ON MULTIPLE ACCESSING FOR FREQUENCY-SELECTIVE MIMO CHANNELS

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ABSTRACT

We consider frequency-selective Multiple-Input Multiple-Output (MIMO) multiple access fading channels under the assumption that each of the users employs Orthogonal Frequency Division Multiplexing (OFDM), the multiple antenna transmitters have no channel knowledge and the multiple antenna receiver has perfect knowledge of all channels. The family of (MIMO) multiple access schemes previously introduced in [1] allows to gradually vary the amount of user collision in frequency by assigning different subsets of the available OFDM tones to different users and hence ranges from FDMA (each OFDM tone is assigned to at most one user) to CDMA (each OFDM tone is assigned to all the users). It was demonstrated in [1] that under joint decoding CDMA (full collision in frequency) outperforms any other multi-access strategy implementing a variable amount of collision. In practice, however, minimizing the amount of user collision in frequency is desirable as this minimizes the receiver complexity incurred by having to separate the interfering signals. In this paper, we systematically study the impact of user collision in frequency on the sum capacity achieved by the multiple access schemes described in [1]. Our analysis shows that the impact of collision on spectral efficiency depends critically on the channel's spatial fading statistics and the number of antennas. We then systematically identify scenarios where the performance gap (in terms of spectral efficiency) between CDMA and FDMA becomes negligible and hence little collision is needed to achieve high sum capacity. Finally, an asymptotic (in the number of users) analysis is used to exactly quantify the performance gap between CDMA and FDMA.

1. INTRODUCTION

The use of multiple-input multiple-output (MIMO) wireless systems has been shown to significantly increase the spectral efficiency of point-to-point wireless links [2] – [6]. The performance limits of MIMO multiple access and broadcast channels are considerably less understood and have recently attracted considerable interest [7] – [10].

Contributions: In this paper, we focus on MIMO multiple access channels with frequency-selective fading (spatially correlated at the receiver) assuming perfect channel state information (CSI) in the multiple antenna receiver and no channel knowledge at the multiple antenna transmitters. Each of the users employs orthogonal frequency division multiplexing (OFDM) [11]. We consider a multiple access scheme, originally described in [1], which implements a variable amount of user collision in frequency (signal space) by assigning subsets of the available OFDM tones to different users. The resulting family of multiple access schemes encompasses the extreme cases of frequency division multiple access (FDMA), where each tone is assigned to at most one user, and code division multiple access (CDMA), where each tone is assigned to all the users. We emphasize that in this paper the terminology CDMA is used solely to indicate that all the users occupy the entire frequency band; the effect of redundancy-introducing spreading will not be considered.

It was shown in [1] that under joint decoding, irrespectively of spatial receive fading correlation and number of antennas, the ergodic capacity region obtained by a fully collision-based (CDMA) scheme is an outer bound to the ergodic capacity region corresponding to any other multiple access strategy, where users collide only on subsets of the available tones or do not collide at all (FDMA). A simple two user example in [1] indicated, however, that for rich scattering and a small number of receive antennas very little collision in frequency is needed to realize a significant fraction of the available sum capacity. Minimizing the amount of user collision in frequency is desirable in practice, as this minimizes the receiver complexity incurred by having to separate the interfering signals.

In this paper, we study the joint decoding performance loss due to suboptimum (i.e., not fully collision-based) multiple accessing for an arbitrary number of users in a systematic fashion. Our analysis is based on the new notion of multi-user multiplexing gain, which is shown to be a simple function of the amount of user collision in frequency. We further quantify our results by finding an asymptotic (large number of users) expression for the sum capacity difference between CDMA (full collision) and FDMA (no collision). Finally, we systematically identify situations where the performance gap (in terms of spectral efficiency) between CDMA and FDMA is negligible and hence little collision is needed to achieve high sum capacity.

Previous work: The (MIMO) multiple access scheme considered in this paper was introduced in [1]. Work on SIMO (i.e., the individual users are equipped with a single transmit antenna and the receiver employs multiple antennas) and MIMO multiple access fading channels has been reported previously in [8, 12] – [15]. Results comparing the information-theoretic performance limits of CDMA and FDMA (two extremes of our multiple access scheme) in single antenna frequency-selective fading multiple access channels can be found in [16] – [20].

