DISTRIBUTED DETECTION BY MULTIPLE TESTS IN SENSOR NETWORKS USING RANGE INFORMATION

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ABSTRACT

We consider the problem of binary distributed detection when the target position is unknown in the context of large-scale, dense sensor networks. We propose to divide the area where the target could be present into smaller ones, performing a log-likelihood ratio fusion rule in each one. We derive the Bayesian and NP fusion rules under using a model of probability of detection that makes no assumptions on the local decision rule. The performances of both tests is analyzed using large deviation bounds on the error probability and a parametric approximation to the probability of detection function. The main conclusions of the analysis of these bounds are that, for designing efficient tests in terms of energy consumption, 1) the exploration area for each test must cover the area in which the target could be present extended by a distance that is less or equal to the range of the local sensors, depending on the type (Bayes or NP) test, and, 2) the pattern for dividing the large area into smaller ones is the area inside the range of a local sensor.

1. INTRODUCTION

In 1986, Chair and Varshney [1] determined the optimum Bayes decision fusion rule for the binary distributed detection problem when the local detection rule is known. Two years later, Tsitsiklis [2] shown that when the number of sensor is arbitrarily large, the optimal binary decentralized detection is achieved by identical local detection rules. From that, many author have proposed and analyzed different optimal fusion rule under a variety of criteria (see [3] and the references therein for a review on this topic).

On the other hand, the recent advances in the development of small and low powered devices that integrate sensing, processing, and wireless communication capabilities [4] make possible the large-scale, dense and random sensor networks where the above algorithms are valuable tools. Nevertheless, some of the hypothesis of the original problem has to be revised. First, the conditional probability distribution under each hypothesis in each sensor can be different due to the spatial variation of the signal (or source, or target) to be detected and, second, if the sensors are identical as suggested in [2], the Probability of False Alarm (PFA) and Probability of Missdetection (PM) must vary from sensor to sensor according to a its position. Authors in [1] have been considered different PFA and PM for each sensor, but no further analysis was given.

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In a previous paper, [5], we proposed a model in which the probability of detection (including false alarms) of the sensor varies as a function p_d of the distance between the sensor and the source or target to be detected, and we consider the simple problem of detecting a target knowing its position and the position of the sensors under both Bayes and Neyman-Pearson (NP) tests. In this paper, we extend these results to the case in which the position of the target is unknown. Given an area where the target could be present, we propose to partition this area into smaller ones, in one of them computing one binary composite hypothesis log-likelihood ratio (LLR) test. Based on large deviation bounds on the error probability and a parametric approximation of p_d, we obtain the optimum size of the partition.

The paper is organized as follows. The statement of the problem and the notation used in the paper is established in Section 2. The LLR tests are derived in Section 3. Section 4 is devoted to the analysis of large deviation bounds on the probability of error. The optimization of the tests is done in Section 5, and the conclusions end the paper.

2. PROBLEM STATEMENT AND NOTATION

A target could be present in some point of the area $\mathscr{D}^* \in \mathbb{R}^2$. If no other information is available, we assume a uniform distribution over $\mathscr{D}^*$ of the target coordinates. $\mathscr{D}^*$ is partitioned into D smaller areas $\mathscr{D}'_j \in \mathbb{R}^2$; i.e. $\mathscr{D}^* = \bigcup_{i=j}^{D} \mathscr{D}'_j$. A target is decided to be present if it is detected in, at least, one area $\mathscr{D}'_j$. The detection of a target in $\mathscr{D}'_j$ is performed by exploring an area $\mathscr{D}_j \in \mathbb{R}^2$ such that $\mathscr{D}'_j \subseteq \mathscr{D}_j$. The exploration of $\mathscr{D}_j$ gives as a result the data set $\{(x_{ji}, y_{ji}) : i = 1, \ldots, l, x_{ji} \in \mathscr{D}_j, y_{ji} \in \{0,1\}\}$, when each pair (x_{ji}, y_{ji}) represents a successful reading of a sensor located at coordinates x_{ji} that can detect ($y_{ji} = 1$) or not ($y_{ji} = 0$) a target. In the sequel, we will omit the dependence respect to j unless necessary.

