MULTI-COMPONENT SIGNAL DENOISING USING UNITARY TIME-FREQUENCY TRANSFORMS

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ABSTRACT

Recently, the analysis of multi-path configurations became a challenging problem for people working in communication or channel characterization fields. In these areas, the received signal is generally expressed as the sum of the transmitted signal and its time–shifted versions. The estimation of the multi–path parameters, typically based on a matched filtering procedure, depends on the noise–level in the received signal.

As shown through experimental results, the usual denoising tools are not always well suited for signals arising from multi-path configuration.

In this paper, a new denoising method is proposed adapted to the case of underwater multi–path configuration. This method, which preserves the distance between arrivals, leads to a better noise robustness of the matched filtering procedure.

1. INTRODUCTION

The problem of analyzing multi–path channels received a great interest in number of applications such as wireless communication, radar, channel characterization, seismic exploration, etc. Generally, the aim of this analysis is the reduction of undesirable multi–path effects. For example, in digital communication, multi–path propagation causes inter–symbol interferences limiting the communication data rates. This phenomenon is strongly accentuated in underwater acoustic channels [1]. In other cases, such as oceanic tomography, the estimation of the multi–path configuration leads to the physical characteristics of the channel (sound velocity profile, salinity, etc) [2].

The standard processing concept in a multi–path configuration is based on the matched filtering [1, 2]. This tool is particularly attractive since it maximizes the signal–to–noise ratio (SNR) under white gaussian noise assumptions. Roughly speaking, the matched filtering is provided by convolving the received signal with the time–reversal copy of the transmitted one. In a real multi–path environment, the matched filter performances can be affected by noise. More precisely, the noise can introduce artefacts on the estimation of the signal parameters. For example, if one is interested in estimating the time–of–arrivals (TOAs) (associated with maximas of the matched filtering), the noise could produce spurious correlation peaks which provides wrong TOAs estimation. A solution consists in denoising the received data before the matched filtering stage. In a multi–path configuration, the main difficulty is to preserve the delays between arrivals in addition to the denoising step.

One of the most popular denoising method [3] is based on the discrimination between signal and noise on an orthogonal basis on which the noisy signal is projected. It has been shown that the wavelet packet decomposition (WPD) is particularly adapted to provide such discrimination [4]. Furthermore, the thresholding of the wavelet coefficients leads to a reduction of the noise. Nevertheless, as shown through experimental results, the HardThresholding [3] procedure affects the temporal structure of arrivals, which damage the performances of the matched filter.

An alternative solution is proposed in this paper. This method is based on the use of another unitary transformation [5]. This transformation projects the signal on a new representation space which allow a compact representation of the signal. In this new space, an appropriate filtering method is applied. The denoised signal is then obtained using the inverse unitary transformation. This leads to a better preservation of the temporal structures and an improved SNR.

The paper is organized as follows. Section 2 investigates the particularity of signal denoising in a multi–path configuration with emphasis on the WPD–based denoising method. An alternative denoising scheme, adapted to this configuration, is then described in section 3. Simulation results are presented in Section 4 and concluding remarks are given in Section 5.

2. DENOISING A SIGNAL IN MULTI–PATH CONFIGURATION

2.1 Model Of A Multi–Path Signal

A multi–path phenomenon causes a signal to arrive at a receiver via two or more different paths with different arrival times. Typically, this effect is modelled as the output of a finite impulse filter (FIR) with an impulse response (IR)[6]

$$h(t) = \sum_{i=1}^{K} a_i \, \delta(t - \tau_i), \quad a_i, \tau_i \in \mathbb{R}, \tag{1}$$

where a_i and τ_i denote the attenuation factor and the time delays of the i^{th} path, K is the FIR length which is equivalent to the number of paths and $\delta(\cdot)$ is the delta function.

For a wideband signal $e(t)$, the received signal is defined by:

$$s(t) = \sum_{i=1}^{K} a_i \, e(t - \tau_i) + n(t) \tag{2}$$

where $n(t)$ stands for the noise. For a low SNR, the time delay estimation process can be dramatically corrupted, even if the transmitted signal is known at the reception. To improve the robustness, several techniques have been proposed [6].

