WAVE FIELD SYNTHESIS SIMULATION BY MEANS OF FINITE-DIFFERENCE TIME-DOMAIN TECHNIQUE

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ABSTRACT

Finite-Difference Time-Domain (FDTD) method was successfully developed to model electromagnetic systems. This technique has been also used in several disciplines, such as optics and acoustics. A new approach for Wave Field Synthesis (WFS) simulation using FDTD instead of finite difference classic method is presented. This software permits to evaluate precision and behaviour of different WFS configurations in time domain and thus in a particular frequency band. Moreover, simulations can be analyzed inside a room or in free space.

1. INTRODUCTION

The final aim of 3D sound systems is the reconstruction of the acoustic sensations that a particular listener would perceive in a real environment. At present time, there is a strong trend towards increasing the realism of the sound reproduction systems. Both spatial sensation as re-creation of acoustic environments are sought. The simplicity of this concept hides a series of important physical and technological complications. In fact, they represent a matter of investigation and constant development in sound engineering.

The most promising system nowadays is the Wave Field Synthesis (WFS), where sound field is synthesized in an extended area by means of arrays of loudspeakers. In order to study this kind of systems, a simulation software is presented. Previous researches propose an analytic model [1]. In this paper an alternative evaluation scheme based in finite-difference time-domain (FDTD) is proposed. This procedure allows to evaluate system response to generic driving signal in time domain, avoiding finite-difference classic method, which processes the field in steps of frequency.

2. FDTD SIMULATION OF ACOUSTIC WAVES

2.1 FDTD formulation

Yee [2] and Taflove [3] developed FDTD method to solve Maxwell equations. This method has been received to solve generic partial differential equations systems. The FDTD method is based on a finite-difference approximation of both space and time derivatives in the wave equation. In this kind of situations, two quantities are chosen. In air acoustics, these quantities are sound pressure and three components of particle velocity.

Assuming small perturbations from rest and negligible viscosity, acoustic equations are given by [4]:

$$\rho_0 \frac{\partial \vec{u}}{\partial t} = -\nabla p, \tag{1}$$

$$\frac{\partial p}{\partial t} = -\rho_0 c^2 \nabla \vec{u}. \tag{2}$$

where $\vec{u}$ is the vectorial gas particle velocity, p is the deviation from ambient pressure, c is sound velocity and ρ_0 is density of the gas at rest.

Using traditional FDTD equations in a 2D-coordinate cartesian grid around acoustic Yee-like unit-cell [5], equations (1) and (2) are discretized as

$$u_x^{i-\frac{1}{2},j,n+\frac{1}{2}} = u_x^{i-\frac{1}{2},j,n-\frac{1}{2}} - \frac{c\Delta t}{\Delta x}\frac{1}{\rho_0 c}(p^{i,j,n} - p^{i-1,j,n}), \tag{3}$$

$$u_y^{i,j-\frac{1}{2},n+\frac{1}{2}} = u_y^{i,j-\frac{1}{2},n-\frac{1}{2}} - \frac{c\Delta t}{\Delta y}\frac{1}{\rho_0 c}(p^{i,j,n} - p^{i,j-1,n}), \tag{4}$$

$$\begin{aligned} p^{i,j,n+1} = p^{i,j,n} &- \frac{\rho_0 c^2 \Delta t}{\Delta x}(u_x^{i+\frac{1}{2},j,n+\frac{1}{2}} - u_x^{i-\frac{1}{2},j,n-\frac{1}{2}}) \\ &- \frac{\rho_0 c^2 \Delta t}{\Delta y}(u_y^{i,j+\frac{1}{2},n+\frac{1}{2}} - u_y^{i,j-\frac{1}{2},n-\frac{1}{2}}).. \end{aligned} \tag{5}$$

where the notation $p(x,y,t) = p(i\Delta x, j\Delta y, n\Delta t) = p^{i,j,n}$ is used.

The solution of the 3D case is trivially obtained if 2D velocity components are replaced by three components.

These equations are updated in time by using a leap-frog scheme. First, u's at time level $n+1/2$ are computed from p's at time level n and previous u's at time level $n-1/2$. Then, p's at time level $n+1$ are computed from u's at time level $n+1/2$ and previous p's at time level n. This process repeats until the temporal simulation is completed.

One important criteria is the stability of the algorithm. Assuming a general wave plane propagated throughout the grid, stability is assured with equation (6).

$$c\Delta t \leq \frac{1}{\sqrt{(\frac{1}{\Delta x})^2 + (\frac{1}{\Delta y})^2}}. \tag{6}$$

2.2 Boundaries

In order to analyze enclosure effect, two approaches are presented in this paper. Firstly, hard surfaces as grid limits are considered when normal particle velocity on the surface vanishes. For that, the condition $u_n = 0$ is required, where $\vec{n}$ is the perpendicular direction at wall. With this boundary condition, it is possible to simulate different geometrical distributions of a room.

