RESTORATION OF IMAGES DEGRADED BY SENSOR NON-LINEARITY

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ABSTRACT

In this paper, a new method based on the particle filtering concept is proposed for restoring images degraded by sensor non-linearity, blurring and noise. The approach is novel and leads to a development of the particle filter for space-variant image restoration problem. The key idea in our approach is to propagate samples corresponding to pixels in the state vector. These samples represent the true state density provided the number of samples is large enough. The interdependencies among the pixels is taken care of by the resampling stage of the algorithm. Our approach is recursive and can handle non-linear/non-Gaussian situations also. This is unlike the Kalman filter which is also recursive in nature but works well only under linear and Gaussian conditions. Also, the particle filter is considerably simpler to implement than the Kalman filter. The proposed method is validated on real images degraded by space-invariant as well as space-variant blur in the presence of sensor non-linearity and noise.

1. INTRODUCTION

The goal of image restoration is to recover an original image from a noisy and distorted version of itself. Image restoration techniques involve modeling the degradations, and applying an inverse procedure to reconstruct the original image [1, 2]. In the image restoration literature, it is customary to model the observed image as a linear convolution of the focused image with the point spread function (PSF) of the blur. The observation noise is usually assumed to be additive, white Gaussian that is independent of the image. However, in its most general form, image distortion is non-linear [2], and the noise can be either non-additive and/or non-Gaussian [3]. For example, the imperfections in recording an original scene arise from the non-linear behavior of most image sensors and scanners [4]. The degradation could also be due to multiplicative noise [5].

A few attempts have been made in the past to incorporate sensor non-linearity into image restoration problems. Andrews and Hunt [6] propose expanding the observation model of the non-linear system into a Taylor series about the mean of the recorded image. Using this expansion an approximate filter is derived for image restoration. However, they recommend that one should process the image as though the non-linearity was not present. This is correct only under the 'low contrast' assumption. Andrews and Hunt [2]also propose incorporating the non-linear response of the photographic film through maximum a posteriori (MAP) restoration but the computational requirements are heavy. In Tekalp et al. [5], the overall process is modeled as a homomorphic system and the Wiener filter is used for restoration. Even though the noise in the density domain is Gaussian, it becomes non-Gaussian in the exposure domain which makes the restoration sub-optimal. The Kalman filter which is a recursive filter is known to be optimal only for linear/Gaussian Bayesian situations [7]. Any opportunistic approximation to the general formulation results in sub-optimality in most cases. The extended Kalman filter (EKF) [7] involves linearization which may result in a gross distortion of the true underlying structure and may lead to filter divergence. It must be noted that most of the earlier works have assumed the blur to be space-invariant. However, the case of space-variant blur, even though difficult to handle, is more realistic as in depth-from-defocus [8].

Our approach to image restoration borrows from some recent interesting works that have been going on in the area of 1-D Particle filters. These filters are being used to address problems in which non-linearity and/or non-Gaussianity is involved in the process model or in the observation model or in both [9, 10]. They offer approximate solutions to a very general class of inference problems, including nonlinear and multi-modal time-series estimation. They are attractive because of their simple form and generality, and because they offer an arbitrarily good approximation, given enough computing power. The main contribution of our work is in proposing a recursive filtering framework that extends the 1-D particle filter to two-dimensions for image restoration. The method is quite general and can handle image degradation due to sensor non-linearity under space-invariant as well as space-variant blurring situations. The non-linear function and blurring are assumed to be known. However, because of the non-linearity in the observation model, the posterior density cannot be modeled as Gaussian. The key idea is to represent the required posterior state density as a set of random samples with associated weights and to compute the estimates based on these samples and weights. As the number of samples becomes very large, this characterization tends to an exact, equivalent representation of the required posterior density. We take the specific case of restoration of scanned photographic images [5] to validate the proposed approach.

2. SENSOR NON-LINEARITY AND IMAGE MODEL

A basic model representing the non-linear relationship between the original image intensity and the recorded values is shown in Figure 1. The given image $s(m,n)$ is degraded by a linear blur having point spread function $h(.,.)$. The PSF could be space variant in general. This blurred image is subjected to a distortion $g(.,.)$ which captures the non-linear behavior of the imaging sensor. The final output is the recorded (degraded) image $d(m,n)$ given by

$$d(m,n) = g[h\{s(m,n)\}] + v(m,n) \quad (1)$$

where $v(m,n)$ is additive white Gaussian noise at the location (m,n). Examples of sensor non-linearity include image scanners and sensors [5, 6].