Organization of the paper: The remainder of this paper is organized as follows. Section 2 introduces the channel and signal model and the multiple access scheme. In Section 3, we define multi-user multiplexing gain and characterize its behavior as a function of collision in frequency (signal space). Section 4 provides results on the asymptotic (large number of users) sum capacity behavior of CDMA and FDMA. We conclude in Section 5.

Notation: $\mathcal{E}$ denotes the expectation operator. The superscripts ^{T,H} and * stand for transposition, conjugate transpose and elementwise conjugation, respectively. $r(\mathbf{A})$, $\mathrm{Tr}(\mathbf{A})$, $\mathrm{span}\{\mathbf{A}\}$ and $\lambda_i(\mathbf{A})$ denote the rank, trace, column space and i-th eigenvalue[1] of the matrix $\mathbf{A}$, respectively. $\mathbf{I}_m$ stands for the $m \times m$ identity matrix.

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[1]Eigenvalues of Hermitian matrices are sorted in descending order.

For equal size matrices $\mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_{K-1}$, $\text{diag}\{\mathbf{A}_i\}_{i=0}^{K-1}$ denotes the block diagonal matrix with $\mathbf{A}_i$ as the i-th block diagonal entry. $\mathbf{A} \otimes \mathbf{B}$ stands for the Kronecker product of the matrices $\mathbf{A}$ and $\mathbf{B}$. $\delta[k] = 1$ for $k = 0$ and 0 otherwise. Let $\mathcal{C}$ denote a set, then $|\mathcal{C}|$ stands for the size of this set. If A and B are random variables, $A \sim B$ denotes equivalence in distribution. An m-variate circularly symmetric zero-mean complex Gaussian random vector is a random vector $\mathbf{z} = \mathbf{x} + j\mathbf{y} \sim \mathcal{CN}_m(\mathbf{0}, \Sigma)$, where the real-valued random vectors $\mathbf{x}$ and $\mathbf{y}$ are jointly Gaussian, $\mathcal{E}\{\mathbf{z}\} = \mathbf{0}$, $\mathcal{E}\left\{\mathbf{z}\mathbf{z}^H\right\} = \mathbf{\Sigma}$, and $\mathcal{E}\left\{\mathbf{z}\mathbf{z}^T\right\} = \mathbf{0}$.

2. SIGNAL AND CHANNEL MODELS

In this section, we describe the multiple access MIMO channel and signal model and the multiple access scheme originally introduced in [1].

2.1 Multiple Access MIMO Channel Model

We consider a multiple access MIMO channel with U users each of which is equipped with M_T transmit antennas, the receiver employs M_R antennas. The individual users' channels are assumed frequency-selective with the i-th user's matrix-valued transfer function given by

$$\mathbf{H}_i(e^{j2\pi\theta}) = \sum_{l=0}^{L-1} \mathbf{H}_{i,l} e^{-j2\pi l\theta}, \quad 0 \leq \theta < 1. \tag{1}$$

We restrict ourselves to purely Rayleigh block-fading channels with the elements of $\mathbf{H}_{i,l}$ $(i = 0, 1, \ldots, U-1; l = 0, 1, \ldots, L-1)$ being circularly symmetric zero-mean complex Gaussian random variables, remaining constant within a block and changing in an independent fashion from block to block [21]. Furthermore, the matrices $\mathbf{H}_{i,l}$ are assumed to be uncorrelated across users (indexed by i) and across taps (indexed by l). Moreover, we assume spatially uncorrelated fading at the transmit arrays. Spatial fading correlation at the receive array is modeled by decomposing the taps $\mathbf{H}_{i,l}$ according to $\mathbf{H}_{i,l} = \mathbf{R}_{i,l}^{1/2} \mathbf{H}_{w,i,l}$ with $\mathbf{H}_{w,i,l}$ denoting a random matrix with i.i.d. $\mathcal{CN}_1(0, 1)$ entries and $\mathbf{R}_{i,l} = \mathbf{R}_{i,l}^{1/2} \mathbf{R}_{i,l}^{1/2}$ is the receive correlation matrix for the l-th tap of the i-th user. We note that the power delay profiles of the individual channels are incorporated into the correlation matrices $\mathbf{R}_{i,l}$. Finally, we assume that the receiver knows all the channels perfectly whereas the transmitters have no CSI.