We also consider a random deployment of sensor with density ρ_s, and that each sensor applies the same binary detection rule, not necessarily based on a LLR test.

The probability of a positive detection ($Y = 1$) in a sensor located at coordinates x when a target is present at coordinates $x^t \in \mathscr{D}'$ is denoted as $p_d(x^t, x, \alpha)$, where α is the probability of false alarm (PFA) of the sensor when no target is present. In other words, $p_d(x^t, x, \alpha) = Pr(Y = 1 | X^t = x^t, X = x)$ when the PFA of the detector is equal to α. $p_d(x^t, x, \alpha)$ has the following properties:

1. $p_d(x^t, x, \alpha) \geq \alpha$
2. $p_d(x^t, x, \alpha) = p_d(\|x^t - x\|_2^2, \alpha)$
3. $p_d(x^t, x, \alpha) \geq p_d(x^t, x', \alpha) \Leftrightarrow \|x^t - x\|_2 \leq \|x^t - x'\|_2$

4. $\lim_{\|x^t - x\|_2 \to \infty} p_d(x^t, x, \alpha) = \alpha$

Given $\mathscr{D}$, we define two hypothesis, H_0 or null hypothesis for the case when no target is present, and H_1 or alternative hypothesis for the case when a target is present.

- Under hypothesis H_0, the joint pdf of X^t, X, and Y is

$$f_{X^t,X,Y|H_0}(x^t,x,y|H_0) = \rho'\rho\left(\alpha\,\delta[y-1] + (1-\alpha)\,\delta[y]\right)$$

where $\rho' = (\int_{\mathscr{D}'} dx)^{-1}$, $\rho = (\int_{\mathscr{D}} dx)^{-1}$, and δ is the Kronecker function.

- Under hypothesis H_1, the joint pdf of X^t, X, and Y is

$$f_{X^t,X,Y|H_1}(x^t,x,y|H_1) = \rho'\rho(p_d(x^t,x,\alpha)\,\delta[y-1] + (1-p_d(x^t,x,\alpha))\,\delta[y])$$

We assume that samples in $\{(x_i,y_i) : i = 1,\ldots,l, x_i \in \mathscr{D}, y_i \in \{0,1\}\}$ are conditionally (under H_0 or H_1) independents.

When necessary, we can assume the following parametric approximation to p_d, that we called the "spanish hat" model:

$$p_d(x^t,x,\alpha) = \begin{cases} (1-\beta) & \text{if } \|x^t - x\|_2 < r_0 \\ \alpha & \text{otherwise} \end{cases}$$

where r_0 is the range of the sensor. This model considers a constant probability of missdetection when the target is located inside the range of the sensor and a constant false alarm probability outside the range of the sensor. This simple model is provided to gain some insight into the performance analysis.

3. HYPOTHESIS DETECTION PROBLEMS

The dependence respect to x^t in the composite hypothesis H_1 is avoided by integrating out x^t over $\mathscr{D}'$, leading to the definition of a new function p_d, p'_d, as

$$p'_d(\mathscr{D}',x,\alpha) = \rho' \int_{\mathscr{D}'} p_d(x^t,x,\alpha)dx^t$$

being now the joint pdf of X and Y under H_1

$$f_{X,Y|H_1}(x,y|H_1) = \rho(p'_d(\mathscr{D}',x,\alpha)\,\delta[y-1] + (1-p'_d(\mathscr{D}',x,\alpha))\,\delta[y])$$