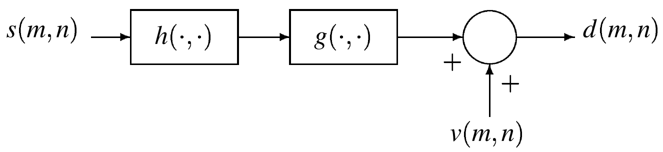


Figure 1: A typical non-linear imaging model.

It must be mentioned that whenever possible most image restoration methods make use of a priori knowledge about the structure of the original image. The development of a suitable model for discrete images requires a trade-off between the accuracy of representation and its utility in image restoration. Causal linear auto-regressive (AR) image models are very conducive for recursive filtering in two-dimensions [11]. In this paper, we assume that the original image can be modeled by a linear AR process as

$$s(m,n) = \sum_{(k,l)\varepsilon\Gamma} c_{m-k,n-l}s(k,l) + \left(1 - \sum c_{m-k,n-l}\right)\bar{s} + w(m,n) \tag{2}$$

where $s(m,n)$ is the image intensity at location (m,n), $c_{m,n}$ are the AR model coefficients (assumed stationary) computed from a prototype image, $w(m,n)$ is uncorrelated zero-mean white Gaussian noise driving the AR process, Γ is the $(L\times L)^{th}$ order causal support at location (m,n), and $\bar{s}$ is the average intensity of the prototype image.

3. THE SCALAR 2-D PARTICLE FILTER

3.1 State-Space Representation

We wish to estimate the original image field I_o from a degraded image field I_d.

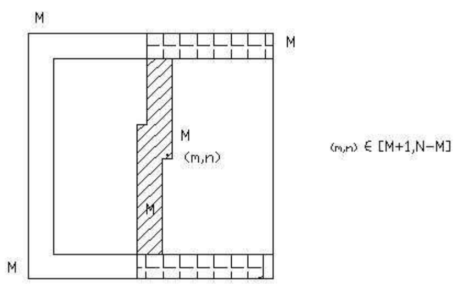


Figure 2: The state vector $\mathbf{S}(m,n)$ at (m,n) (shaded region).

We employ the state-space approach developed by Woods *and* Ingle [12] in their work on the 2-D Kalman filter. The Markov chain assumption must be valid even as the filter makes a transition from the end of a column to the beginning of the next column in the image field I. The state vector $\mathbf{S}(m,n)$ at the location (m,n) contains all the pixels shown in Fig. 2. The inclusion of boundary pixels in the state vector for the sake of 'continuous Markovianity' facilitates the realistic case of estimating even the pixels in the boundary field B. This conception of the state for recursive filtering easily incorporates the image formation model (equation (2)) in the form of a Markov chain. In fact, in reality, only a small number of pixels near the current location in the state vector are actually involved in the computations. This is equivalent to a reduced update particle filter (RUPF).

3.2 Algorithm for Image Recovery

Our objective is to construct the posterior probability density function (pdf) of the current pixel $s(m,n)$, given all the available information $D_{m,n}$: $p(s(m,n)/D_{m,n})$. We can then calculate an optimal estimate of the pixel at the current location from this pdf. Suppose we have an old sample-set $\{\mathbf{S}(m-1,n,k), k = 1,2,\ldots,N\}$. The proposed two-dimensional scalar Particle filter propagates and updates these samples to obtain a new sample-set $\{\mathbf{S}(m,n,k), k = 1,2,\ldots,N\}$ which is approximately distributed as $p(\mathbf{S}(m,n)/D_{m,n})$. we construct the k^{th} of N new samples as follows:

1. Predict by sampling from

$$p(\mathbf{S}(m,n)/[\mathbf{S}(m-1,n) = \mathbf{S}(m-1,n,k)]) \tag{3}$$

to obtain each $\mathbf{S}^*(m,n,k)$. Since we use a linear 2-D AR model for the original image,

$$s^*(m,n,i) = \sum_{k,l\varepsilon\Gamma} c_{m-k,n-l}s(k,l,i) + (1-\sum c_{m-k,n-l})\bar{s} + w(m,n,i) \tag{4}$$

The sample set $\{\mathbf{S}^*(m,n,k), k = 1,2,\ldots,N\}$ is distributed as $p(\mathbf{S}(m,n)/D_{m-1,n})$, the effective prior density at location (m,n).