2.2 Signal Model

We assume that each of the users employs OFDM [11] with N tones (subcarriers) and the length of the cyclic prefix satisfies $L_{cp} \geq L$. The latter assumption guarantees that each of the frequency-selective MIMO fading channels decouples into a set of parallel MIMO frequency-flat fading channels. The receive signal vector for the k-th tone is consequently given by

$$\mathbf{r}_k = \sum_{i=0}^{U-1} \mathbf{H}_i(e^{j2\pi\frac{k}{N}}) \mathbf{c}_{i,k} + \mathbf{n}_k, \;\; k = 0, 1, \ldots, N-1, \tag{2}$$

where $\mathbf{c}_{i,k} = [c_{i,k}^{(0)} \; c_{i,k}^{(1)} \; \ldots \; c_{i,k}^{(M_T-1)}]^T$ with $c_{i,k}^{(l)}$ denoting the data symbol transmitted by the i-th user from the l-th antenna on the k-th tone and $\mathbf{n}_k \sim \mathcal{CN}_{M_R}(\mathbf{0}, \mathbf{I}_{M_R})$ is white noise satisfying

$$\mathcal{E}\{\mathbf{n}_k \mathbf{n}_{k'}^H\} = \mathbf{I}_{M_R} \delta[k - k'].$$

We shall next state a result which will be used frequently in what follows. Under the assumptions in Sec. 2.1, using (1) we can conclude that the $M_R \times M_T$ channel matrices for user i are identically distributed for all tones $k = 0, 1, \ldots, N-1$ [6], i.e.,

$$\mathbf{H}_i(e^{j2\pi\frac{k}{N}}) \sim \mathbf{H}_i, \; i = 0, 1, \ldots, U-1, \; k = 0, 1, \ldots, N-1. \tag{3}$$

In particular, we have

$$\mathbf{H}_i = \mathbf{R}_i^{1/2} \mathbf{H}_{i,w}, \tag{4}$$

where $\mathbf{R}_i = \sum_{l=0}^{L-1} \mathbf{R}_{i,l}$ and $\mathbf{H}_{i,w}$ is a random matrix with i.i.d. $\mathcal{CN}_1(0, 1)$ entries independent across i.

We note that the assumption of the individual users employing OFDM modulation essentially results in a periodic signal model, or more precisely the action of the channel on the transmitted signal is described by circular convolution rather than linear convolution. Our results are therefore not restricted to OFDM modulation, but hold more generally.

2.3 Multiple Access Scheme

We consider a family of multiple access schemes obtained by assigning each OFDM tone $k = 0, 1, \ldots, N-1$ to a subset of users $\mathcal{U}_k$. A fully collision-based[2] multiple access scheme where all tones are assigned to each user (i.e., $\mathcal{U}_k = \{0, 1, \ldots, U-1\}$ for $k = 0, 1, \ldots, N-1$) is referred to as CDMA. FDMA is characterized by $|\mathcal{U}_k| \leq 1$, $k = 0, 1, \ldots, N-1$. We emphasize that the capacity region obtained for a fully collision-based scheme (denoted CDMA in this paper) provides an outer bound to the capacity region of CDMA systems employing (redundancy-introducing) spreading, such as multi-carrier CDMA [22].

For fixed total user powers P_i $(i = 0, 1, \ldots, U-1)$ the ergodic capacity region of any multiple access scheme falling into the framework of [1] is outer bounded by the ergodic capacity region achieved with full collision (CDMA) and uniform power allocation across tones and transmit antennas [1]. The corresponding ergodic sum capacity[3] is given by [1]

$$C_S = \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \sum_{i=0}^{U-1} \frac{P_i}{NM_T} \mathbf{H}_i \mathbf{H}_i^H\right)\right\} \tag{5}$$

and equals the ergodic sum capacity of the underlying multiple access channel [23]. The main conclusion in [1] is as follows: In order to maximize system performance in terms of ergodic capacity, every user should split its total available transmit power uniformly between all tones and transmit antennas and the receiver has to perform joint decoding. In practice, however, minimizing the amount of user collision in frequency is desirable as this minimizes the receiver complexity incurred by having to separate the interfering signals. It is therefore important to understand the joint decoding performance loss resulting from suboptimal (i.e., not fully collision-based) multiple accessing. In this paper, we will be concerned only with performance degradation in terms of sum capacity. For a complete characterization of performance loss in terms of the entire ergodic capacity region, the interested reader is referred to [23]. We finally note that throughout the paper we assume that irrespectively of the tone assignment (and hence the multiple-access scheme used) all the users perform uniform power allocation across their assigned tones and the M_T transmit antennas.