Given $\{(x_i,y_i) : i = 1,\ldots,l, x_i \in \mathscr{D}, y_i \in \{0,1\}\}$, the log-likelihood ratio between both hypothesis can be computed as

$$\lambda = \sum_{i=1}^{l} \ln \Gamma_i$$

where

$$\begin{aligned} \Gamma_i &= \frac{p'_d(\mathscr{D}',x_i,\alpha)\,\delta[y_i-1] + (1-p'_d(\mathscr{D}',x_i,\alpha))\,\delta[y_i]}{\alpha\,\delta[y_i-1] + (1-\alpha)\,\delta[y_i]} \\ &= \begin{cases} \frac{1-p'_d(\mathscr{D}',x_i,\alpha)}{1-\alpha} & \text{if } y_i = 0 \\ \frac{p'_d(\mathscr{D}',x_i,\alpha)}{\alpha} & \text{if } y_i = 1 \end{cases} \end{aligned}$$

Under Bayes criteria, the threshold τ is easily set as

$$\tau = \ln \frac{\pi_0(C_{10} - C_{00})}{\pi_1(C_{01} - C_{11})}$$

and, under the NP criteria, the determination of the threshold can be done using the asymptotic normality of λ when the number of sensors is high, as in [5]. However, the analysis of the performance of both tests (the mean probability of error in the Bayes test and the power in the NP test) is hard to determine analytically, and we will make use of large deviation bounds to perform it.

Prior to proceed with the performance analysis, lets determine the form that takes function p'_d when using the "spanish hat" approximation of p_d. In order to simplify the analysis, lets assume that the area $\mathscr{D}' \in \mathbb{R}^2$ is a circle of radius r' centered on the origin, and we obtain

$$p'_d(\mathscr{D}',x,\alpha) = \begin{cases} (1-\beta) & \text{if } \|x\|_2 \le r_0 - r' \\ (1-\beta)A(x) + \alpha(1-A(x)) & \text{if } r_0 - r' < \|x\|_2 < r_0 + r' \\ \alpha & \text{otherwise} \end{cases}$$

where

$$A(x) = \frac{1}{2\pi r'^2}\left(r_0^2\theta(x) - r_0^2 sin\theta(x) + r'^2\phi(x)sin\phi(x)\right)$$

and

$$\theta(x) = 2\arccos\left(\frac{r_0^2 - r'^2 + \|x\|_2^2}{2r_0\|x\|_2}\right)$$

$$\phi(x) = 2\arccos\left(\frac{r'^2 - r_0^2 + \|x\|_2^2}{2r'\|x\|_2}\right)$$

that is represented in Figure 1 for $r_0 = 1$, $\alpha = \beta = 0.1$, and different values of r'. When necessary, function A can be

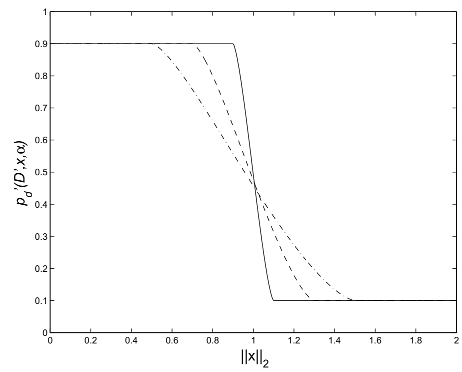


Figure 1: Different p'_d functions.

linearly approximated by

$$A(x) = \frac{1}{2}\left(\frac{r_0 - \|x\|_2}{r'} + 1\right) \quad (1)$$

4. LARGE DEVIATION BOUNDS

When the number of sensor, l, tends to infinity, the probability of error of both Bayes and NP tests can be bounded using large deviation bounds in the form of error exponents [6, 7]. If ε_l is the probability of error (of some kind) obtained with l observation, the error exponent is defined as

$$\lim_{l\to\infty} -\frac{1}{l}\ln\varepsilon_l$$

In NP test, the best error exponent is given by the Stein's lemma, that applied to our problem says that for any $\alpha_n \in (0,1)$

$$\lim_{l\to\infty} -\frac{1}{l}\ln\beta_l = D(f_{X,Y|H_0}\|f_{X,Y|H_1})$$

where D is the Kullback-Leibler (KL) divergence. We will denote $D(f_{X,Y|H_0}\|f_{X,Y|H_1})$ as $D(H_0\|H_1)$ for short.