Nevertheless, the most popular tools for time delay estimation assume that the noise is characterized as white Gaussian and uncorrelated with the arrivals. These hypotheses are not always satisfied. One example is the underwater channel where the noise may be non–gaussian and correlated with the arrivals [1].

2.2 Denoising Via WPD

The performances of the standard time delay estimation process may be improved by denoising the signal before the estimation step. Usual tools for signal denoising are based on wavelet packet decomposition. Typically, the denoising via WPD–based method can be done in three steps [3] which consist in

1. Decomposing the signal on an orthogonal wavelet basis. In the wavelet domain, nonzero coefficients are concentrated in the neighborhood of the signal transitions.
2. Thresholding the wavelet coefficients in this basis. This allows to reduce the noise which is generally uniformly distributed in the wavelet subspaces.
3. Reconstructing the signal with the remained coefficients.

However, the non–linearity of the thresholding procedure affects the useful part of the received signal. Namely, the WPD is sensitive to the signal transitions and the interferences terms, induced by multi–path, are amplified while they are not related to a physical phenomenon.

This effect can be highlighted by considering two signals. The former is a noisy chirp signal and the latter is a rough simulation of a three–path configuration with close arrival times. For each test signal, the correlation function is estimated before and after the denoising step. Considering a 6^{th} Symmlet–type wavelet, simulation results are depicted Fig. 1.

In the left part, one can see that the denoising method effectively leads to a noise reduction which is clearly visible on the Wigner–Ville distribution (WVD) plane (Fig. 1a and 1c). Hence, the correlation performance (Fig. 1e) has been improved since the denoising leads to a peak with a higher magnitude and a reduction of the side lobes.

In contrary, the right side of Fig. 1 shows that the correlation function (Fig. 1f) associated to the denoised signal exhibits a peak with a higher magnitude than the correct one.

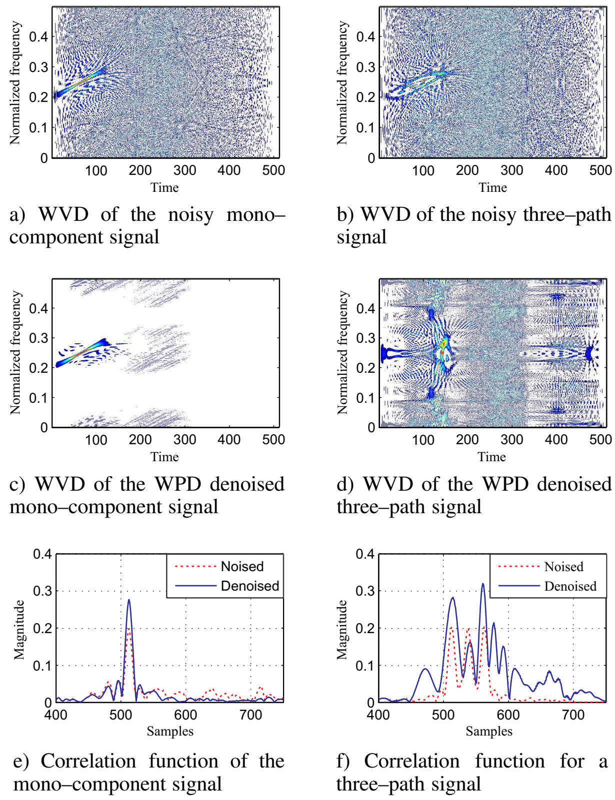
This example clearly demonstrates the limitation of the WPD–based denoising method in the case of signal made of very close arrival times. To overcome these limitations an alternative denoising method is proposed in the next section.

