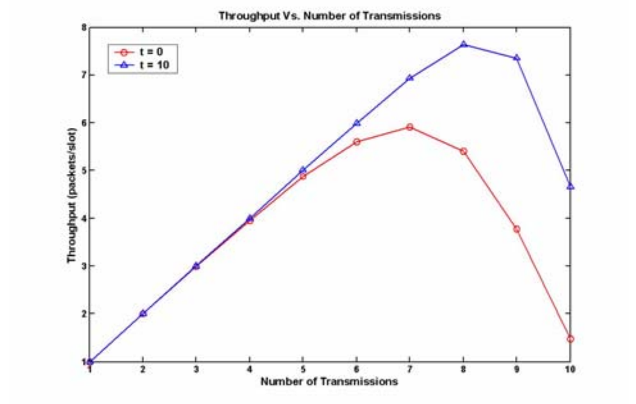

Figure 1: Throughput Vs. Number of Transmissions for t=0 and t=10

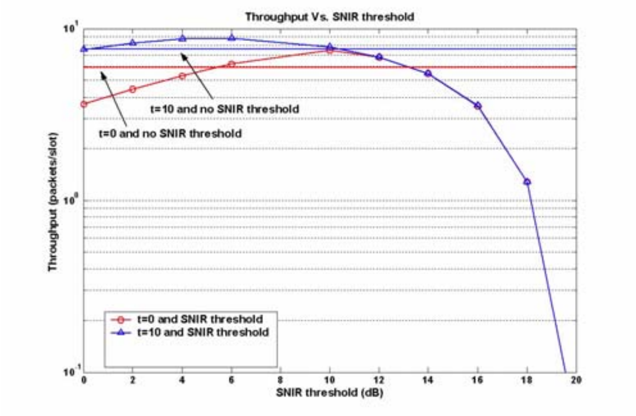

Figure 2: Throughput Vs. SNIR threshold for t=0 and t=10

6.1 PHY-MAC Optimization

We still have to answer the following question, why the maximum throughput achieved by the instantaneous scheduling system can be greater than that for the system that averages over the channel statistics? Clearly, these simulations highlight the differences between an optimization from a PHY layer point of view and an optimization from a MAC layer point of view. PHY optimization is usually based on the instantaneous realization of the channel whereas MAC optimization is based on averages of such channel statistics. The main reason for these two different approaches is, as it has been stated in section 2, two fold. First, the MAC layer must usually deal with traffic issues. And second, MAC optimization parameters (throughput, delay, etc.) correspond to long term system behavior evaluations. From figure 2, we see that if the PHY level schedules transmissions in order to guarantee a minimum $SNIR$, this might not lead to the best performance of the system and depends on the election of the $SNIR_{th}$. Similarly, at MAC layer, if the access to the medium is controlled without accounting for any possible improvement based on instantaneous information at PHY layer, the best system performance can not be ensured. It is worth mention that, although for some values of $SNIR_{th}$ the throughput achieved by the scheduling system is greater than that in the average system, it has not been demonstrated to be the maximum achievable throughput. Actually, the scheduling performed at PHY level modifies the channel statistics and hence, the MAC should be aware of the new channel statistics (cross-layer information). Then, one could propose a joint PHY-MAC optimization using the so called a-posteriori p.d.f of the channel. Considering equation (11), the relationship between $SNIR_{th}$ and α is straightforward and an optimal pair $(k, SNIR_{th})$ could be obtained using

$$\overline{PSR}_{k,SNIR_{th}} = \frac{\int_{\alpha(SNIR_{th})}^{+\infty} PSR_k(\alpha)\ p(\alpha)\ d\alpha}{\int\limits_{\alpha(SNIR_{th})}^{\infty} p(\alpha)\ d\alpha}$$

7. CONCLUSIONS

A general approach to the Multipacket Reception modelling has been given in this article. This tool is useful for the design of Multipacket MAC protocols. We have seen that if no optimization is used at PHY level, an optimal number of simultaneous transmissions that maximize throughput can be obtained by averaging the PSR over the channel statistics. Our simulation results have been compared with a system where scheduling of transmissions is based on a minimum $SNIR$ requirement. Such, comparisons point out that a joint PHY-MAC optimization should be foreseen.

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