CLASSIFICATION AND SAMPLING OF SHAPES THROUGH SEMIPARAMETRIC SKEW-SYMMETRIC SHAPE MODEL

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ABSTRACT

We present a novel method for shape modeling using an extended class of semiparametric skew-symmetric (SSS) distributions. Given several realizations of a simple shape, the proposed method models it as a joint distribution of angle and distance from the centroid of all points on the boundary. The model, called “Semiparametric Skew-Symmetric Shape Model” (SSSM), is capable of capturing inherent variability of shapes provided the realization contours remain within a certain neighborhood range around a ‘mean’ with high probability. As this paper, we will discuss SSSM learning, classification through SSSM and sampling shapes from the learned models.

1. INTRODUCTION

The goal of shape modeling is to find mathematical representations that capture the intrinsic morphologies of various shapes and simultaneously account for their variability. In recent years, various methods have been proposed to solve the problem including rigid models [3]. Although rigid models have been popular for many applications, they are still not able to reflect the inherent variability of shapes, e.g., anatomical shapes, that has prompted researchers to look for other more flexible approaches [7, 4, 5, 9, 6].

In contrast to these approaches, we view the variability of shape as one that allows realization contours to remain within a certain neighborhood range around a mean. This in turn suggests that for any given angle, a probability density function may be found to capture the corresponding potential excursion of the curves at that angle. We specifically exploit a class of semiparametric skew-symmetric distributions [8] because of potential skewness of data which may arise in practical problems. The method works equally well for non-skewed data since the distribution class includes the non-skewed distributions.

The paper is organized as follows. We start with a problem statement section given next. In Section 3, we describe the probabilistic model we wish to develop. Section 4 and 5 respectively describe classification using SSSM and shape sampling. In Section 6, we provide illustrating examples and finally give some conclusions in Section 7.

2. PROBLEM STATEMENT

Any shape $S_i$ can be represented by a curve $C_{S_i}(t)$ in the parameterized form given below:

$$C_{S_i} : I \subseteq \mathbb{R}^+ \rightarrow \mathbb{R} \times [0, \pi]$$  \hspace{1cm} (1)

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and for convenience and clarity, we take $I \subset \mathbb{N}$, which corresponds to a sampled curve. A given sampled curve, $\{C_{S_i}(t)\}_{i=1,...,N}$, we are asked to provide a probabilistic model for $S_i$ in terms of its radius (from the centroid) and angle around. Alternatively, we may view $C_{S_i}(t)$ as a parametric representation $(x(t), y(t))$ or $(\sqrt{x^2(t) + y^2(t)}, \arctan(y(t)/x(t)))$.

To better illustrate the overall approach, we discuss a specific template of a heart. Given several realizations of a heart, we are asked to learn and to subsequently capture all objects having similar shape realizations. By learning, we mean to compute from training data, the relevant parameters of the underlying distribution of the shape. We assume that, after normalization, the boundaries of realizations can deviate from the mean shape only by some multiple of standard deviation $\sigma$ and lie within a tubular region shown in Fig. 1(c) with probability close to 1. Combining all such realizations and then sampling them is tantamount to assembling a cloud of points in the neighborhood of the mean boundary as shown in Fig. 1(c). The points in the cloud may be assumed “i.i.d” or at most may be modeled as a first order Markov process. The boundary of any shape realization will be a subset of points within a tubular cloud, which is interpreted as a permissible shape space, as shown in Fig. 1(c).

$$S_i \sim \{\sqrt{x^2 + y^2} + n_i, \theta_i + n_\theta\}$$  \hspace{1cm} (2)

Based on these realizations, the points in the cloud at a given angle may be distributed around the template boundary according to a skewed distribution $p(r|\theta)$. Note that the non-skewed densities are a particular case of skewed representation.

