

# CODE-AIDED JOINT CHANNEL AND FREQUENCY ESTIMATION FOR A ST-BICM DS-CDMA SYSTEM

*M. Guenach, F. Simoens, H. Wymeersch and M. Moeneclaey*

Department of Telecommunications and Information Processing  
Ghent University, St.-Pietersnieuwstraat 41, B-9000 GENT, Belgium  
E-mail: {guenach,fsimoens,hwymeersch,mm}@telin.UGent.be

## ABSTRACT

We consider the problem of joint multi-user detection and channel parameter estimation in a space-time bit-interleaved coded modulation (ST-BICM) scheme for an asynchronous DS-CDMA uplink transmission over frequency selective channels. The performance of standard coherent detectors relies on the availability of accurate estimates of the channel parameters and Doppler shifts. Conventionally, these are estimated using pilot symbol in the burst, a technique that reduces both the energy- and bandwidth efficiency. We will derive an iterative estimation technique, based on the SAGE algorithm, that combines pilot symbols and information from the detector in an elegant and efficient manner. We show through computer simulation that the proposed receiver considerably outperforms conventional channel estimation schemes using the same number of pilot symbols.

## 1. INTRODUCTION

Direct-Sequence Code-Division Multiple-Access (DS-CDMA) systems have the ability to accommodate multiple users in multi-path fading environments. Recently developed coding and detection schemes allow a reliable transmission of multiple users at very high data rates. However, these complex detection schemes are very sensitive to synchronization errors.

Currently, a lot of effort is being devoted to developing powerful parameter estimation algorithms. In a multi-user context, several algorithms have been presented to jointly synchronize and detect the different users. Most of them are based on the Expectation-Maximization (EM) [1,2] or Space Alternating Generalized Expectation Maximization (SAGE) algorithm [3–5] and have been shown to have excellent performance in a wide variety of scenarios. However, these algorithms only exploit information from training symbols. To achieve a satisfactory performance, a non-negligible part of the data burst should be occupied by training symbols which significantly decreases the bandwidth efficiency. Only recently, algorithms that also exploit information of the underlying error-correcting code have started to surface (see [6–10]). These estimation algorithms operate by iterating between decoding and estimation, where improved decoding leads to more reliable parameter estimates, leading to improved decoding etc. Although such techniques are by now accepted for simple scenarios, some major modifications are required for more complex situations.

How one may include code properties for multi-user ST-BICM frequency and channel parameter estimation is the topic of the current paper. This paper is a continuation of our work from [11–14] where we have applied the SAGE algorithm to code-aided estimation in a variety of scenarios.

Here this work is extended to include carrier frequency estimation and combines problems related to channel estimation using multiple-antennas, supporting multiple users for a static multi-path system.

## 2. SYSTEM MODEL

We consider an uplink DS-CDMA with  $K_u$  users. The transmitter-end of the  $k$ -th user encodes a block of  $M_b$  bits ( $\mathbf{b}_k$ ), interleaves and groups them into blocks of  $q$  bits. The resulting block of coded bits ( $\mathbf{c}_k$ ) is mapped to a sequence of  $M_d$  symbols, belonging to a  $2^q$ -point complex constellation. Multiplexing with  $M_t$  pilot symbols yields the sequence  $\mathbf{d}_k = [d_k[-M_t], \dots, d_k[M_d - 1]]$ . The complex symbols  $d_k[m]$  are shaped by a normalized spreading waveform  $k(t)$ .

The resulting signal propagates through a multi-path fading channel, with  $L$  paths, supposed to be constant over one block of data and varying independently from block to block. We consider a system with receive diversity, where the receiver is equipped with an array of  $n_R$  antennas. The channel impulse response, as seen by the  $p$ -th receive antenna for the signal of the  $k$ -th user is given by