2. Evaluate the likelihood of each prior sample $s^*(m,n,k)$ in the light of the degraded image intensity $d(m,n)$, and normalize the weights.

$$\Omega_{m,n}(k) = \frac{p(d(m,n)/[\mathbf{S}(m,n) = \mathbf{S}^*(m,n,k)])}{\sum_{l=1}^{N} p(d(m,n)/[\mathbf{S}(m,n) = \mathbf{S}^*(m,n,l)])} \tag{5}$$

For a non-linear sensor with space-variant blurring, we have

$$\Omega_{m,n}(i) = \frac{p_v(d(m,n) - g[h\{s(m,n,i)\}])}{\sum_{j=1}^{N} p_v(d(m,n) - g[h\{s(m,n,j)\}])}$$

3. Once the sample-set $\{s^*(m,n,k), \Omega_{m,n}(k)\}$ has been constructed, estimate moments for the pixel intensity at the current location as

$$E[f(s(m,n))] = \sum_{k=1}^{N} \Omega_{m,n}(k)\, f(s^*(m,n,k)). \tag{6}$$

For instance, an optimal estimate (mean) of the pixel intensity can be obtained using $f(s(m,n)) = s(m,n)$.

4. Update by resampling from the prior sample-set $\{\mathbf{S}^*(m,n,k), k = 1,2,\ldots,N\}$ to obtain $\mathbf{S}(m,n,k)$ such that

$$P[\mathbf{S}(m,n,k) = \mathbf{S}^*(m,n,j)] = \Omega_{m,n}(j) \tag{7}$$

This can be implemented through the following steps.

(a) Generate a uniform random number $r \varepsilon [0,1]$

(b) Find the smallest j for which $c(j) \geq r$

(c) Set $\mathbf{S}(m,n,i) = \mathbf{S}^*(m,n,j)$

Here, $c(0) = 0$ and $c(k) = c(k-1) + \Omega_{m,n}(k)$, $(k = 1,2,\ldots,N)$.

The samples $s(m,n,k)$ are approximately distributed as the required posterior density $p(s(m,n)/D_{m,n})$.

4. EXPERIMENTAL RESULTS

In order to demonstrate and validate our approach, we restore real images degraded by a non-linear sensor and compare the performance of our algorithm with an existing one [5]. The blur function is assumed to be known. The nature of the blur is assumed to be Gaussian (one can use any blur kernel) and the noise is assumed to be additive and white Gaussian. The original image is modeled as an AR process and the AR coefficients are obtained by solving equation (2) using a prototype image.

To show that the particle filter can be effectively used for handling non-linearity, we consider the specific case of restoring scanned photographic images [5]. When a photographic film is used as the image recording medium, there is a well known non-linear relationship between the incoming light intensity (exposure) and the silver density deposited on the film. It is a mapping from the exposure domain to the density domain. This relationship is usually provided by the manufacturer as a $D - \log E$ curve where E is the exposure and D is the optical density. The $D - \log(E)$ characteristic curve is given by $D(E) = \alpha \log_{10}(E) + \beta$ where the subscript α represents the slope of the linear region and β is the density offset of the extrapolated region. Thus, following equation (1) the degraded image $d(m,n)$ recorded in the density domain, can be expressed as

$$d(m,n) = \alpha \log_{10}[h\{s(m,n)\}] + \beta + v(m,n) \qquad (8)$$

The problem is to restore the original image $s(m,n)$ given the degraded image $d(m,n)$. The sensor non-linearity is pointwise and logarithmic in nature. This practical example is interesting because on the one hand the degradation (in the density domain) can be looked upon as a non-linear function in the presence of additive noise. In the exposure domain, the model becomes linear but the noise manifests as multiplicative noise. The aim is to show that the proposed method can handle sensor non-linearity/multiplicative noise. In our experiments, β is assumed to be -1 and α is calculated such that the mean value of the given image is mapped to the mean value of the degraded image for display purpose.