Figure 1: (a) Heart shape (b) Some realizations superimposed on each other (c) Sampled superimposed realizations (d) Constituent realizations.
3. SHAPE ANALYSIS

In this paper, we will be investigating a class of closed simple shapes similar to those given in Fig. 4. For convenience, we adopt a modified polar coordinate system \((r, \theta)\), where \(\theta \in [0, \pi]\). In addition, we choose to translate the origin to shape centroid and scale the shape to a pre-specified area. We begin with sampling a shape at angles \(\Theta \in [0, \pi]\) shape centroid and scale the shape to a pre-specified area. Using the prior, 3.2 Overall Shape Model

3.1 Prior Distribution

Because of the random nature of \(R\) and \(\Theta\), a complete shape representation requires assigning a prior on \(\Theta\), whose choice is influenced by the presence of shape singularities, for instance a cusp shown in Fig. 1(a). Provided that such events are present, their nonuniform occurrence throughout, suggests a piecewise uniform or piecewise uniform tapered (PUT) distribution for the prior as shown in Fig. 2.

3.2 Overall Shape Model

Using the prior, \(p(\theta)\), together with a conditional density for \(r\), \(p(r|\theta)\), we can construct the overall density for a shape. Based on \(m\) angle samples, \(\theta = (\theta_1, \ldots, \theta_n)^T\), drawn according to the prior, the overall shape has conditional likelihood:

\[
p(Z_1, \ldots, Z_n | \theta) = \prod_{i=1}^{m} \prod_{j=1}^{n} p(r_j | \theta_i) p(\theta_j), \tag{4}
\]

where \(\{Z_j\}_{j=1}^m\) is the point with radius \(r_j\) and angle \(\theta_j\), \(N_\epsilon^\theta\) is the \(\epsilon\)-neighborhood of \(\theta = \theta_i\) and \(Z_j = (r_j, \theta_j) \in N_\epsilon^\theta\).

3.2.1 Template Learning

We define a template as the best realization of the ideal shape \(S\) in MLE sense. In other words, the template, \(R_{\text{template}}\), maximizes the conditional likelihood function (4) over a set of all possible realizations \(\{R_i; i = 1, \ldots, l\}\):

\[
R_{\text{template}} = \arg\max_{R_i} p(Z_1, \ldots, Z_n | R_i) |_{R_i}. \tag{5}
\]

This is equivalent to using modes of the bimodal distribution as the most likely representative for the boundary points. Hence, corresponding to each angle, we estimate the modes of the posterior, which will then be assumed to lie on the template boundary. In the limiting case, where the number of angle samples goes to infinity, the set of modes will constitute the closed contour of the learned template. This is illustrated for Mickey face in Fig. 3.

3.3 Prior performance assessment

In order to quantify the goodness of a learned template and the quality of a prior selection, we evaluate the cumulative deviation between an ideal shape and learned template with specific priors. At a given angle, \(\theta\), the deviation of the ideal boundary points \(r_1(\theta)\) and \(r_2(\theta)\) from the estimated ones is given by:

\[
dr(\theta) = r_1(\theta) - r_2(\theta). \tag{6}
\]

As a performance measure, we may use \(l_2\)-norm of the difference between two shapes. Discretizing angle space and considering some \(\epsilon\)-neighborhood of \(\theta\), the departure of learned template from the ideal shape is given by:

\[
D = \sqrt{\sum_{\theta \in [0,\pi]} \sum_{\epsilon \in N^\theta} d_{r,\theta}^2}, \tag{7}
\]

where \(N^\theta\) is some \(\epsilon\)-neighborhood of \(\theta\) and \(d_{r,\theta}\) is the deviation for the \(j\)-th mode at a given angle \(\theta_j \in N^\theta\).

4. CLASSIFICATION OF SHAPES

With the probabilistic model (4) in hand, we may now employ maximum likelihood classifier. Suppose that we have learned models \(p_5\) for \(l\) shapes \(\{S_i; i = 1, \ldots, l\}\). Given a test shape, \(S_0\), we compute \(l\) likelihoods \(p_5(S_0 | S_i; i = 1, \ldots, l)\) and assign \(S_0\) to the class that achieves the largest likelihood.