$$g_k^{(p)}(t) = \sum_{l=1}^L g_{k,l}^{(p)}(t - \tau_{k,l}^{(p)}), \quad (1)$$

where  $g_{k,l}^{(p)}$  and  $\tau_{k,l}^{(p)}$  are the complex gain and path delay of the  $l$ -th propagation path. We group these parameters  $\mathbf{h}_{kl}^{(p)} = [g_{kl}^{(p)}, \tau_{kl}^{(p)}]$ . We further assume that each user is affected by a different frequency offset  $F_k$  caused by an oscillator mismatch or Doppler shift, independent of the antenna index. The latter is an acceptable assumption since all antennas can share the same oscillator. The  $2 \times n_R \times L + 1$  parameters of the  $k$ -th user are contained in  $\mathbf{h}_k = [F_k, \{\mathbf{h}_{kl}^{(p)}\}, \forall p, l]$ . The data frames are corrupted by a vector of independent additive white Gaussian noise  $\mathbf{n}(t)$  with power spectral density  $2N_0$ . Hence, the equivalent baseband signal on the different antennas at the base station is given by the  $(n_R \times 1)$  vector:

$$\begin{aligned} \mathbf{r}(t) &= \sum_{k=1}^{K_u} \sum_{m=-M_t}^{M_d} d_k[m] \mathbf{u}_k(t - mT_d) e^{j2\pi F_k t} + \mathbf{n}(t) \\ &= \sum_{k=1}^{K_u} \sum_{l=1}^L \mathbf{s}_{k,l}(t, \mathbf{d}_k, \mathbf{h}_{k,l}, F_k) + \mathbf{n}(t) \end{aligned} \quad (2)$$

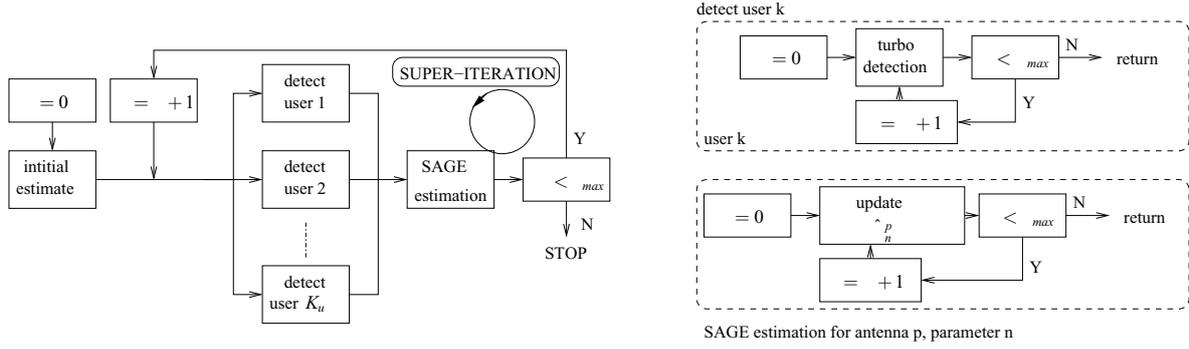


Figure 1: Receiver operation flow chart.

or, with obvious notations

$$\mathbf{r}(t) = \sum_{k=1}^{K_u} \mathbf{s}_k(t, \mathbf{d}_k, \theta_k) + \mathbf{n}(t) \quad (3)$$

$$= \mathbf{s}(t, \mathbf{D}, \boldsymbol{\theta}) + \mathbf{n}(t) \quad (4)$$

where  $T_d$  denotes the symbol period and  $\mathbf{u}_k(t) = [u_k^I(t) \dots u_k^{NR}(t)]^T$ . Here,  $u_k^p(t)$  results from the convolution of  $g_k^p(t)$  with the normalized spreading waveform  $\phi_k(t)$ . We have introduced  $\mathbf{d}_k = [\{d_{k,l}\}, \forall k]$  and  $\mathbf{D} = [\mathbf{d}_1^T, \dots, \mathbf{d}_{K_u}^T]^T$ .

The receiver consists of two main blocks: an iterative detector (with iteration indexed by  $i$ ), an iterative estimator (with iterations indexed by  $j$ ). The receiver iterates between these blocks, with corresponding iteration index  $i, j$ . A flow chart describing the operation of the receiver is presented in Fig. 1. The remainder of this paper is devoted to the operation of the estimator block.

### Detector

We assume an iterative detector, based on [15], consisting of a MMSE interference canceling equalizer, a soft demapper and a decoder. As the equalizer requires knowledge of the channel, it is imperative that accurate estimates of the unknown channel parameters  $\theta_k$  are available. As this paper will focus solely on the estimation problem, the exact type of detector is irrelevant. The only important aspects are (i) that it is iterative and (ii) that at each iteration it computes approximations of the marginal a posteriori probabilities (APPs) of the coded symbols in  $\mathbf{D}$ .