Using the "spanish hat" approximation of p_d with $\mathscr{D}$ and $\mathscr{D}'$ being circles centered on the origin and radius of, respectively, R and r', and employing the linear approximation (1), $D(H_0\|H_1)$ is

$$D(H_0\|H_1) = -\begin{cases} H(\alpha)+\ln\left(\beta\left(\frac{1-\beta}{\beta}\right)^{\alpha}\right) & \text{if } R\le r_0-r' \\ H(\alpha)+\frac{F(R)-F(r_0-r')+(r_0-r')^2\ln\left(\beta\left(\frac{1-\beta}{\beta}\right)^{\alpha}\right)}{R^2} & \text{if } r_0-r'<R<r_0+r' \\ \frac{(r_0+r')^2H(\alpha)+F(r_0+r')-F(r_0-r')+(r_0-r')^2\ln\left(\beta\left(\frac{1-\beta}{\beta}\right)^{\alpha}\right)}{R^2} & \text{if } R>r_0+r' \end{cases} \quad (2)$$

where H is the entropy function, and

$$F(r) = a\frac{\ln(k_0r+k_1)(k_0^2r^2-k_1^2)-(k_0r-k_1)^2}{k_0{}^2}+$$
$$+(1-a)\left(\frac{\ln(-k_0r-(k_1-1))(k_0^2r^2-(k_1-1)^2)}{k_0{}^2}+\frac{-(k_0r-(k_1-1))^2}{2k_0{}^2}\right)$$

being

$$k_0 = -\frac{1}{2r'}(1-\beta-\alpha)$$
$$k_1 = \alpha - k_0(r_0+r')$$

In Bayes tests (assuming that $C_{10}-C_{00}=C_{01}-C_{11}$), the best achievable error exponent is the Chernoff information, $C(f_{X,Y|H_1}, f_{X,Y|H_0})$ or $C(H_1,H_0)$ for short, defined as

$$C(f_{X,Y|H_1}, f_{X,Y|H_0}) = D(f_{X,Y|s_0}\|f_{X,Y|H_1}) = D(f_{X,Y|s_0}\|f_{X,Y|H_0}) \quad (3)$$

where

$$f_{X,Y|s}(x,y|H_0) = \frac{f^s_{X,Y|H_1}(x,y|H_1)\,f^{1-s}_{X,Y|H_0}(x,y|H_0)}{\sum\int f^s_{X,Y|H_1}(x',y|H_1)\,f^{1-s}_{X,Y|H_0}(x',y|H_0)\,dx'}$$

and s_0 the value of s such that (3) is satisfied.

The Chernoff information can also be obtained as minus the minimum of the cumulant generating function (cgf) of the log-likelihood ratio per sample under hypothesis H_0 (or H_1), i.e.,

$$C(H_1,H_0) = -\frac{1}{l}\min_{0\le s\le 1}\mu_{\lambda,0}(s)$$

that in our problem takes the form

$$\mu_{\lambda,0}(s) = l\ln\left[\int_{\mathscr{D}}\rho\left(\frac{p'_d(\mathscr{D}',x,\alpha)^s}{\alpha^{s-1}}+\frac{(1-p'_d(\mathscr{D}',x,\alpha))^s}{(1-\alpha)^{s-1}}\right)dx\right]$$

Unfortunately, no analytical expression could be found using the "spanish hat" model, even using the linear approximation (1). However, numerical results will be presented in the next section.

5. OPTIMIZATION OF THE TESTS

The sensors are assumed to be battery powered, and the wireless transmissions from sensors to the fusion center is the most energy consuming operation [4]. For elongating the life of the sensor network, a reasonable criteria is to read the minimum number of sensors to achieve a probability of error in the problem of detecting the target less or equal a given arbitrarily small value, ε[1]. Also, as the power-related quantity that is assumed to be constant is the number of deployed sensor per area unit, the above criteria must be transformed to the minimum number of sensors per area unit.