3. MULTI-USER MULTIPLEXING GAIN

The aim of this section is to first introduce the notion of multi-user multiplexing gain and then quantify the impact of spatial receive fading correlation, number of transmit and receive antennas and collision in frequency on multi-user multiplexing gain.

3.1 Definition of Multi-User Multiplexing Gain

Our definition of multi-user multiplexing gain is a straightforward generalization of multiplexing gain for point-to-point links as defined in [24]. Before stating the formal definition, we define the total user power as $\bar{P} = \sum_{i=0}^{U-1} P_i$ and assume $P_i = d_i \bar{P}$, where $d_i > 0$

[2] Note that collision takes place in frequency.

[3] Throughout the paper rates are specified in bps/Hz.

is a constant not depending on $\bar{P}$, and $\sum_{i=0}^{U-1} d_i = 1$. Consequently, $\bar{P} \to \infty$ implies $P_i \to \infty$ $(i = 0, 1, \ldots, U-1)$ and d_i describes the fraction of total power assigned to user i.

Definition 1 *For a given tone assignment $\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}$ (and hence multiple access scheme) denote the corresponding sum capacity as $C_S(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})$. The multi-user multiplexing gain realized by this tone assignment is defined as*[4]

$$m(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}) = \lim_{\bar{P} \to \infty} \frac{C_S(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})}{\log_2(\bar{P})}. \quad (6)$$

As already stated in Sec. 2.3, for fixed P_i $(i = 0, \ldots, U-1)$, the sum capacity and hence $m(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})$ is maximized for full collision (CDMA), i.e., $\mathcal{U}_k = \{0, 1, \ldots, U-1\}$ for $k = 0, 1, \ldots, N-1$. The multi-user multiplexing gain achieved by CDMA will henceforth be denoted as m_{CDMA} and serves as a reference when computing the multiplexing gain for a variable amount of user collision in frequency.

3.2 Multi-User Multiplexing Gain for CDMA

The sum capacity corresponding to CDMA is given by

$$C_S = \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \bar{P} \sum_{i=0}^{U-1} \frac{d_i}{NM_T} \mathbf{H}_i \mathbf{H}_i^H\right)\right\}. \quad (7)$$

Denoting the eigenvalues of the matrix $\sum_{i=0}^{U-1} \frac{d_i}{NM_T} \mathbf{H}_i \mathbf{H}_i^H$ as $\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{M_R-1}$, we can rewrite (7) as

$$C_S = \mathcal{E}\left\{Q \log_2(\bar{P}) + \sum_{l=0}^{Q-1} \log_2\left(\frac{1}{\bar{P}} + \lambda_l\right)\right\} \quad (8)$$

with the random variable $Q = r\left(\sum_{i=0}^{U-1} \frac{d_i}{NM_T} \mathbf{H}_i \mathbf{H}_i^H\right)$. Using (6), it follows immediately from (8) that

$$m_{CDMA} = \mathcal{E}\{Q\}.$$

We can simplify this result by writing

$$\sum_{i=0}^{U-1} \frac{d_i}{NM_T} \mathbf{H}_i \mathbf{H}_i^H = \mathbf{H}\mathbf{D}\mathbf{H}^H,$$

where

$$\mathbf{H} = [\mathbf{H}_0 \;\; \mathbf{H}_1 \;\; \ldots \;\; \mathbf{H}_{U-1}] \quad \text{and} \quad \mathbf{D} = \frac{1}{NM_T} \mathrm{diag}\{d_i\}_{i=0}^{U-1} \otimes \mathbf{I}_{M_T}.$$

Since $r\left(\mathbf{H}\mathbf{D}\mathbf{H}^H\right) = r\left(\mathbf{H}\mathbf{D}^{1/2}\right)$ and $d_i > 0$ $(i = 0, 1, \ldots, U-1)$ which implies $r\left(\mathbf{H}\mathbf{D}^{1/2}\right) = r(\mathbf{H})$, we can conclude that $m_{CDMA} = \mathcal{E}\{r(\mathbf{H})\}$. It is shown in [23] that the circularly symmetric complex Gaussian assumption on the $\mathbf{H}_i$ $(i = 0, 1, \ldots, U-1)$ implies that $r(\mathbf{H})$ is a constant with probability 1 (w.p.1) and hence the multi-user multiplexing gain m_{CDMA} is given by the value that $r(\mathbf{H})$ takes on w.p.1.