3. SIGNAL DENOISING VIA FRACTIONAL FOURIER TRANSFORM

Let $\mathcal{U}$ be a one–to–one map of the Hilbert space $L^2(\Omega \subseteq \mathbb{R})$ to itself. The necessary and sufficient condition for $\mathcal{U}$ to be an unitary operator is [7]:

$$\mathcal{U}^*\mathcal{U} = \mathcal{U}\mathcal{U}^* = \mathcal{I}, \tag{3}$$

where $\mathcal{I}$ is the identity operator. Unitary transformations have useful property for time–frequency analysis. Most relevant ones are the preservation of the signal norm in the transformed space ($\|\mathcal{U}x\| = \|x\|, \quad with \quad x \in L^2(\Omega)$) and the existence of an inverse transformation [7].



a) WVD of the noisy mono–component signal

b) WVD of the noisy three–path signal

c) WVD of the WPD denoised mono–component signal

d) WVD of the WPD denoised three–path signal

e) Correlation function of the mono–component signal

f) Correlation function for a three–path signal

Figure 1: Comparison of the WPD denoising method effects on a mono–component signal and a 3 path signal.

3.1 Fractional Fourier Transform

A well–known exemple of such operator class is the fractional Fourier transform (FrFT) of the signal $x(t) \in L^2(\Omega)$ given by [8, 9]

$$(\mathcal{F}^\alpha x)(\xi) = \int_t h^\alpha(t,\xi)\, x(t)\, dt,$$

where the kernel $h^\alpha(t,\xi)$ of the fractional Fourier operator $\mathcal{F}^\alpha$ is defined by

$$h^\alpha(t,\xi) = \frac{\exp\left(i\,\frac{\alpha}{2}\right)}{\sqrt{i\,\sin(\alpha)}} \cdot \exp\left(i\pi\frac{(t^2+\xi^2)\cos(\alpha) - 2t\xi}{\sin(\alpha)}\right). \tag{4}$$

Note that for $\alpha = 0$, $(\mathcal{F}^0 x)(\xi) = x(t)$ and for $\alpha = \frac{\pi}{2} + 2k\pi$, $k \in \mathbb{N}$ the classical expression of the Fourier Transform (FT) is recovered.

The FT is a projection operator which maps the time space to the frequency space. It corresponds to a counterclockwise rotation of the marginals over the time–frequency plane with an angle of $\frac{\pi}{2}$. From this definition, the FrFT can be seen as a generalization of the FT for an arbitrary angle of rotation α.

Since $\mathcal{F}^\alpha$ is a unitary operator it follows, from (3) and (4), that the inverse FrFT (IFrFT) exists and is given by

$$(\mathcal{F}^\alpha)^{-1} = (\mathcal{F}^\alpha)^* = \mathcal{F}^{-\alpha}.$$

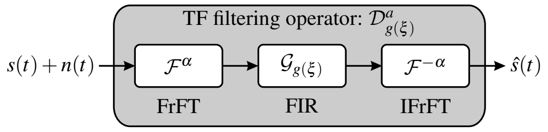


Figure 2: The time–frequency filtering operator.

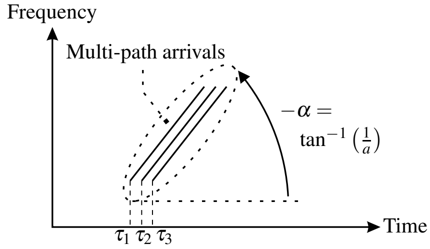


Figure 3: Optimal choice of the angle of rotation α.

3.2 Time–Frequency Denoising Operator

Let's consider the class of (time) signals which has a linear instantaneous frequency behavior

$$s(t) = \exp\left(2i\pi\left(\frac{a}{2}t+b\right)t\right), \quad a,\ b\ \in \mathbb{R}, \tag{5}$$

were a denotes the chirprate and b is the start frequency. For any nonzero value of a, it is well–known that this class of signals is spread over the frequency domain since $s(t)$ contains all frequencies. To reduce this effect, we suggest to use the fractional domain. Indeed, one can prove that it is possible to obtain an arbitrarily compact support of this class on the fractional domain. For an angle of rotation $\alpha = \tan^{-1}(-1/a)$, one obtain

$$\left(\mathcal{F}^{\tan^{-1}(-1/a)}s\right)(\xi) = \sqrt{i\ \csc(\alpha)}\cdot\delta\left(\xi - \frac{\mathrm{sign}(b)}{\sqrt{b^2+1}}\right),$$

where $\csc(\cdot)$ stands for the cosecant function and $\mathrm{sign}(\cdot)$ is the signum function. Obviously, for a signal $s(t)$ corrupted by an additive noise, it is possible to separate the signal and the noise on the fractional domain by one of the numerous stationary filtering method. With no loss of generality the use of FIR filtering is considered for this purpose.