### Estimator

Estimation of the channel parameters and frequency offsets is conventionally performed by a data-aided estimation, exploiting only the presence of the pilot symbols [5, 16]. As the system under consideration may operate at (very) low SNR, a large amount of pilots are required for accurate estimates, resulting in a reduction of the overall spectral efficiency. In the next section we will derive an estimator, based on the SAGE algorithm, that exploits both the pilot symbols *and* the coded symbols in an efficient manner.

## 3. SAGE ESTIMATION

We will project all signals onto a suitable basis, so that  $\mathbf{r}(t)$  is represented by a vector  $\mathbf{r}$ ,  $\mathbf{s}(t, \mathbf{D}, \boldsymbol{\theta})$  by  $\mathbf{s}$ ,  $\mathbf{s}_k(t, \mathbf{d}_k, \theta_k)$  by

$\mathbf{s}_k$ ,  $\mathbf{n}(t)$  by  $\mathbf{n}$  and so forth.

### 3.1 Principle

The Maximum Likelihood estimation of  $\boldsymbol{\theta}$  is obtained by maximizing (w.r.t.  $\boldsymbol{\theta}$ ) the likelihood function:

$$\hat{\boldsymbol{\theta}}_{ML} = \arg \max_{\boldsymbol{\theta}} E_{\mathbf{D}} [p(\mathbf{r} | \mathbf{D}, \boldsymbol{\theta})]. \quad (5)$$

Since both the maximization and the expectation in (5) are practically impossible to compute, we resort to the SAGE algorithm to find an estimate of  $\boldsymbol{\theta}$ : we take a subset of  $\boldsymbol{\theta}$ , say  $\theta_k$  and define  $\bar{\boldsymbol{\theta}} = \boldsymbol{\theta} \setminus \theta_k$ . With  $\theta_k$  we associate a so-called hidden data space  $\mathbf{z}_k$ . Starting from an estimate  $\hat{\boldsymbol{\theta}}(0)$ , we iteratively compute

$$Q(\theta_k | \hat{\boldsymbol{\theta}}(\bar{\boldsymbol{\theta}})) = E_{\mathbf{z}_k} [\log p(\mathbf{z}_k | \theta_k, \hat{\boldsymbol{\theta}}(\bar{\boldsymbol{\theta}})) | \mathbf{r}; \hat{\boldsymbol{\theta}}(\bar{\boldsymbol{\theta}})]$$

and then update the estimate of  $\theta_k$  as follows

$$\hat{\theta}_k(i+1) = \arg \max_{\theta_k} Q(\theta_k | \hat{\boldsymbol{\theta}}(\bar{\boldsymbol{\theta}}))$$

which is a maximization problem of a dimensionality of  $\theta_k$ . We update the different parameters  $\theta_k, \forall k$  in a successive manner.

### 3.2 Signal Decomposition

Let us decompose our estimation problem. The noise at the  $p$ -th receive antenna ( $\mathbf{n}^{(p)}$ ) can be written as the sum of weighted  $K_u L$  zero-mean, mutually independent AWGN components  $\mathbf{n}_{k,l}^{(p)}$ , such that

$$\mathbf{n}^{(p)} = \sum_{k=1}^{K_u} \sum_{l=1}^L \sqrt{\gamma_{k,l}^{(p)}} \mathbf{n}_{k,l}^{(p)}$$

subject to the constraint  $\gamma_{k,l}^{(p)} \geq 0$ . The received signal at the  $p$ -th receive antenna, can be written as

$$\mathbf{r}^{(p)} = \sum_{k=1}^{K_u} \sum_{l=1}^L \mathbf{x}_{k,l}^p \quad (6)$$

with

$$\mathbf{x}_{k,l}^p = \mathbf{s}_{k,l}^p + \sqrt{\gamma_{k,l}^{(p)}} \mathbf{n}_{k,l}^{(p)}$$