In case of simulating free space conditions, spurious reflections must be suppressed. Several methods are been implemented, as Mur finite-difference scheme [6] and Highdon second order operator [7], [8], to provide acceptable engineering values.

3. WAVE FIELD SYNTHESIS FUNDAMENTALS

3.1 Principles

Wave Field Synthesis (WFS) is a sound reproduction technique that, by analogy to holography and in base to the Huygens principle, reproduces an acoustic field inside a volume from the stored signals recorded in a given surface. Huygens principle tells that the wave front radiated by a source behaves like a distribution of sources that are in the wave front, named secondary sources.

Wave field synthesis was first proposed with application to 3D sound by Berkhout [9], [10]. The synthetic wave front is created by loudspeaker arrays that substitute the individual loudspeakers. Ideal situation would be when an area, which is completely surrounded by loudspeakers, is fed with signals that create a volumetric velocity proportional to the particle velocity normal component of the original wave front. The main advantage of these systems is the great extension of the useful listening area; since all the loudspeakers surrounded a volume compose an accurate wave field reproduction zone.

In practice it is not necessary to surround completely the listener by a surface in three dimensions, it is enough to consider a linear loudspeaker array located in front of the listener. In Figure 1, a typical WFS configuration is presented, where a virtual sound source is synthesized in the location of the listener by using a loudspeaker array. However, unlike stereo systems, the synthesized field is not only valid in this location, but also in the rest of the room.

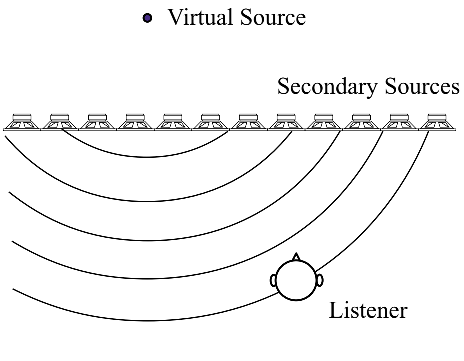

Figure 1: Simulated wave field of a virtual source behind the loudspeaker array.

3.2 Formulation

It is known that an arbitrary sound field within a closed volume can be generated with a distribution of monopole and dipole sources on the surface of this volume [9]. For practical purposes, this method has been adapted to make use of linear loudspeakers arrays surrounding the listening area, rather than planes of loudspeakers. Using the geometry of Figure 2, the sound field created by a virtual source can be synthesized by the array of loudspeakers at the analysis point [11], according to:

$$P(\vec{r},\omega) = \sum_{n=1}^{N} \left[Q(r_n,\omega) H(\phi_n,\omega) \frac{e^{-jkr'_n}}{r'_n} \right] \Delta x, \qquad (7)$$

where N is the number of loudspeakers in the array, ω is the angular frequency, k is the wave number, $Q(r_n,\omega)$ is the driving signal of the n^{th} loudspeaker, ϕ_n is the angle between the main axis of the n^{th} loudspeaker and its connection line to the analysis point, $H(\phi_n,\omega)$ is the directivity index of the n^{th} loudspeaker, Δx is the spatial interval between the array loudspeakers, r_n is the distance between the virtual source and the n^{th} loudspeaker, r'_n is the distance between the n^{th} loudspeaker and the analysis point and $\vec{r}$ defines the analysis point.

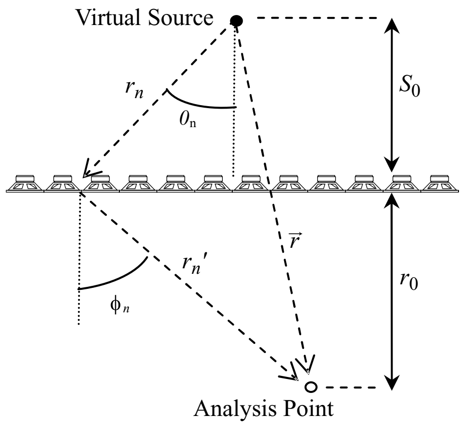

Figure 2: Geometric description for WFS.

Derivation of the driving signals for a line array of loudspeakers is found in [12]:

$$Q(r_n,\omega) = S(\omega) \frac{\cos\theta_n}{G(\theta_n,\omega)} \sqrt{\frac{jk}{2\pi}} \sqrt{\frac{r_0}{s_0 + r_0}} \frac{e^{-jkr_n}}{\sqrt{r_n}}, \qquad (8)$$

where $S(\omega)$ is the virtual source radiated signal, θ_n is the angle between the virtual source and the main axis of the n^{th} loudspeaker, and $G(\theta_n,\omega)$ is the directivity index of the virtual source. The driving signal is only a delayed version of the virtual source signal multiplied by a factor. This depends on the distance between virtual source and each element of the loudspeaker array r_n and the angle θ_n between them. The factor $\sqrt{jk}$ introduces an emphasis of 3 dB per octave. Also notice that the driving signal depends on the listener position. This dependence is very weak and it is a consequence of the two-dimensional approximation [11].