Following this procedure the Time–frequency denoising operator (TFDO) $\mathcal{D}^{a}_{g(\xi)}$ can be represented as in Fig. 2, where $\mathcal{G}_{g(\xi)}$ is the operator related to the FIR filter characterized by a frequency response $g(\xi)$. In short, the denoising procedure consists in projecting the signal on the fractional domain, applying a classical stationnary filtering procedure (e.g. FIR) and, using IFrFT, returning to the time domain.

The corresponding operator may be expressed in its integral form

$$\left(\mathcal{D}^{a}_{g(\xi)}x\right)(t') = \frac{1}{\sqrt{\sin(\alpha)}}\int_{\xi}\int_{t} g(\xi)\,x(t)$$
$$\cdot\exp\left(i\pi\frac{(t^2-t'^2)\cos(\alpha)+2\xi(t'-t)}{\sin(\alpha)}\right)\ dt\ du.$$

Let's consider an emitted signal with a linear instantaneous frequency law (see (5)) propagating in the multipath channel defined in (1). A schematic representation of the received signal is depicted in Fig. 3. As shown, the optimal FrFT angle α is given by the opposite of the angle between the time axis and the chirp cluster. The problem of estimating this angle can be easily addressed with the help of a time–frequency representation of the Cohen class [10]. A more elegant solution consists in the estimation of the principal axes as discussed in [9] since only few FrFT are needed for the estimation of the FrFT angle.

In the remaining of this paper, it is assumed that a previous estimation stage has been performed and a valid estimation of the angle of rotation α is available.

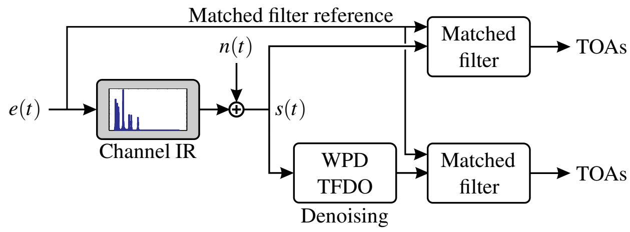


Figure 4: Schematic representation of the simulation procedure.

4. SIMULATION RESULTS

In this section, the behavior of the algorithm using underwater simulated data is studied. The simulation procedure is depicted Fig. 4. The channel IR (see (1)) is obtained by a numerical simulation of a realistic physical configuration. It is made of 7 different propagation paths with a sampling rate of 3 kHz. The transmitted pulse $e(t)$ is 2 seconds in duration and has a linear sweep of frequencies from 300 Hz to 800 Hz. The model of the received signal $s(t)$ is as defined in (2). Note that no noise assumption has been made for the derivation of the TFDO. However, to fairly compare the denoising methods it is assumed, in our simulations, $n(t)$ to be a white Gaussian stationary noise for which the WPD–based denoising method performs optimally. The TOAs are estimated by a matched filtering method with the transmitted signal as reference. Note that following results have been obtained with 50 different noise realizations and have to be taken in an average sense.

Fig. 5 shows the input signal–to–noise ratio ($\mathrm{SNR_{in}}$) versus the output signal–to–noise ratio ($\mathrm{SNR_{out}}$)

$$\mathrm{SNR_{in}} = 10\cdot\log_{10}\frac{P_i}{P_n}\ (dB) \quad \mathrm{SNR_{out}} = 10\cdot\log_{10}\frac{P_o}{P_n}\ (dB),$$

where P_i and P_o are respectively the signal power before and after denoising, and P_n the noise power. Both WPD–based denoising and TFDO lead to an improvement of the $\mathrm{SNR_{in}}$. Still, the TFDO achieves a better noise reduction than the WPD–based method.