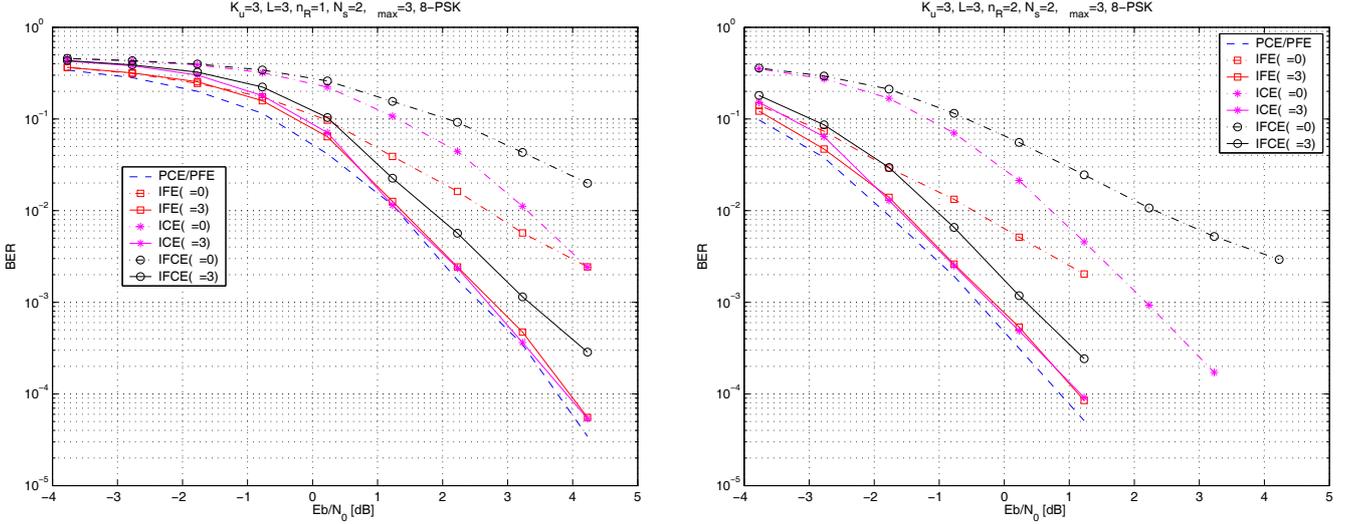


Figure 2: BER-performance of the different estimation schemes for one receive antenna (left) and two receive antennas (right). (PCE/PFE: perfect synchronization, IFE: estimation of frequency offset only (channel known), ICE: estimation of channel only (frequency known), IFCE: both frequency and channel estimation)

Clearly,

$$\log p(\mathbf{x} | \mathbf{D}) = \log p(\mathbf{x}_{k,l}^{(p)} | \mathbf{h}_{k,l}^{(p)}, F_k, \mathbf{D}) \quad (7)$$

where

$$\begin{aligned} & \log p(\mathbf{x}_{k,l}^{(p)} | \mathbf{h}_{k,l}^{(p)}, F_k, \mathbf{D}) \\ &= \frac{1}{2^{(p)N_0}} \int_{-}^{+} \left| x_{k,l}^{(p)}(t) - s_{k,l}^{(p)}(t, \mathbf{d}_k, \mathbf{p}_k, \mathbf{h}_{k,l}^{(p)}, F_k) \right|^2 dt. \end{aligned}$$

### 3.3 Hidden Data selection

#### 3.3.1 Estimation of $F_k$

Select as hidden data  $\mathbf{z} = \left[ \left\{ \mathbf{x}_{k,l}^{(p)} \right\}_{\forall p,l}, \mathbf{D} \right]$  so that

$$\begin{aligned} & \hat{F}_k(\cdot + 1) = \\ & \arg \max_{F_k} E_{\mathbf{z}} \left[ \log p(\mathbf{x}_{k,l}^{(p)} | \hat{\mathbf{h}}_{k,l}^{(p)}, F_k, \mathbf{D}) | \mathbf{r}; \hat{(\cdot)} \right]. \end{aligned}$$

The convergence rate can be maximized by setting  $\frac{(p)}{k,l} = 1/L$  and  $\frac{(p)}{k',l'} = 0$ , for  $k' \neq k$  or  $l' \neq l$ .