The signal to noise ratio (SNR) expressed in dB of the given degraded image is defined by

$$\mathrm{SNR} = 10 \log_{10}\left(\frac{\text{variance of}\{\alpha \log_{10}[h\{s(m,n)\}] + \beta\}}{\text{variance of}\{v(m,n)\}}\right)$$

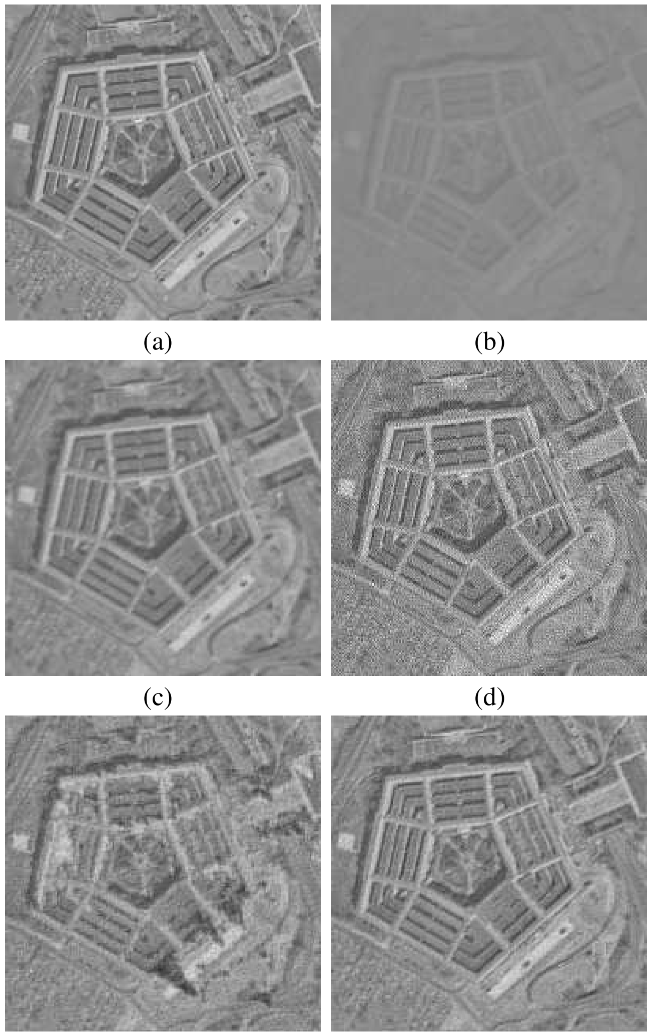
The restored image is usually objectively evaluated by using the quadratic signal-to-noise ratio improvement (ISNR) measure given by

$$\mathrm{ISNR} = 10 \log_{10}\left(\frac{\sum_{i,j}(d(i,j) - s(i,j))^2}{\sum_{i,j}(r(i,j) - s(i,j))^2}\right) (\mathrm{dB})$$

where $d(\cdot,\cdot)$, $s(\cdot,\cdot)$, and $r(\cdot,\cdot)$ represent the degraded observation, the original image and the recovered image, respectively.

Case I. Space-invariant blur with sensor non-linearity

The 'Pentagon' image (size 200×200) is selected as the original image (Fig. 3(a)). To this non-textured real image, we apply space-invariant Gaussian blur with blur parameter $\sigma = 1$. The blurred image is then acted upon by sensor non-linearity corresponding to $\alpha = 65.14$. Independent measurement noise corresponding to SNR of 15 dB is then added in the density domain. The observed degraded image



(a) (b) (c) (d) (e) (f)

Figure 3: (a) Original 'Pentagon' image. (b) Degraded image in density domain for $\sigma = 1$, $\alpha = 65.14$, and SNR=15 dB. (c) Degraded image in exposure domain. (d) Recovered image using modified Wiener filter (ISNR=4.73 dB). Recovered image using Particle filter corresponding to (e) $N = 5$ (ISNR = 3.37 dB), and (f) $N = 200$ (ISNR = 5.93 dB).

corresponding to density and exposure domain is shown in Fig. 3(b) and 3(c) respectively. The image is first restored by a modified linear Wiener filter proposed in [5]. The recovered image is shown in the exposure domain (Fig. 3(d)). While the blur has been removed reasonably well, this is at the cost of noise and the image appears very spotty. The ISNR value is 4.75 dB. Next we restore the image using the proposed method. For $N = 5$, the quality of the recovered image (Fig. 3(e)) is poor, as expected (the ISNR is 3.37 dB). When the number of samples is increased to 200, the recovered image improves considerably (Fig. 3(f)) and is , in fact, much better than the output of [5]. The ISNR value is 5.93 dB. We noted that the improvement in ISNR is only marginal beyond 200 samples. Since the particle filter is not constrained to be linear unlike the filter in [5], it outperforms [5].