Fig. 6 shows the normalized correlation coefficient, defined as:

$$Corr = \frac{\langle s(t), \widehat{s}(t)\rangle}{\|s(t)\|_2\cdot\|\widehat{s}(t)\|_2}.$$

This criterion is meaningful to describe how the denoising method affects the useful part of te signal. As introduced in section 2.2, the WPD–based denoising method is very sensitive to the signal interferences. Such interferences are interpreted as fast transitions and are amplified in the denoised

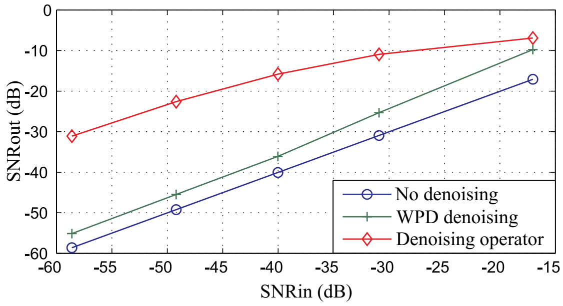


Figure 5: SNR_{out} versus SNR_{in}.

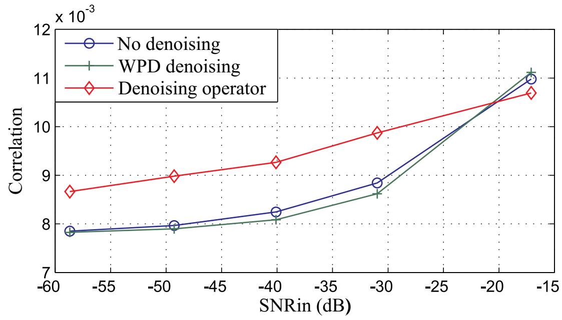


Figure 6: Normalized correlation coefficient versus SNR_{in}.

signal. This effect degrades the useful part of the signal and so its correlation with the original signal.

On the contrary, the TFDO has a better signal shape–preserving behavior. Indeed, the signal is not distorted by the use of a FIR filter and the denoised signal is still highly correlated with the original signal. However, the fractional Fourier transform introduces false transients which are due to a side–effect. This may explain the poor correlation performances of the TFDO above -20 dB since this effect cannot be neglected anymore for high SNR_{in}. This should be easily compensate by means of a zero–padding method.

Fig. 7 shows the mean estimation error of the TOA, defined as the number of false TOA detection, versus the SNR_{in}.

As previously explained, the WPD–based denoising method leads to a poorest estimation of the TOA than with no denoising procedure. Since the useful part of the signal is affected by the denoising, the matched filtering is corrupted by false peaks.

The TFDO gives the best result. The mean estimation error is improved which confirms the better signal shape preservation behavior of the TFDO.

5. CONCLUSION

In this paper we have addressed the problem of denoising signals in a multi–path environment. The starting point was the usual denoising methods based on wavelet techniques. As shown through an example, the WPD–based denoising method is limited in the case of close multi–component signals.

As an alternative, we have proposed a new denoising tool based on a unitary transform. In this new space an efficient filtering method was applied.

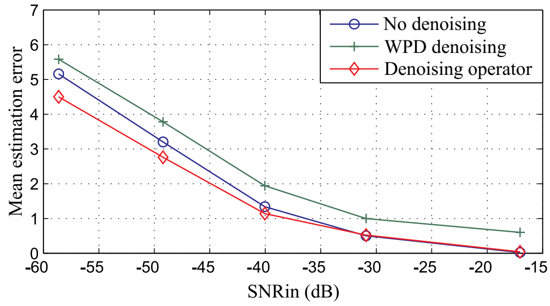


Figure 7: Mean estimation error versus SNR_{in}.

The performances show improvements of matched filtering processing when the signal has previously been denoised by the proposed method.

In this work, we have voluntary focused on the class of linear instantaneous frequency signals. In the future work, we will generalize this concept for arbitrary time–frequency contents.

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