4. ALGORITHMS FOR SIMULATING WAVE FIELD SYNTHESIS

4.1 Developed Software

The presented software is an adaptation of an earlier software based on an analytical model. This software is able to simulate up to three loudspeaker arrays (central, left and right) in different configurations: linear, Open U and Closed U. The simulation is carried out with FDTD technique using an uniform grid. This means $\Delta s = \Delta x = \Delta y$ in the proposed formulation. The maximum frequency to be simulated can be chosen by changing the cells size, according to $f_s = c/(\sqrt{2}\Delta s)$.

An example of a generated sound field can be seen in Figure 3.

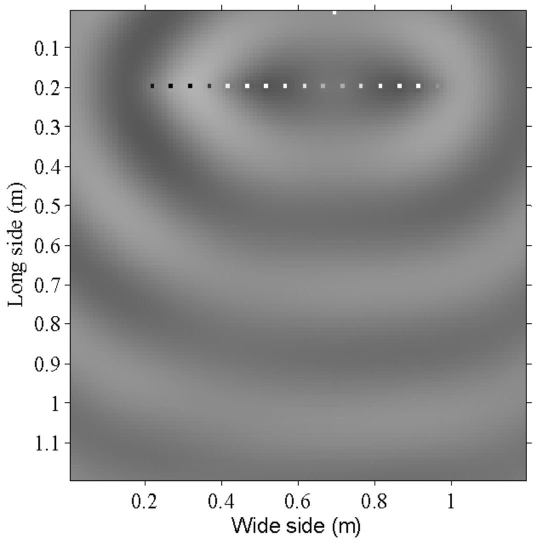

Figure 3: Wave field simulated with FDTD.

4.2 Simulating WFS with FDTD techniques

Up to now, the synthesized sound field simulations were based on an analytical model whose best advantage was its simplicity. Unfortunately, this simplicity is its main weakness, as it will be seen in this section.

From a virtual sound source viewpoint, FDTD technique is more versatile than analytical model since there is no need for calculating the virtual sound source equation. Instead, it is enough to perform the virtual source signal convolution with the impulse response for each loudspeaker. Only the virtual source temporal evolution is required.

With FDTD, simulations are performed in time domain as in the analytical model. This has the advantage of simulating the whole spectrum at a time. Another clear advantage of FDTD over analytical models is the ability to simulate the wave field construction step by step. This is crucial to evaluate the sound field accuracy when the virtual source is inside the room, which is the most difficult case to synthesize.

In other words, FDTD makes possible and easy to simulate the reproduction room effect over the synthesized sound field (see Figure 4). This is extremely difficult with the previous analytical model, if accuracy is needed. With this technique, room effects can be canceled dynamically by generating the proper signals. For doing so, not only the room impulse response, but the wave front amplitudes, orientations and shapes are needed [10].

As drawback, sound sources with FDTD technique are point sources and, by definition, any point source has an omnidirectional directivity characteristic. Thus it is not possible to simulate the loudspeakers directivity comprising an array

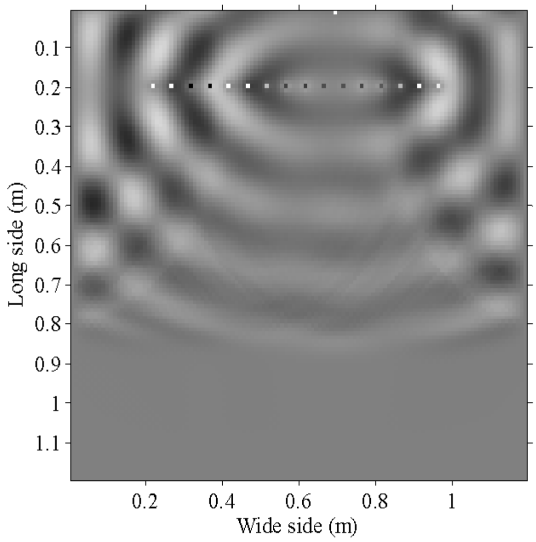

Figure 4: Simulation with room effects.

for frequencies above $ka > 0.5$. This can be corrected by replacing the point sources with a piston, simulated with a set of point sources.

5. CONCLUSION

An introduction of the FDTD technique for WFS is presented. The main advantage in using this technique is the ability of observing the sound field in time steps which contains an extended frequency range components. With classical techniques, each frequency component must be calculated to consider the working range. This time processing permits to study phenomena in boundary conditions in detail.

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