For simplicity, lets consider the area $\mathscr{D}^*$ to be square with side of length d. Lets consider also an overlapping partition of $\mathscr{D}^*$ using circular cells $\mathscr{D}'$ with radius r', in such way that $\mathscr{D}^*$ is covered using $\left\lceil \frac{d}{\sqrt{2}r'}\right\rceil^2$ cells. Assuming independent readings on each cell, and assuming that the number of read sensor by cell, l, is large enough or, equivalently, that the sensors are densely deployed, the number of read sensor to achieve a probability of error less or equal to ε, l, is

$$l \ge \frac{\ln\varepsilon}{D}\left\lceil\frac{d}{\sqrt{2}r'}\right\rceil^2$$

where $D = D(H_0\|H1)$ for NP tests, and $C(H_1,H_0)$ for Bayes tests. The number of read sensor per area unit to achieve a probability of error less or equal to ε, $l\rho$, is

$$l\rho \ge \frac{\ln\varepsilon}{D/\rho}\left\lceil\frac{d}{\sqrt{2}r'}\right\rceil^2 \quad (4)$$

By minimizing the right side of (4), we want to answer the following questions:

1. Given r' and d, are they any optimum configuration of the exploration area $\mathscr{D}$?.
2. Given $\mathscr{D}$ and d, are they any optimum value of r'?.

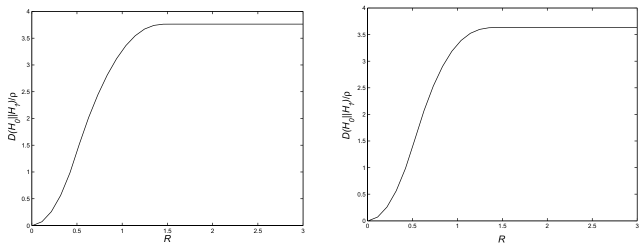
The analysis will be performed using the "spanish hat" model for p_d and circular regions $\mathscr{D}$. Analytical solutions will be given for NP test (using the linear approximation (1)), and only numerical results will be provided for Bayes tests.

For NP tests, the minimum of (4) is equivalent to the maximum of $\pi R^2 D(H_0\|H_1)$, where $D(H_0\|H_1)$ is as in (2). $\pi R^2 D(H_0\|H_1)$ is a non-decreasing function that achieves its maximum at $R = r_0 + r'$, as shown in Figure 2.a for specific values of r_0, r', α and β, and we can conclude that the exploration area, $\mathscr{D}$, must cover, at least, the range of the sensor, having no penalty for exploring big areas (other than the managing larger amount of data). Figure 2.b shown the numerical evaluation of the exact function (without the linear approximation (1)) that is indistinguishable from 2.a.

For Bayes tests, the minimum of (4) is equivalent to the maximum of $\pi R^2 C(H_1,H_0)$, whose numerical evaluation is shown in Figure 3 for specific values of r_0, r', α and β. This function achieves its maximum at a value of R between r_0 and r_0+r' that coincides approximately with the effective range of function p'_d, r_e defined as

$$r_e = \sqrt{\frac{\int(p'_d(\mathscr{D}',x,\alpha)-\alpha)\lceil x}{\pi(p'_d(\mathscr{D}',x^{\sqcup},\alpha)-\alpha)}}$$

[1]For the NP tests, ε is the power of the test, and for Bayes tests, ε is the mean probability of error.



(a) (b)

Figure 2: $\pi R^2 D(H_0\|H_1)$ as a function of R for the "spanish hat" model of p_d (a) using the linear approximation (1) and (b) exact. $r_0 = 1$, $r' = 0.5$, $\alpha = \beta = 0.1$.