#### 3.3.2 Estimation of $\mathbf{h}_{kl}^{(p)} = [g_{kl}^{(p)}, \mathbf{h}_{kl}^{(p)}]$

Select as hidden data  $\mathbf{z} = \left[ \mathbf{x}_{k,l}^{(p)}, \mathbf{D} \right]$ , so that

$$\begin{aligned} & \hat{\mathbf{h}}_{k,l}^{(p)}(\cdot + 1) = \\ & \arg \max_{\mathbf{h}_{k,l}^{(p)}} E_{\mathbf{z}} \left[ \log p(\mathbf{x}_{k,l}^{(p)} | \mathbf{h}_{k,l}^{(p)}, \hat{F}_k, \mathbf{D}) | \mathbf{r}; \hat{(\cdot)} \right]. \end{aligned}$$

The convergence rate can be maximized by setting  $\frac{(p)}{k,l} = 1/L$  and  $\frac{(p)}{k',l'} = 0$ , for  $k' \neq k$  or  $l' \neq l$ .

### 3.4 Practical computation

Performing the above expectations with respect to  $\mathbf{z}$  are evaluated as follows: first of all, expectation w.r.t.  $\mathbf{x}$ , conditioned on  $\mathbf{D}$  is very straightforward [5, 16]. For a purely DA algorithm, this defines the entire algorithm, since expectation with respect to  $\mathbf{D}$  is trivial. In our case, we will perform the expectation with respect to  $\mathbf{D}$  not only for the pilot symbols, but also for the coded symbols. It turns out that this requires the marginal posterior probabilities, the APPs  $p(d_k[m] | \mathbf{r}, \hat{(\cdot)})$  of all users, i.e.  $\forall k$ . Due to space limitations, the mathematical details are omitted here.

### 3.5 Complexity Reduction

Although the proposed algorithm provides a systematic way to exploit code properties for channel estimation, its computational complexity is still very high. We have introduced two modifications to the receiver to alleviate these problems, as detailed in [14], resulting in a low-complexity receiver, where the estimator is now also iterative (with iteration index  $\cdot$ ), as depicted in Fig. 1. The key idea lies in maintaining the state information in the detector while performing the estimation steps, and thus providing multiple parameter updates for fixed APPs<sup>1</sup>.

## 4. NUMERICAL RESULTS

In this section we will provide numerical results to evaluate the performance of the proposed iterative multiuser receiver. We have carried out computer simulations for a system with  $K_u = 3$  users, using a rate  $R = 1/2$  convolutional code and polynomial generators  $(23)_8$  and  $(35)_8$  with 8-PSK signaling using Gray mapping. Frames consist of 120 coded data

<sup>1</sup>The additional iterations  $(\cdot)$  correspond to the estimation iterations without updating the APPs

symbols and 10 training symbols to initialize the SAGE algorithm. The spreading codes are Gold codes of length  $N_c = 7$ . The channels consist of  $L = 3$  taps. We will evaluate the proposed estimation scheme for  $\mu_{max} = 3$  in terms of BER performance.

Fig. 2 shows plots for different estimation scenarios of the SAGE algorithm using  $n_R = 1$  (left) and  $n_R = 2$  (right) receive diversity. We observe that when the channel state is known, the detector requires  $\mu_{max} = 3$  iterations to converge. When we estimate the channel parameters with  $\mu_{max} = 3$ , the initial estimate (corresponding with  $\mu = 0$ ) results in a substantial BER degradation. With subsequent iterations between detection and estimation (i.e., increasing  $\mu$ ), the BER performance improves. The overall system converges after  $\mu = 3$  iterations. The resulting BER degradation is below 0.5 dB for all considered SNRs and all configurations (IFE, ICE and IFCE). The complexity of the receiver can be further reduced by decreasing  $\mu_{max}$  to 1 at the cost of more iterations between detection and estimation (i.e., higher  $\mu$ ). Therefore it can be concluded for all scenarios, compared to the DA estimation ( $\mu = 0$ ), a substantial performance gain is observed when exploiting the soft information provided by the turbo-detector to improve the performance.

## 5. CONCLUSION

We have investigated a DS-CDMA multi-antenna receiver with bit-interleaved coded modulation, performing iterative multiuser detection and joint channel and frequency offset estimation. Different approaches for the estimation were considered, and we demonstrated that the estimator based on the low-complexity SAGE algorithm is most suited for this scenario. The estimator operates by accepting soft information from the detector, in the form of a posteriori probabilities (APPs) of the coded symbols. The computational overhead of the estimator is minimized by embedding the estimation stages in the detection stages so that a form of joint detection and estimation is performed.