Since m_{CDMA} is determined by the rank of a sum of matrices, a general expression for m_{CDMA} in terms of $\mathbf{R}_i$, M_T, M_R and U can not be given. However, for $M_R \leq M_T$, decomposing $\mathbf{H}$ as

$$\mathbf{H} = \tilde{\mathbf{R}}\tilde{\mathbf{H}},$$

where $\tilde{\mathbf{R}} = [\mathbf{R}_0^{1/2} \;\; \mathbf{R}_1^{1/2} \;\; \cdots \;\; \mathbf{R}_{U-1}^{1/2}]$ and $\tilde{\mathbf{H}} = \mathrm{diag}\{\mathbf{H}_{i,w}\}_{i=0}^{U-1}$ and noting that $\tilde{\mathbf{H}}$ is of full row rank w.p.1, it follows that $r(\mathbf{H}) = r(\tilde{\mathbf{R}})$ w.p.1. Since $r(\tilde{\mathbf{R}}) = r(\tilde{\mathbf{R}}\tilde{\mathbf{R}}^H) = r\left(\sum_{i=0}^{U-1} \mathbf{R}_i\right)$, the multi-user multiplexing gain for $M_R \leq M_T$ is given by

$$m_{CDMA} = r\left(\sum_{i=0}^{U-1} \mathbf{R}_i\right). \quad (9)$$

For $M_R > M_T$, a trivial lower bound on m_{CDMA} follows from the fact that $r(\mathbf{H}_i) = \min(r(\mathbf{R}_i), M_T)$ w.p.1 [25] and hence

$$m_{CDMA} \geq \max_i \left(\min(r(\mathbf{R}_i), M_T)\right). \quad (10)$$

Still assuming that $M_R > M_T$, an upper bound on m_{CDMA} is obtained by noting that $r\left(\mathbf{H}_{i,w}\right) = \min(M_R, M_T)$ w.p.1 [25] and hence $r(\tilde{\mathbf{H}}) = UM_T$ w.p.1, which using $r(\mathbf{H}) \leq \min(r(\tilde{\mathbf{R}}), r(\tilde{\mathbf{H}}))$ [26, p. 13] finally yields

$$m_{CDMA} \leq \min\left(r\left(\sum_{i=0}^{U-1} \mathbf{R}_i\right), UM_T\right). \quad (11)$$

Both the exact expression (9) and the lower and upper bounds for the case $M_R > M_T$ show that the multi-user multiplexing gain can be significantly higher than the single-user multiplexing gain that would be obtained if only one of the users were present. This is due to the fact that in the multi-user case all the users contribute to the multiplexing gain (when sum capacity is the quantity of interest). More specifically, the presence of multiple users increases the effective number of transmit antennas from M_T to $M_T U$. On the receiver side, as evidenced by (9) and (11), the limiting factor for multiplexing gain is $r\left(\sum_{i=0}^{U-1} \mathbf{R}_i\right)$ rather than $r(\mathbf{R}_i)$ in the single-user case (assuming that the i-th user is present). Since in practice the number of users U is in general large, $r\left(\sum_{i=0}^{U-1} \mathbf{R}_i\right)$ typically determines m_{CDMA}.

In order to obtain a high-rank sum-correlation matrix $\sum_{i=0}^{U-1} \mathbf{R}_i$, we either need the receive antenna spacing to be large so that the individual correlation matrices $\mathbf{R}_i$ are high-rank or alternatively the $\mathbf{R}_i$ have to span different subspaces. In practice, the latter requirement tends to be satisfied if the individual users are well separated in space, which is typically the case in a cellular system. We finally note that for large M_R and rich scattering/large user separation, m_{CDMA} can be U times higher than the single-user multiplexing gain that would be obtained if only one of the users were present.