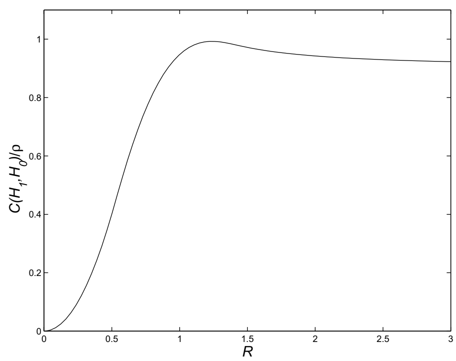


Figure 3: $\pi R^2 C(H_1,H_0)$ as a function of R for the "spanish hat" model of p_d. $r_0 = 1$, $r' = 0.5$, $\alpha = \beta = 0.1$.

This fact is maintained in extensive series of numerical simulations using different values of r_0, r', α and β.

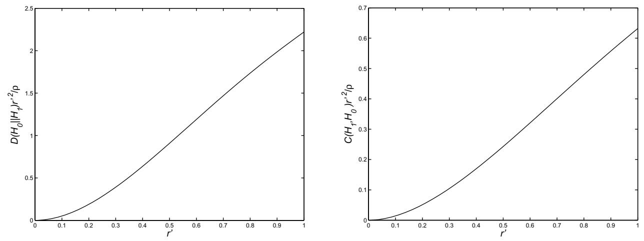
These conclusions coincides with the results presented in [5] for a fixed location of the target, taking now p'_d the role of p_d in [5]. This coincidence is not surprising because p'_d is the convolution of p_d and $f_{X^t}(x^t)$.

Now we will proceed to answer the question of how big $\mathscr{D}'$ must be. In order to guarantee that intersection between $\mathscr{D}'$ and the area inside the range of each sensor (for the value of R previously obtained) is not empty, r' must be less or equal to r_0. For demonstrating that the optimum value of r' is equal to r_0 we must show only that $\pi R^2 D(H_0\|H_1)$ and $\pi R^2 C(H_1,H_0)$ are increasing functions of r'. Taking into account that $R = r_0 + r'$ for NP tests and the expression of $D(H_0\|H_1)$ in (2) it is easy to show the increasing nature of $\pi R^2 D(H_0\|H_1)$. Figure 4.a shown an example of such function for specific values of r_0, α and β.

The demonstration is not easy for Bayes tests due to the nonexistence of analytic expression for $C(H_1,H_0)$. However, all the performed numerical evaluations exhibit this behavior, being one of them the one that is shown in Figure 4.b.

6. CONCLUSIONS

In this paper we extend previous results on distributed detection from the known to the unknown target position. We propose to divide the problem into smaller composite hypothesis tests, and we derived such tests for the Bayesian and Neyman-Pearson case. The probability of error of both kind of tests were analyzed using Stein's lemma (NP test) and Chernoff information (Bayes test), and a simple parametric



(a) (b)

Figure 4: $\pi R^2 D(H_0\|H_1)$ (a), and $\pi R^2 C(H_1,H_0)$ (b) as a function of r' for the "spanish hat" model of p_d. $r_0 = 1$, $\alpha = \beta = 0.1$, $R = r_0 + r'$ in (a), and $R = r_e$ in (b).

approximation to the detection function p_d.

Using as a criteria of efficiency the minimum number of read sensors per area unit to achieve a probability of error less or equal that a given value, the analysis revealed two main facts. The first is that given an area where the target could be present, $\mathscr{D}'$, and given also the range of the detector, r_0, the area of exploration must cover, in the case of NP tests, at least an area that results from extending $\mathscr{D}'$ in all directions by a distance r_0. In the case of Bayes tests, there exists an optimum area that is, in general, contained in the minimum area for NP test and whose definition depends only on $\mathscr{D}'$ and r_0. Contrary to NP tests, in the Bayes test there exist a efficiency penalty when exploring bigger areas.

The second fact is that $\mathscr{D}'$ must coincide with the area inside the range of a single sensor. So, when exploring a large area, this one must be partitioned using the pattern of $\mathscr{D}'$. Note that $\mathscr{D}'$ defines the area of uncertainty about the position of the target, being the exploration area associated with the area of uncertainty always bigger.

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