5. SAMPLING FROM MODELS

Shapes can be sampled from the distribution given by (4). In order to sample a cloud of shapes, we need to learn the conditional distributions, \(p(r|\theta)\), for \(m\) given angles \(\Theta = \theta\) drawn according to a PUT distribution. Once the learning phase is complete, we proceed to generate \(n >> m\) additional
In this section, we present some results that demonstrate generality and effectiveness of the proposed method.

6.1 Model Learning

Using different priors, we learn models for three shapes, identified as star, brain, and heart, which are acquired from real images (Fig. 4). We used 20 angle samples in each case except for Star shape, which was learned with 10 samples. A third order polynomial was employed in (3) with skewed normal distribution. Learned templates are given in Fig. 5 and Fig. 6. Templates learned with 100 samples are shown in Fig. 7. Performance measures are tabulated in Table 1. It is clear, both visually and quantitatively by the performance measures, that the PUT prior gives better results than uniform prior.

<table>
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<tr>
<th>Case Study</th>
<th>Prior for angle Θ</th>
<th>D (×10^6)</th>
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<td>2.1</td>
</tr>
<tr>
<td></td>
<td>PUT</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>100 Samples</td>
<td>1.6</td>
</tr>
<tr>
<td>Brain</td>
<td>Uniform</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>PUT</td>
<td>0.4</td>
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<td></td>
<td>PUT</td>
<td>0.211</td>
</tr>
<tr>
<td></td>
<td>100 Samples</td>
<td>0.198</td>
</tr>
</tbody>
</table>

6.2 Classification

In this section, we present classification results that were obtained for a database of 143 car and 68 banana shapes, some of which are shown in Fig. 10 and Fig. 11 respectively. Templates learned from SSSM for the two shapes are presented in Fig. 10 and Fig. 11. Given an observation S₀, we test the following two hypotheses:

\[ H₀ : S₀ \text{ is a car.} \quad (11) \]
\[ H₁ : S₀ \text{ is a banana.} \]

Likelihoods \( p_{S₀} \text{car} \) and \( p_{S₀} \text{banana} \) for 143 car and 68 banana shape realizations were computed. The likelihoods were then used to classify with 97.6% success rate.

6.3 Sampling from Model

A cloud of points simulated for Mickey face is shown in Fig. 8. Angle sample size was 100 while the number of points was 20000. Some sampled shapes are given in Fig. 9.

7. CONCLUSIONS

In this paper, we presented semiparametric model for shape representation, whose statistical nature accounts for variations present in different realizations. The method is sufficiently general to capture possible skewness that might appear in data. Computer simulations demonstrate that semiparametric skew-symmetric template learning is quite effective and robust for capturing variability inherent to shapes. It can capture shape singularities to some extent and may be applied to complex multi-loop templates using higher order polynomial \( P_k \) in (3). We also presented classification results and a way to sample shapes from the model. The method can also be extended to 3-D shapes, which we are currently investigating.

REFERENCES

Figure 4: (a) Actual shapes: star, brain, and heart; (b) Corresponding realizations.

Figure 5: Template learned using uniform prior: (a) Star; (b) Brain; (c) Heart.

Figure 6: Template learned using piecewise tapered uniform prior: (a) Star; (b) Brain; (c) Heart.

Figure 7: Template learned with 100 angle samples: (a) Star; (b) Brain; (c) Heart.

Figure 8: Shape simulation of Mickey face according to Eq. 4: (a) Realizations; (b) Simulated realizations.

Figure 9: Star shapes simulated according to Eq. 10: (a) Without Markov chain; (b) $\sigma_0 = 1$; (c) $\sigma_0 = 0.09$; (d) $\sigma_0 = 0.04$.

Figure 10: Car Shape: (a) Some car images from different viewing angles; (b) Corresponding outer contours; (c) Learned template.

Figure 11: Banana Shape: (a) Some sample bananas; (b) Corresponding outer contours; (c) Learned template.