Case II. Space-variant blur with sensor non-linearity

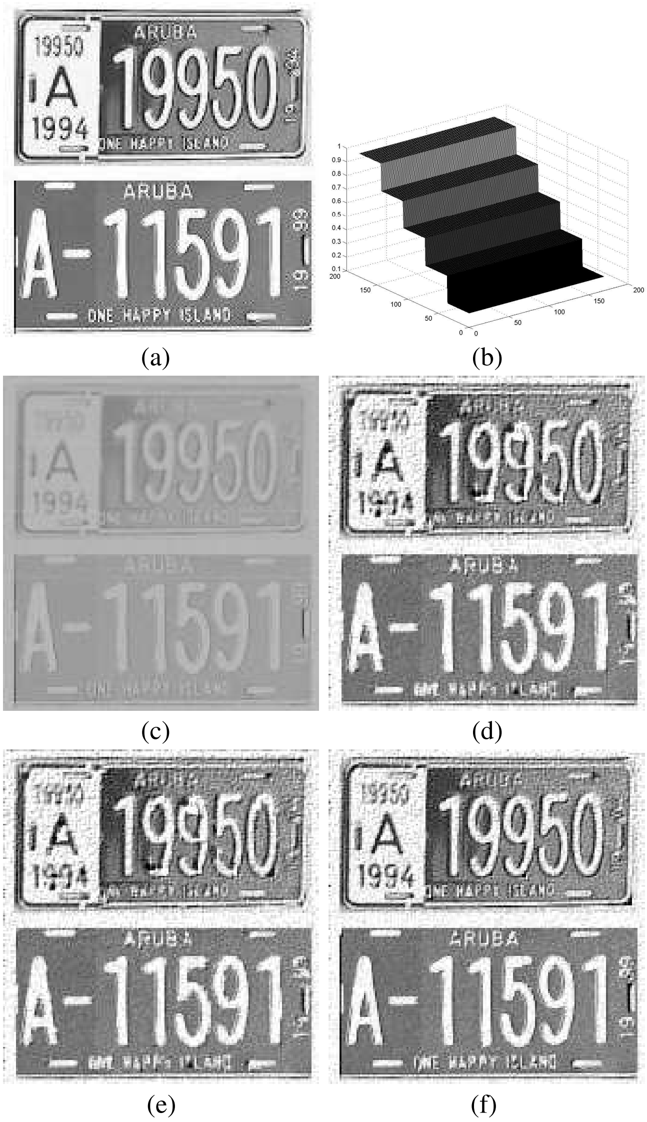


Figure 4: (a) Original 'License plate' image. (b) 3D plot of blur parameter. (c) Corresponding degraded image. Restored image corresponding to (d) $N = 5$ (ISNR = 1.46 dB), (e) $N = 10$ (ISNR = 3.33 dB), and (f) $N = 200$ (ISNR = 7.72 dB).

We now consider the case when the sensor is non-linear but the blur is space-variant. The blurring is assumed to be Gaussian with a known spatial staircase-type distribution as shown in Fig. 4(b). This can result for example when a real aperture camera images an object with discontinuous depths. The original image (of size 200×200) to be restored is shown in Fig. 4(a). For this experiment, $\alpha = 75.97$ and measurement noise variance was 5. The recovered images for N=5, 10, and 200 are shown in Fig. 4(d - f). We could not compare performance with [5] as the method in [5] is limited to space-invariant situations only. The proposed method is general and has been able to recover the image quite satisfactorily when the number of samples used is sufficiently large, even though the blur is space-variant.

5. CONCLUSIONS

We have proposed a general two-dimensional recursive filtering framework for image restoration in the presence of sensor non-linearity based on Particle filtering theory. The technique was validated by applying it to restore real images degraded by sensor non-linearity and under both space-invariant and space variant blurring conditions. The proposed algorithm is compared with an existing scheme and is found to perform better.

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