3.3 Multi-User Multiplexing Gain for General Tone Assignments

Next, we characterize the impact of user collision in frequency (or lack thereof) on multi-user multiplexing gain. We start by introducing some notation. For $\mathcal{D} \subseteq \mathcal{U} = \{0, 1, \ldots, U-1\}$, $r(\mathcal{D})$ denotes the rank that $\sum_{i \in \mathcal{D}} \mathbf{H}_i \mathbf{H}_i^H$ assumes[5] w.p.1. As before, we define the total user power as $\bar{P} = \sum_{i=0}^{U-1} P_i$ with $P_i = d_i \bar{P}$ and $d_i > 0$ for $i = 0, 1, \ldots, U-1$. Denoting the power assigned to the k-the tone of the i-th user as $P_{i,k}$, we let $P_{i,k} = b_{i,k} P_i$ $(i = 0, 1, \ldots, U-1, k = 0, 1, \ldots, N-1)$, where $\sum_{k=0}^{N-1} b_{i,k} = 1$ for $i = 0, 1, \ldots, U-1$. Since we assumed that the total available power P_i is split uniformly between the N_i tones assigned to user i, we have

$$b_{i,k} = \begin{cases} \frac{1}{N_i}, & i \in \mathcal{U}_k \\ 0, & i \notin \mathcal{U}_k. \end{cases}$$

With these definitions the sum capacity for a general tone assignment $\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}$ can now be written as [1]

$$C_S = \frac{1}{N} \sum_{k=0}^{N-1} \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \frac{\bar{P}}{M_T} \sum_{i=0}^{U-1} d_i b_{i,k} \mathbf{H}_i \mathbf{H}_i^H\right)\right\}.$$

[4] Recall that the noise variance was fixed to equal 1 so that taking $\bar{P}$ in (6) to infinity is equivalent to taking the SNR to infinity.

[5] The existence proof of $r(\mathcal{D})$ $\forall D \subseteq \mathcal{U}$ is provided in [23].

Denoting the l-th eigenvalue of the matrix $\frac{1}{M_T}\sum_{i=0}^{U-1} d_i b_{i,k} \mathbf{H}_i \mathbf{H}_i^H$ as $\lambda_{l,k}$, it follows that

$$
\begin{aligned}
C_S &= \frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{r(\mathcal{U}_k)-1} \mathcal{E}\left\{\log_2\left(1+\bar{P}\lambda_{l,k}\right)\right\} \\
&= \frac{\log_2(\bar{P})}{N}\sum_{k=0}^{N-1} r(\mathcal{U}_k) + \frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{r(\mathcal{U}_k)-1} \mathcal{E}\left\{\log_2\left(\frac{1}{\bar{P}}+\lambda_{l,k}\right)\right\}.
\end{aligned} \tag{12}
$$

Applying (6) to (12), we obtain the multi-user multiplexing gain for general tone assignment $\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}$ as

$$
m(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}) = \frac{1}{N}\sum_{k=0}^{N-1} r(\mathcal{U}_k). \tag{13}
$$

Since $r(\mathcal{U}_k) \le r(\mathcal{U})$, it follows immediately from (13) that $m(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}) \le r(\mathcal{U}) = m_{CDMA}$ showing that the multi-user multiplexing gain for any tone assignment (and hence any amount of collision) is upper-bounded by the multi-user multiplexing gain obtained for CDMA (full collision). However, (13) also shows that one does not have to enforce full collision in frequency to achieve m_{CDMA}; it suffices to choose tone assignments that result in $r(\mathcal{U}_k) = r(\mathcal{U})$ for $k = 0, 1, \ldots, N-1$. The extent to which this is possible depends on the channel's spatial fading statistics and the number of transmit and receive antennas. It is shown in [23] that collision in frequency between users i and j does not affect the multi-user multiplexing gain if and only if $r(\mathbf{R}_i) = r(\mathbf{R}_j) \le M_T$ and $\text{span}\{\mathbf{R}_i\} = \text{span}\{\mathbf{R}_j\}$, which corresponds to the case where the channel does not provide any spatial separation between users i and j. This observation suggests a simple strategy for optimum (in the sense of multi-user multiplexing gain) tone assignment when[6] $r(\mathbf{R}_i) \le M_T$ for $i = 0, 1, \ldots, U-1$. On a given tone, only users with different $\text{span}\{\mathbf{R}_i\}$ should collide. Intuitively this ensures that the spatial degrees of freedom offered by the multi-access channel (and responsible for multi-user multiplexing gain) are indeed exploited. Recall that in point-to-point MIMO links, in order to realize spatial multiplexing gain it is crucial that the signals transmitted from the individual antennas are co-channel (or equivalently collide in signal space).