The performance of the proposed algorithm is compared in terms of BER with a DA SAGE algorithm. It turns out that the SAGE algorithm, exploiting information from all the data symbols significantly outperforms the DA SAGE algorithm.

## Acknowledgment

This work has been supported by the Interuniversity Attraction Poles Program P5/11- Belgian Science Policy. The second author also gratefully acknowledges the support from the Fund for Scientific Research in Flanders (FWO-Vlaanderen).

## REFERENCES

- [1] A. Kocian and B.H. Fleury. "EM-based joint data detection and channel estimation of DS-CDMA signals". *IEEE Trans. Comm.*, 51(10):1709–1720, October 2003.
- [2] A. P. Dempster, N. M. Laird and D. B. Rubin. "Maximum likelihood from incomplete data via the EM algorithm". *Journal of the Royal Statistical Society*, 39(1):1–38, 1977. Series B.
- [3] J.A. Fessler and A.O. Hero. "Space alternating generalized expectation maximization algorithm". *IEEE Trans. Comm.*, 42(10):pp.2664–2677, October 1994.
- [4] A.A. D'Amico, U. Mengali and M. Morelli. "Channel estimation for the Uplink of a DS-CDMA system". *IEEE Trans. on Wireless Comm.*, 2(6):1132–1137, November 2003.
- [5] B.H. Fleury, M. Tschudin, R. Heddergott, D. Dahlhaus and K.I. Pedersen. "Channel parameter estimation in mobile radio environments using the SAGE algorithm". *IEEE J. Sel. A. Comm.*, 17(3):pp.434–450, March 1999.
- [6] V. Lottici and M. Luise. "Carrier phase recovery for turbo-coded linear modulations". In *IEEE ICC'02*, April 2002.
- [7] A. Burr and L. Zhang. "A novel carrier phase recovery for turbo-coded QPSK". In *EW'02*, Florence, February 2002.
- [8] W. Oh and K. Cheun. "Joint decoding and carrier phase recovery". *IEEE comm. letters*, 5(9):pp.375–377, Sept. 2002.
- [9] M. Kobayashi, J. Boutros, G. Caire. "Successive interference cancellation with SISO decoding and EM channel estimation". *IEEE Journal On Selected Areas In Communications*, 18(8):pp.1450–1460, August 2001.
- [10] N. Noels, C. Herzet, A. Dejonghe, V. Lottici, H. Steendam, M. Moeneclaey, M. Luise and L. Vandendorpe. "Turbo-synchronization: an EM algorithm interpretation". In *IEEE ICC'03*, 2003.
- [11] M. Guenach, H. Wymeersch and M. Moeneclaey. "Joint estimation of path delay and complex gain for coded systems using the EM algorithm". In *Proc. International Zürich Seminar (IZS'04)*, Zürich, Switzerland, February 2004.
- [12] M. Guenach, H. Wymeersch and M. Moeneclaey. "Iterative joint timing and carrier phase estimation using the SAGE algorithm for a coded DS-CDMA system". In *Proc. International Symposium on Control, Communications and Signal Processing (ISCCSP)*, Hammamet, Tunisia, March 2004.
- [13] M. Guenach, H. Wymeersch and M. Moeneclaey. "On DS-CDMA uplink channel parameter estimation for bit-interleaved-coded modulation". In *Proc. SPWC*, London, UK, June 2004.
- [14] M. Guenach, H. Wymeersch and M. Moeneclaey. "On soft multiuser channel estimation of DS-CDMA uplink using different mapping strategies". In *Proc. IEEE ICC*, Seoul, Korea, June 2005.
- [15] X. Wang and H. V. Poor. "Iterative (turbo) soft interference cancellation and decoding for coded CDMA". *IEEE Trans. Comm.*, 47(7):pp.1046–1061, July 1999.
- [16] M. Feder and E. Weinstein. "Parameter estimation of superimposed signals using the EM algorithm". *IEEE Trans. on Acoustics, Speech and Signal Processing*, 36:477–489, April 1988.