We finally consider two extreme cases further illustrating the role of user collision in frequency. For $\text{span}\{\mathbf{R}_0\} = \text{span}\{\mathbf{R}_1\} = \cdots = \text{span}\{\mathbf{R}_{U-1}\}$ and $r(\mathbf{R}_0) \le M_T$, the lower and upper bounds (10) and (11) (for $M_R > M_T$) and the exact expression (9) (for $M_R \le M_T$) all meet and yield $m_{CDMA} = r(\mathbf{R}_0)$. Using (13) it can be shown [23] that in this case $m_{FDMA} = m_{CDMA}$, i.e., orthogonal multi-accessing achieves full multi-user multiplexing gain. On the other hand, when $\mathbf{R}_i\mathbf{R}_j = \mathbf{0}\ \forall i \ne j$ (the extreme case of perfect spatial separation between all the users and large number of receive antennas), $m_{FDMA} = \frac{1}{U} m_{CDMA}$ [23].

We conclude this section by noting that in practice, for good spatial separation between the users and for large M_R, collision in frequency (signal space) is critical to achieve a high multi-user multiplexing gain. On the other hand, for poor spatial separation and/or small M_R little or no collision is needed to achieve m_{CDMA}. It should be noted that in the latter case m_{CDMA} will be smaller than in the former case.

4. ASYMPTOTIC ANALYSIS

The results in the previous sections can be further quantified through an asymptotic (in the number of users) comparison of FDMA and CDMA inspired by the approach in [17] used to analyze the sum capacity gap between FDMA and CDMA in single-antenna multiple access fading channels.

In the following, for the sake of simplicity, we assume $\mathbf{R}_0 = \mathbf{R}_1 = \ldots = \mathbf{R}_{U-1} = \mathbf{I}_{M_R}$ and equal user powers, i.e., $P_0 = P_1 = \cdots = P_{U-1} = P$. Furthermore, we set $N = KU$ with $K \in \mathbb{N}$. With these assumptions, the sum capacity for CDMA (full collision) is given by

$$
C_{S,CDMA} = \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \frac{P}{NM_T}\sum_{i=0}^{U-1}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}. \tag{14}
$$

Since $N = KU$ and $\frac{1}{U}\sum_{i=0}^{U-1}\mathbf{H}_i\mathbf{H}_i^H \xrightarrow{\text{w.p.1}} M_T\mathbf{I}_{M_R}$ as $U \to \infty$, we have

$$
\mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \frac{P}{NM_T}\sum_{i=0}^{U-1}\mathbf{H}_i\mathbf{H}_i^H\right)\right\} \longrightarrow M_R\log_2\left(1+\frac{P}{K}\right)
$$

as $U \to \infty$. Therefore, in the large user limit for P large, we have

$$
C_{S,CDMA} \approx M_R \log_2(P/K).
$$

For FDMA, assuming that each user employs K tones, the sum capacity is obtained as

$$
C_{S,FDMA} = \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \frac{P}{KM_T}\mathbf{H}_0\mathbf{H}_0^H\right)\right\}.
$$

Again, considering the large P regime, we have [27]

$$
\begin{aligned}
C_{S,FDMA} \approx{}& \min(M_T,M_R)\log_2\left(\frac{P}{KM_T}\right) \\
&+ \frac{1}{\ln 2}\left(\sum_{j=1}^{\min(M_T,M_R)}\sum_{p=1}^{\max(M_T,M_R)-j}\frac{1}{p} - \gamma\min(M_T,M_R)\right),
\end{aligned}
$$

where $\gamma \approx 0.5772$ denotes Euler's constant.

Combining our results it follows that in the large P regime asymptotically in the number of users

$$
\begin{aligned}
C_{S,CDMA} - C_{S,FDMA} \approx{}& (M_R - \min(M_T,M_R))\log_2(P/K) \\
&+ \min(M_T,M_R)\log_2(M_T) \\
&- \frac{1}{\ln 2}\left(\sum_{j=1}^{\min(M_T,M_R)}\sum_{p=1}^{\max(M_T,M_R)-j}\frac{1}{p} - \gamma\min(M_T,M_R)\right).
\end{aligned} \tag{15}
$$

In the SISO case ($M_T = M_R = 1$) Eq. (15) specializes to $C_{S,CDMA} - C_{S,FDMA} \approx \gamma/\ln 2$, which was found previously in [17]. Fig. 1 shows $C_{S,CDMA} - C_{S,FDMA}$ for $M_R = 2, 4$ and $P/K = 20$dB as a function of M_T. In the regime $M_T < M_R$ we observe that the asymptotic sum capacity performance benefits significantly from collision in frequency. As M_T increases the performance gap between CDMA and FDMA closes. For $M_T \ge M_R$ the quantity $C_{S,CDMA} - C_{S,FDMA}$ no longer depends on P which implies that FDMA achieves the same (asymptotic in U) multi-user multiplexing gain as CDMA. This is due to the fact that for $M_T \ge M_R$ the multi-user multiplexing gain is "bottle-necked" by M_R and collision of the transmit signals across the M_T antennas of an individual user is sufficient to achieve full multi-user multiplexing gain. For fixed $M_T < M_R$, the performance gap in (15) increases with M_R, which can be explained as follows: Increasing M_R opens up more spatial dimensions and hence collision in frequency becomes mandatory to achieve full multi-user multiplexing gain.

5. CONCLUSION

We studied the family of MIMO multiple access schemes previously introduced in [1], which allows to gradually vary the amount of user collision in frequency (signal space) by assigning different subsets of the available OFDM tones to different users. The performance of the proposed class of multiple access schemes, ranging from FDMA to CDMA, was assessed by computing the corresponding multi-user multiplexing gains. We further quantified the performance gap between CDMA and FDMA through asymptotic (in the number of users) expressions for the corresponding sum capacities in the high SNR regime.

Our main findings are summarized as follows. The multi-user multiplexing gain is typically limited by the richness of scattering

[6] Strategies for optimum tone assignment in the general case are discussed in [23].

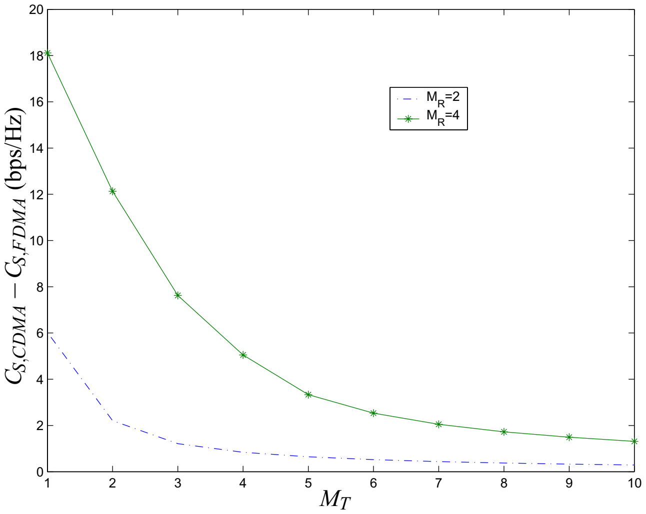

Figure 1: Asymptotic sum capacity difference between CDMA and FDMA for different values of M_R.

at the receiver or the number of receive antennas. Depending on the propagation conditions and the number of transmit and receive antennas, the multi-user multiplexing gain can be significantly higher than the multiplexing gain that would be obtained if only one of the users were present. We showed that for good spatial separation between the users and for large M_R, collision in frequency is crucial to achieve high multi-user multiplexing gain. On the other hand, for poor spatial separation and/or small M_R little or no collision is needed to achieve full multi-user multiplexing gain. We finally note that even though from a sum capacity point-of-view the number of receive (base station) antennas is typically the limiting factor, there is still strong motivation for using multi-antenna transmitters (terminals) since this will result in higher individual data rates.

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